A Study of Ellipticity of Atomic Nucleus and other related Parameters

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Received: May 11, 2018 Accepted: June 19, 2018

ABSTRACT *Generally atomic nuclei are considered spherically symmetric for simplification of deduction, calculation as well for explanation of various problems. From experimental and theoretical point of view for different nuclear properties, it is observed that spherical symmetric shape of nuclei is not always true. The existence of electric quadrupole moment of nuclei indicates the deviation of nucleus from spherical symmetry and the magnitudes and also sign of quadrupole moment reflects about the actual shape of the nucleus. Though numerical values of electric quadrupole moment of the nuclei is very small (in the units of barns, i.e.in terms of area) but plays a vital role for explaining different nuclear properties, like nuclear binding energy and supports a particular nuclear model, nuclear shell model. Ellipticity of the nucleus explains whetherthe nucleus is spherically symmetric or prolate or oblate ellipsoidal in shape. Non zero value of ellipticity also predicts information about the semi-major and semi minor axes of the nucleus.*

Keywords: *Nuclearquadrupole moment,spherically symmetric, charge distribution,ellipticity.*

I. INTRODUCTION

Electrical quadrupole moment of nuclei first observed in the nucleus of deuteron ($_1D^2$) from the study of hyperfine structure of atomic spectra of atomic spectrum [1].It arises due to asymmetric distribution of positive charge in the nucleus. To achieve information about nuclear quadrupole moment total charge, shape, size and charge distribution of the nuclei is very much essential to know. It may be estimated by studying the interactions between nuclear field and the applied or with the atomic field. For classical approach, total positive charge and charge distribution as well as the size of the nuclei are important to get numerical value of electric quadrupole moment of the nucleus. For quantum mechanical treatment, nuclear charge distribution, total nuclear spin and the projection of spin in a particular direction are essential. Quadrupole moment ofan ellipsoidal nucleus, i.e. the nucleus due to asymmetric distribution of charge is also related with semi-major and semi-minor axes of the nucleus, hence ultimately it is expressed in terms of radius, atomic number and ellipticity of the nucleus. Nuclear ellipticity can be explained as the degree of deviation of a nucleus from a sphere of an ellipsoidal shape that can be measured as the ratio of the semi-major to semi-minor axes. Finally using the standard well known values of quadrupole moment and radius of the nucleus, ellipticity of the nucleus can be estimated out. The numerical value of ellipticity of the nucleus is very small and it may be zero, positive as well as negative. Positive value of itindicates, the nucleus is extended along Z-axis i.e. semi-major axis is greater than semi-minor axis and the nuclei are called prolate spheroid. Whereas negative value provides the contraction nature of a nucleus along Z- axis and semi-minor axis is greater than semi -major axis, these are called oblate spheroid[2],[3].From the estimated value of ellipticity and average value of radius, the values of semi-major axis and semi-minor axis can be calculated out.In the present work standard values of electric quadrupole moment of nuclei and average value of radius are used and from these, ellipticity of nucleus and numerical values of semi-major axis (a) as well as semi-minor axis (b) are estimated. Finally the variation of ellipticity, how the difference between a and b changes and also the ratio between the two axis occurs, are studied for different atomic nucleus (from Z=2 to Z=80).

II Theory and calculation

Classically when the charge is not situated at the centre, but it is located at a point having co-ordinates (x,y, z),the net quadrupole moment is expressed as $Q = \frac{e}{2}(3z^2 - r^2)$ -------(1). Where e is total charge of the nucleus, Z is distance of the point along Z axis. It is zero when the charges (positive) placed evenly about all the three axes i.e. it is zero for spherically symmetric charge distribution. When numerical value of Q is positive, indicates that the nuclei are extended along Z-axis. But when Q is negative, nuclei are contracted along Z-axis [1],[2]. Quantum mechanical form of EQM of the nucleus may be expressed as

$Q(m_I) = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0 \quad Q_0 = \int \rho(3Z^2 - r^2)dv$ Where Q_0 is the intrinsic quadrupole moment of the nucleus,

I is nuclear total spin and K is the projection of spin in Z- direction[4].

