A study on some of the contributions of Ibn Turk In Algebra

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ABSTRACT *The book of Ibn Turk dealt with quadratic equations of Al- Khwarizmi's type (i) Squares equal to roots, that is* $ax^2 = bx$ *(iv) Squares and roots equal to numbers, that is* $ax^2 + bx = c$ *(v) Squares and numbers equal to roots, that is* $ax^2 + c = bx$ *(vi) Roots and numbers equal to square, that is* $ax^2 = bx + c$*, where a, b, c are given positive numbers. In the case of type (v), Ibn Turk gave geometric versions for all possible cases. He gave the geometric justification for the solution of the equation* $x^2 + c = bx$*. He also discussed the "intermediate case", where the root of the square is exactly equal to half the number of roots. On this context, he gave the geometric justification of the impossibility of solving the equation* $x^2 + 30 = 10x$*.*

Keywords: *Quadratic equations, Geometric justification for the solution of the equation, Intermediate case, Geometric justification for the impossibility of the solution of the equation.*

1. Introduction

Mathematics has developed into a world- wide language with a special kind of logical structure.By studying the development of mathematical ideas, it is possible to obtain a view of the intellectual progress of mankind. Mathematics is a study in which old ideas are continuously replaced by new ones. The study of history of Mathematics is treated as a developing movement.

Algebra is the study of mathematical problems involving in one more unknowns. We come across problems in Algebra knowingly or unknowingly in our life. When these problems are translated with the help of unknowns, they get reduced to equations or in equations in the unknowns.

As the Arabs traded freely in India, they obtained some knowledge of the works of the Hindu writers as early as 750 AD. After this time, the arithmetic and algebra of the Hindus, which was copied by the Arabs used them in their works. The Arabs were mainly transmitters and brokers. They brought together Hindu and Greek ideas, fertilizing the ones with the others and revolutionizing arithmetic, algebra and trigonometry. Their contributions in these branches of mathematics, especially in algebra, was considerable.

Abd al-Hamid ibnWasiibn Turk al-Jili was a contemporary of Al- Khwarizmi, who was a scholar at the "House of Wisdom" (Dar al- Hikma), a kind of academy of scientists, set up at Baghdad during the reign of the Caliph Al- Ma'mun.Very little is known about him. The sources even differ as to whether he was from Iran, Afghanistan, or Syria.

2. Related Work

As per [5], Ibn Turk's book dealt with Quadratic Equations of al-Khwarizmi's type (i), (iv), (v) and (vi). This includes a much more detailed geometric description of the method of solutionthan is found in al-khwarizmi's work. In the case of type (v), that is $ax^2 + c = bx$,Ibn Turk gave geometric versions for all possible cases. His first example, $x^2 + 21 = 10x$ is the same as that of Al-Khwarizmi. His geometric justification for one case of $x^2 + c = bx$ is given in the following figure:-

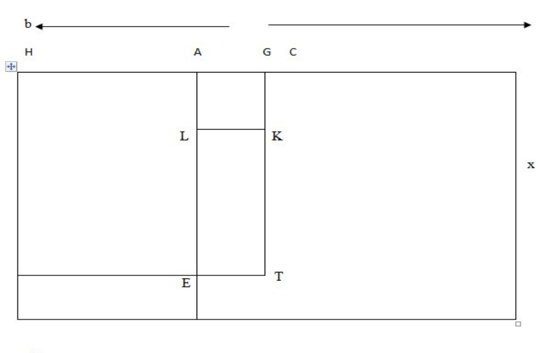

Figure*1:-Ibn Turk's geometric justification for the solution of the equation*$x^2 + c = bx$

He began the geometrical demonstration by taking G, the mid-point of CH as shown in the above figure, may be either on the line segment AH, as in al-Khawarizmi's diagram or on the line segment CA of the above figure. Complete the squares and rectangles as in the figure of al-Khwarizmi[5]. But here the solution x = AC is now given as CG + GA.

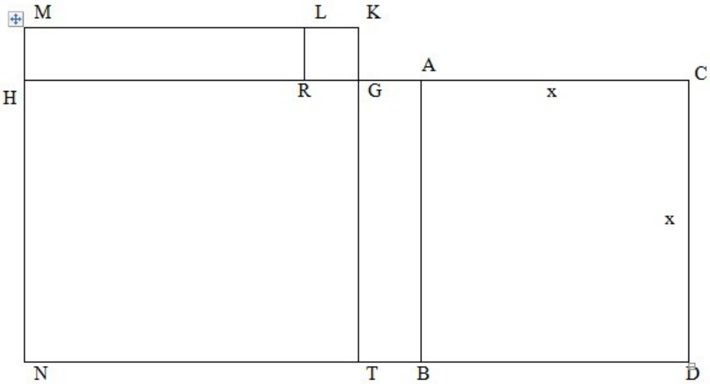

***Figure*2**:- *Al- Khwarizmi's geometric justification for the solution of the equation $x^2 + c = bx$*

Ibn Turk also discussed what he called the "intermediate case" where the root of the square is exactly equal to half the number of roots. He gave the example $x^2 + 25 = 10x$.

In this case, the geometric diagram consists of a rectangle divided into two equal squares.

He noted that "there is a logical necessity of impossibility in this type of equation, when the numerical quantity is greater than(the square of) half the number of roots".[5]

Example:- [5]

$$x^2 + 30 = 10x$$

He resorted to a geometric argument as in the figure given below. Assuming that G is located on the segment AH, we know that the rectangle KMNT is greater than the rectangleHABN. But the conditions of the problem show that the rectangle HABN equals 30 and the former rectangle KMNT is only equals 25. A similar argument works in the case where G is located on CA.

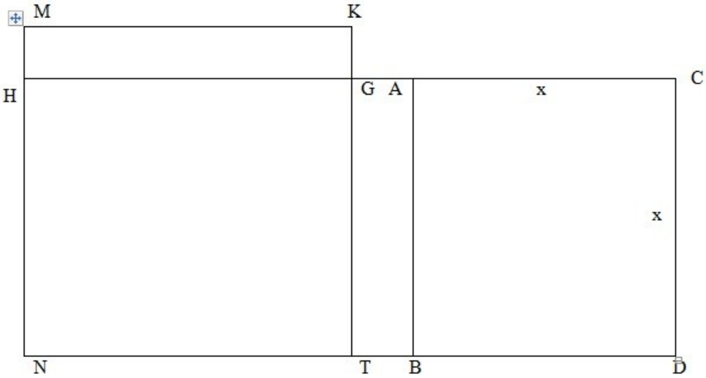

***Figure*3** *:Ibn Turk's geometric justification of the impossibility of solving the equation $x^2 + 30 = 10x$*

3. References

1. Berggren J.L., Episodes in the Mathematics of Medieval Islam, Springer-Verlag, New York (1986).
2. Boyer C.B., A History of Mathematics, John Wiley & Sons Inc., New York (1968).
3. Fauvel.J and Gray. J. (Ed.) , The History of Mathematics: A Reader, The Open University (1987).
4. Flegg G., The Solution of Equations, The History of Mathematics UNIT 4, The Open University Press, Walten Hall, Milton Keynes (1975)
5. Katz V.J., A History of Mathematics- An Introduction, Harper Collins College Publishers, NewYork (1993).
6. Sanford V., A Short History of Mathematics, George. G., Harrap and Co. Ltd, London (1930).
7. Struik D.J., A Concise History of Mathematics, G.Bell and Sons Ltd, London (1954).
8. Suzuki J., A History of Mathematics, Prentice Hall, New Jersey (2002).