An EOQ Model with Exponential Amelioration and Two Parametric Weibull Deterioration

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ABSTRACT *Most of the works on classical inventory models are based on the assumptions that the utility of inventory items remains constant over time. In this paper, an EOQ model from the perspective to solve the upbringing of the livestock and sales decision problem of the livestock or their by products by incorporating two opposite physical characteristics, namely amelioration of the livestock and deterioration of them in terms of their food cost and their treatment when they are ill has been studied. Time-varying demand patterns are commonly used to reflect sales in different phases of a product life cycle in the market. For example, the demand for inventory items increases over time in the growth phase and decreases in the decline phase. Many researches are done on inventory with constant demand rate, time varying demand patterns but very few of the researchers have considered the demand of the items as power demand pattern. In this paper, an inventory model with exponential amelioration, two parameter Weibull deterioration, time-dependent holding cost and power demand patterns is analysed. Shortages are allowed to occur. The derived model is illustrated by a numerical example and its sensitivity analysis is carried out.*

Keywords: *Amelioration,Deterioration, Power Demand, Exponential Distribution, Two parameter Weibull distribution, Time dependent holding cost, Fully backlogged shortages.*

INTRODUCTION

Young or fast growing animals such as fish, chickens, ducks and pigs are examples of ameliorating items. The manufacturers purchase these livestock when they are small and raise them [4] and hence treat them as the raw materials. When these livestocks are kept in farm, they will increase in value due to their growth and decrease due to feeding expenses and/or if diseased. When these livestocks are grown up they are treated as finished goods. To be more realistic the livestock's growth is faster in the beginning and declines at the later stage [4,5].

Hwang considered an inventory model for items with Weibull ameliorating for developing an economic order and selling quantity model [4]. Law and Wee developed an integrated production-inventory model for ameliorating and deteriorating items taking account of time discounting [7].

Deterioration of items in inventory systems has become an interesting feature for its practical importance. Deterioration refers to damage, spoilage, vaporization or obsolescence of the products. Hadley and Whitin was the first to consider the deterioration of inventory items, he dealt with the deterioration of fashion goods at the end of prescribed storage period [6].

Time-varying demand patterns are commonly used to reflect sales in different phases of a product life cycle in the market. For example, the demand for inventory items increases over time in the growth phase and decreases in the decline phase. Wu Kun-Shan revised the model of Wu, Lin, Tan and Lee [10] and developed a deterministic inventory model for items with time-varying demand, Weibull deterioration along with shortages [10, 11]. In this model inventory was allowed to start with shortages and end without shortages by considering holding cost continuous, non-negative and non-decreasing function of time.

A number of researchers have worked on inventory with constant demand rate, time varying demand patterns. A few of the researchers have considered the demand of the items as power demand pattern. Datta and Pal presented an EOQ model with the demand rate dependent on instantaneous stock displayed until a predefined maximum level of inventory L is achieved [3]. They used special form of Weibull density function to sidetrack the mathematical complications in deriving a compact EOQ model. In this paper, the models of Datta and Pal and Hwang are extended to analyse an inventory model for ameliorating and deteriorating items along with power demand patterns and time-dependent holding cost [3, 4, 5]. Exponential amelioration and two parametric Weibull deterioration are considered. Shortages are allowed to occur. The derived model is illustrated by a numerical example and its sensitivity analysis is carried out.

NOTATIONS

We use the following notations for the mathematical model:

1. $Q\ (t)$: The instantaneous state of the inventory level at any time t. $(0 \le t \le T)$

2. $R(t)$: Demand rate.
3. $\theta(t)$: Rate of deterioration per unit time.
4. $A(t)$: Rate of amelioration per unit time.
5. A: Ordering cost per order.
6. C_h: Inventory holding cost per unit per unit time.
7. C_d: Deterioration cost per unit per unit time.
8. C_s: Shortage cost due to lost sales per unit.
9. C_a : Amelioration cost per unit.
10. C_p: Purchase cost per unit $(Cp > Ca)$.
11. S: Initial inventory level at time t=0.
12. S_1: Total shortages.
13. t_1: The time at which the inventory level reaches zero (decision variable).
14. T: Length of cycle time (decision variable).
15. TC: Total cost per unit time.