For an ellipsoidal nucleus, electric quadrupole moment, if the positive charge density is considered uniform throughout the ellipsoid, can be expressed as $Q = \frac{2}{5}Z(a^2 - b^2) = \frac{4}{5}\varepsilon Z R^2$ ------------ (2) where a, b are semi-major and semi-minor axis and ε iscalled ellipticity of the nucleus. Existence of ellipticity indicates non spherical symmetry of positive distribution of charge in the nucleus. Q is zero when a=b. One can express the ellipticity of a nucleus ε in terms of semi-major axis and semi-minor axis which is $\varepsilon = \frac{2(a-b)}{(a+b)}$ and

$\frac{(a+b)}{2} = R --- (3)$ (R is mean radius of the nucleus) [3].

When $a \rangle b$, the nucleus is called prolate spheroid and for these nuclei ε is positive, on the other hand for oblate nuclei ε is negative and $b \rangle a$. So using equations 1, 2 and 3 the values can be written in a simplified form like

$$\varepsilon = 1.25 \frac{Q_0}{ZR^2} ------(4)$$

$$a = \frac{(2R + R\varepsilon)}{2} ------(5)$$

$$b = \frac{(2R - R\varepsilon)}{2} ------(6)$$

$$a - b = R\varepsilon --------(7)$$

$$\frac{a}{b} = \frac{(2R + R\varepsilon)}{(2R - R\varepsilon)} ------(8)$$

III Results

Standard values of Q_0 and radius of particular atomic nucleus are utilised to estimate ellipticity by using the equation (4),than these values are used to get a, b , their differences and their ratio by using other equations [3],[4],[5],[7]. From the calculations, it is foundboth positive and negative values of ellipticity in addition to zero value. Semi major axis of nucleus is greater than semi-minor axis for positive values of ellipticity for positive values of ellipticityand reverse is true for negative values of ellipticity. For the study of variation of ratio with atomic number, only magnitudes are considered.All these numerical data are placed in the table.Variations of ellipticity with atomic no. are studied starting from Z=2 to Z=80 and graphically presented in the fig.1. Difference between semi-major axis and semi-minor axis are studied and the variations are plotted in the fig.2, where zero, positive and negative values are obtained. Zero value indicates perfectly spherical symmetric nuclei.Fig.3 represents the variation of the ratio semi-major axis to semi-minor axis (a/b) with the atonic no. of the nucleus.

Table No.1

Serial No.	Atomic No.(Z) of the nuclei.	Quadrupole moment of the nuclei in barns.	Nuclear radius in fm.	Ellipticity of the nucleus	Semi major axis(a) of the nuclei in fm.	Semi minor axis(b) of the nuclei in fm.	Difference between semi major to semi-minor axis	Ratio a and b
1	2	0.0028	1.6755	0.062337	1.727723	1.623277	0.104446	1.064343
2	3	-0.0406	2.444	-0.28321	2.097914	2.790086	-0.69217	0.751918
3	4	0.052	2.519	0.256093	2.841549	2.196451	0.645097	1.2937