ASSUMPTIONS

1. The inventory system deals with single item.
2. Demand follows the power demand pattern, i.e., demand at time t is assumed to be $d\left(\frac{t}{T}\right)^{\frac{1}{n}}$, where d is demand size during the planning horizon T, n ($0<n<\infty$) is the pattern index.
3. $\theta(t) = \alpha\beta t^{\beta-1}$, the two parameter Weibull deterioration rate where α is the scale parameter ($0 < \alpha << 1$) and β is the shape parameter ($\beta > 0$).
4. Amelioration rate $A(t)$ is constant which is derived from exponential distribution.
5. Holding cost is a linear function of time and it is $C_h = h + rt$ (h, r > 0).
6. The lead time is zero.
7. Time horizon period is infinite.
8. No repair or replacement of the deteriorated items takes place during a given cycle.
9. Total inventory cost is a real and continuous function which is convex to the origin.
10. The second and higher power of α and θ are neglected in the analysis of the derived model.

MATHEMATICAL MODEL AND ANALYSIS

The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible. Replenishment is made at time t=0 and the inventory level is at its maximum level S. During the period $(0, t_1)$ the inventory level is decreasing and at time t_1 the inventory reaches at zero level, where the shortages start and demands in the period (t_1, T) are allowed to get backlogged. The pictorial presentation is shown in Figure 1.

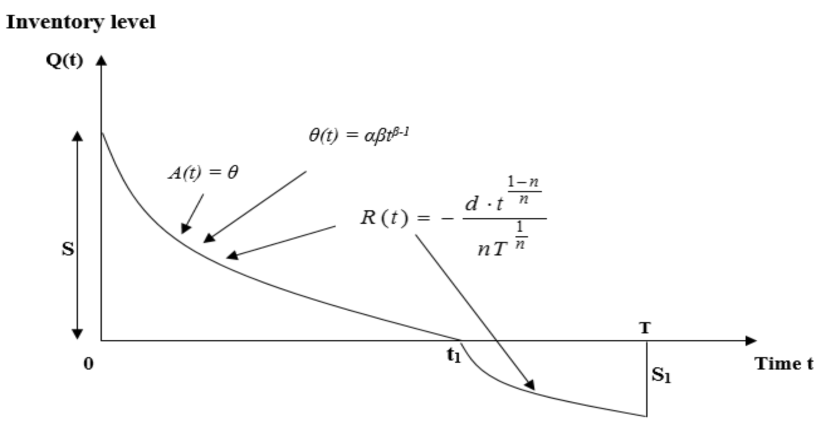

Figure 1: Graphical presentation of the inventory system

The differential equations which describe the instantaneous state of $Q(t)$ over the period $\left[0,T\right]$ are given by:

$$\frac{dQ(t)}{dt}+\left(\alpha\beta t^{\beta-1}-\theta\right)Q(t)=-\frac{d\cdot t^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}} \qquad \left(0\le t\le t_1\right) \qquad (1)$$

$$\frac{dQ(t)}{dt}=-\frac{d\cdot t^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}} \qquad \left(t_1\le t\le T\right) \qquad (2)$$

Under the boundary conditions Q (0) = S and Q (t_1) = 0 the solutions of equations (1) and (2) are given by

$$Q(t)=\frac{d}{T^{\frac{1}{n}}}\left[\begin{array}{l}\left(1+\theta t-\alpha t^{\beta}\right)\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right)\\ +\frac{\alpha}{1+n\beta}\left(t_1^{\frac{1}{n}+\beta}-t^{\frac{1}{n}+\beta}\right)\\ -\frac{\theta}{1+n}\left(t_1^{\frac{1}{n}+1}-t^{\frac{1}{n}+1}\right)\end{array}\right] \qquad \left(0\le t\le t_1\right) \qquad (3)$$

$$Q(t)=\frac{d}{T^{\frac{1}{n}}}\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right) \qquad \left(t_1\le t\le T\right) \qquad (4)$$

Putting $Q(0)=S$ in equation (3), we get

$$S=\frac{d}{T^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}}+\frac{\alpha}{1+n\beta}t_1^{\frac{1}{n}+\beta}-\frac{\theta}{1+n}t_1^{\frac{1}{n}+1}\right] \qquad (5)$$

In equation (4), by taking $Q(T)=-S_1$ we get

$$S_1=\frac{d}{T^{\frac{1}{n}}}\cdot\left(T^{\frac{1}{n}}-t_1^{\frac{1}{n}}\right) \qquad (6)$$

THE TOTAL COST COMPRISES OF THE FOLLOWING COSTS

1. Operating Cost (OC)

The operating cost over the period [0, T] is

$OC = A$ (7)

2. Deterioration Cost (DC)

The deterioration cost during the time period [0, t_1] is

$$DC=C_d\cdot\alpha\beta\cdot\left[\int_0^{t_1}t^{\beta-1}Q(t)dt\right]$$

$$\Rightarrow DC = C_d \cdot \alpha \cdot \frac{d}{T^{\frac{1}{n}}}\left(\frac{t_1^{\frac{1}{n}+\beta}}{1+n\beta}\right) \tag{8}$$

3. Inventory Holding Cost (*IHC*)

The inventory holding cost during the time period [0, t_1] is

$$IHC = \int_0^{t_1}(h+rt)Q(t)dt$$

$$\Rightarrow IHC = \frac{d}{T^{\frac{1}{n}}}\cdot\left\{\begin{array}{l} h\left[\frac{1}{1+n}t_1^{\frac{1}{n}+1} - \frac{\theta}{2(1+2n)}t_1^{\frac{1}{n}+2} + \frac{\alpha\beta}{(\beta+1)(1+n\beta+n)}t_1^{\frac{1}{n}+\beta+1}\right] \\ +r\left[\frac{1}{2(1+2n)}t_1^{\frac{1}{n}+2} - \frac{\theta}{6(1+3n)}t_1^{\frac{1}{n}+3} + \frac{\alpha\beta}{2(\beta+2)(1+n\beta+2n)}t_1^{\frac{1}{n}+\beta+2}\right]\end{array}\right\} \tag{9}$$

4. Shortage Cost (*SC*)

The shortage cost over the period [t_1, T] is

$$SC = -C_s\int_{t_1}^{T}Q(t)dt$$

$$\Rightarrow SC = C_s \cdot \frac{d}{T^{\frac{1}{n}}}\left[\frac{1}{1+n}t_1^{\frac{1}{n}+1} + \left(\frac{n}{n+1}T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right)T\right] \tag{10}$$

5. Amelioration Cost (*AMC*)

The Amelioration cost during the time period [0, t_1] is

$$AMC = C_a\int_0^{t_1}\theta Q(t)dt$$

$$\Rightarrow AMC = C_a\theta\frac{d}{T^{\frac{1}{n}}}\left(\frac{t_1^{\frac{1}{n}+1}}{1+n}\right) \tag{11}$$

6. Purchase Cost (*PC*)

The purchase cost over the time period [0, T] is

$$PC = C_p\cdot(S+S_1)$$

$$\Rightarrow PC = C_p\frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}} + \frac{\alpha}{1+n\beta}t_1^{\frac{1}{n}+\beta} - \frac{\theta}{1+n}t_1^{\frac{1}{n}+1}\right) + \left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right)\right] \tag{12}$$

Hence, the total cost TC per unit time is given by

$$TC = \frac{1}{T}(OC + DC + AC + IHC + SC + PC)$$

$$\Rightarrow TC = \frac{1}{T}\left\{\begin{array}{l} A + C_d\cdot\alpha\cdot\dfrac{d}{T^{\frac{1}{n}}}\left(\dfrac{t_1^{\frac{1}{n}+\beta}}{1+n\beta}\right) + C_a\cdot\theta\cdot\dfrac{d}{T^{\frac{1}{n}}}\left(\dfrac{t_1^{\frac{1}{n}+1}}{1+n}\right) \\ + \dfrac{d}{T^{\frac{1}{n}}}\cdot\left\{\begin{array}{l} h\left[\dfrac{1}{1+n}t_1^{\frac{1}{n}+1} - \dfrac{\theta}{2(1+2n)}t_1^{\frac{1}{n}+2} + \dfrac{\alpha\beta}{(\beta+1)(1+n\beta+n)}t_1^{\frac{1}{n}+\beta+1}\right] \\ +r\left[\dfrac{1}{2(1+2n)}t_1^{\frac{1}{n}+2} - \dfrac{\theta}{6(1+3n)}t_1^{\frac{1}{n}+3} + \dfrac{\alpha\beta t_1^{\frac{1}{n}+\beta+2}}{2(\beta+2)(1+n\beta+2n)}\right] \end{array}\right\} \\ + C_s\cdot\dfrac{d}{T^{\frac{1}{n}}}\left[\dfrac{1}{1+n}t_1^{\frac{1}{n}+1} + \left(\dfrac{n}{n+1}T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right)T\right] \\ + C_p\cdot\left\{\dfrac{d}{T^{\frac{1}{n}}}\left[\begin{array}{l}\left(t_1^{\frac{1}{n}} + \dfrac{\alpha}{1+n\beta}t_1^{\frac{1}{n}+\beta} - \dfrac{\theta}{1+n}t_1^{\frac{1}{n}+1}\right) \\ + \left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right)\end{array}\right]\right\} \end{array}\right\} \qquad (13)$$

First order and second order partial derivatives of the average total cost function (TC) w.r.t. t_1 and T are obtained with the help of Maple software.

Our objective is to determine optimum value t_1^* and T^* of t_1 and T respectively to minimize the average total cost TC. The values of t_1^* and T^* can be obtained by solving the equations $\dfrac{\partial TC}{\partial t_1} = 0 \;\; and \;\; \dfrac{\partial TC}{\partial T} = 0$ (14)

such that the following sufficient conditions are satisfied:

$$\left.\begin{array}{l} \left|\left(\dfrac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)\left(\dfrac{\partial^2 TC(t_1,T)}{\partial T^2}\right) - \left(\dfrac{\partial^2 TC(t_1,T)}{\partial t_1 \partial T}\right)^2\right|_{t=t_1^*, T=T^*} \\ \left|\dfrac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right|_{t=t_1^*, T=T^*} \end{array}\right\} > 0 \qquad (15)$$

NUMERICAL EXAMPLE

Let us consider the following example to illustrate the above developed model. By taking A=200, h=2, r=0.5, θ=0.0001, α=0.0001, β=8, n=2, d=4, C_d=6, C_p=10, C_s=4, C_a=3 (with appropriate units), we get the optimum values of t_1 and T as t_1^*=5.444753456 units, T^*=263.4788632 units respectively and optimal value of TC = 9.436933458.

SENSITIVITY ANALYSIS

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes or errors in its input parameter values.

In this section, the sensitivity of the total cost per time unit TC with respect to the changes in the values of the parameters A, α, β, h, r, θ, n, d, C_d, C_s, C_p and C_a is analysed.

The sensitivity analysis is performed by considering 10% and 20% increase or decrease in each one of the above parameters keeping all other parameters the same. The results are presented in Table 1.

Table 1: *Partial Sensitivity Analysis.*

Parameter	% Change	t_1	T	TC
A	– 20	5.227550738	194.3164575	9.260212914
	– 10	5.339624687	227.6980413	9.355285908
	+ 10	5.543896461	301.6178569	9.507874586
	+ 20	5.637822470	342.0774521	9.570134984
α	– 20	5.587722919	258.0252942	9.407019724
	– 10	5.511708231	260.8836993	9.422900283
	+ 10	5.385013989	265.8577851	9.449489667
	+ 20	5.331149177	268.0557050	9.460839256
β	– 20	6.219132291	245.1475254	9.294053648
	– 10	5.584488563	260.7345423	9.437542484
	+ 10	4.704032704	287.7599718	9.580882390
	+ 20	4.166831039	311.1592143	9.697605237
θ	– 20	5.444753826	263.4794033	9.436933914
	– 10	5.444753641	263.4791333	9.436933916
	+ 10	5.444753271	263.4785931	9.436933018
	+ 20	5.444753086	263.4783231	9.436932568
h	– 20	5.554069837	260.5302498	9.416314298
	– 10	5.495943998	262.0037127	9.427120879
	+ 10	5.399005246	264.9397480	9.445930209
	+ 20	5.357648088	266.3781387	9.454243828
r	– 20	5.381834224	263.6860211	9.446179336
	– 10	5.411994836	263.5663087	9.441706090
	+ 10	5.480619284	263.4361461	9.431815959
	+ 20	5.520273932	263.4563133	9.426295885
n	– 20	5.881630030	469.9322634	9.405521054
	– 10	5.603259751	326.7683373	9.473452870
	+ 10	5.346750732	229.9330114	9.336024175
	+ 20	5.283320027	210.2668744	9.196263941
d	– 20	5.683029372	363.1665780	7.678804993
	– 10	5.554577773	305.9994262	8.563670381
	+ 10	5.349452801	230.8523454	10.29953785
	+ 20	5.265749407	205.1743758	11.15232088
C_d	– 20	5.444753389	263.4788101	9.436933387
	– 10	5.444753423	263.4788366	9.436933430
	+ 10	5.444753490	263.4788897	9.436933505
	+ 20	5.444753523	263.4789163	9.436933559
C_s	– 20	5.581373660	395.1483004	7.721622611
	– 10	5.508546344	318.8353845	8.589066816
	+ 10	5.388171737	221.9940864	10.26572336
	+ 20	5.337474043	190.0669627	11.07591791
C_p	– 20	5.509319076	245.3674269	9.383500027
	– 10	5.474756414	254.3736762	9.411064603
	+ 10	5.418495262	272.6870817	9.461320810
	+ 20	5.395360830	282.0019060	9.484398126
C_a	– 20	5.444753267	263.4787128	9.436933237
	– 10	5.444753361	263.4787880	9.436933345
	+ 10	5.444753551	263.4789384	9.436933585
	+ 20	5.444753646	263.4790136	9.436933705

GRAPHICAL PRESENTATION

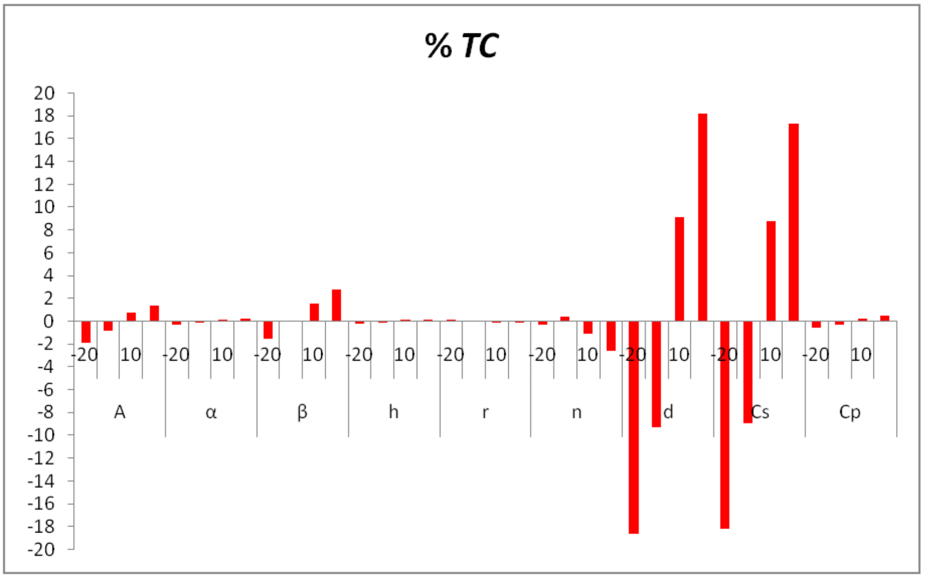

Fig. 2: *Graphical Presentation of the Partial Sensitivity Analysis for all the Parameters of TC.*

CONCLUSION

- From Table 1 of partial sensitivity analysis and Figure 2 showing partial sensitivity analysis for all the parameters of TC, it is observed that as parameters A, α, β, θ, h, d, C_d, C_s, C_p and C_a increases the average total cost also increases and for parameter 'r' the average total cost decreases.
- From Table 1 and Figure 2, it is observed that the total cost per time unit (TC) is highly sensitive to changes in the values of d and C_s.
- From Table 1 and Figure 2, it is also observed that the total cost per time unit (TC) is moderately sensitive to the changes in the values of A and β.
- From Table 1 and Figure 2, the total cost per time unit (TC) is less sensitive to changes in the values of α, h, r, θ, C_d, C_p and C_a.
- Again, total cost (TC) is attaining a maximum value when the pattern index n is increased upto 10%. Hence, it is economical to increase the value of n up to 10% only.

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