4	5	0.0407	2.406	0.175769	2.617451	2.194549	0.422901	1.192705
5	6	0.06	2.4702	0.204855	2.723216	2.217184	0.506032	1.228232
6	7	0.0193	2.5582	0.052662	2.62556	2.49084	0.134721	1.054087
7	8	0	2.6991	0	2.6991	2.6991	0	1
8	9	-0.23	3.0055	-0.35364	2.474067	3.536933	-1.06287	0.699495
9	11	0.1045	2.9936	0.132509	3.19194	2.79526	0.39668	1.141912
10	12	-0.29	3.057	-0.32325	2.562915	3.551085	-0.98817	0.721727
11	13	0.1466	3.061	0.150444	3.291254	2.830746	0.460508	1.162681
12	14	0.16	3.1224	0.146529	3.351162	2.893638	0.457524	1.158114
13	16	-0.15	3.2611	-0.11019	3.081425	3.440775	-0.35935	0.895561
14	17	0.085	3.3654	0.055183	3.458257	3.272543	0.185713	1.056749
15	18	0.01	3.4274	0.005912	3.437531	3.417269	0.020262	1.005929
16	19	0.0585	3.4346	0.326257	3.994881	2.874319	1.120563	1.389853
17	20	0	3.4776	0	3.4776	3.4776	0	1
18	21	-0.156	3.5439	-0.07394	3.41289	3.67491	-0.26202	0.9287
19	22	-0.177	3.5921	-0.07794	3.452115	3.732085	-0.27997	0.924983
20	23	-0.043	3.6002	-0.01803	3.567744	3.632656	-0.06491	0.982131
21	24	-0.08	3.6452	-0.03136	3.588047	3.702353	-0.11431	0.969126
22	25	0.33	3.7057	0.120155	3.92833	3.48307	0.44526	1.127836
23	26	-0.19	3.7377	-0.06539	3.615504	3.859896	-0.24439	0.936684
24	27	0.35	3.7875	0.112956	4.00141	3.57359	0.427821	1.119717
25	28	-0.1	3.8125	-0.03071	3.753952	3.871048	-0.1171	0.969751
26	29	-0.211	3.8823	-0.06034	3.765168	3.999432	-0.23426	0.941426
27	30	-0.124	3.9283	-0.03348	3.862538	3.994062	-0.13152	0.96707
28	31	0.165	3.9845	0.041907	4.067989	3.901011	0.166978	1.042804
29	32	-0.25	4.0742	-0.05883	3.954353	4.194047	-0.23969	0.942849
30	33	0.3	4.0968	0.067706	4.235489	3.958111	0.277378	1.070078
31	34	-0.31	4.14	-0.0665	4.002354	4.277646	-0.27529	0.935644
32	35	0.313	4.1629	0.064505	4.297164	4.028636	0.268528	1.066655
33	36	0	4.1884	0	4.1884	4.1884	0	1
34	37	0.277	4.2036	0.05296	4.314911	4.092289	0.222621	1.0544
35	38	0.33	4.224	0.06084	4.352495	4.095505	0.25699	1.062749
36	40	-0.51	4.2694	-0.08744	4.082752	4.456048	-0.3733	0.916227
37	41	-0.37	4.324	-0.06033	4.19356	4.45444	-0.26088	0.941434
38	42	-0.26	4.4091	-0.0398	4.321349	4.496851	-0.1755	0.960972
39	44	-0.68	4.4809	-0.09621	4.265339	4.696461	-0.43112	0.908203
40	46	-0.51	4.5318	-0.06748	4.378895	4.684705	-0.30581	0.934722
41	48	0.35	4.6087	0.042912	4.707585	4.509815	0.197769	1.043853
42	49	0.8	4.601	0.096405	4.82278	4.37922	0.443559	1.101287
43	50	0.022	4.6519	0.002542	4.657812	4.645988	0.011823	1.002545
44	51	0.167	4.6802	0.018686	4.723928	4.636472	0.087456	1.018863
45	52	0.06	4.7346	0.006434	4.749832	4.719368	0.030463	1.006455
46	53	-0.71	4.752	-0.07415	4.575808	4.928192	-0.35238	0.928496
47	54	0.01	4.7859	0.001011	4.788318	4.783482	0.004837	1.001011
48	55	-0.00355	4.8041	-0.00035	4.80326	4.80494	-0.00168	0.99965
49	56	-0.14	4.8378	-0.01335	4.805502	4.870098	-0.0646	0.986736
50	57	0.2	4.855	0.018607	4.90017	4.80983	0.090339	1.018782
51	58	0.35	4.8771	0.031712	4.954432	4.799768	0.154664	1.032223
52	59	-0.077	4.8919	-0.00682	4.875226	4.908574	-0.03335	0.993206
53	60	0.32	4.9123	0.027627	4.980157	4.844443	0.135714	1.028014
54	62	-1.666	5.0819	-0.13006	4.751426	5.412374	-0.66095	0.877882
55	63	2.71	5.1064	0.20621	5.632895	4.579905	1.052989	1.229915
56	64	-2.01	5.1569	-0.14762	4.776266	5.537534	-0.76127	0.862526
57	65	1.432	5.06	0.107557	5.332119	4.787881	0.544238	1.11367
58	67	3.58	5.2022	0.246799	5.84415	4.56025	1.2839	1.281542
59	68	-2.7	5.2516	-0.17996	4.779055	5.724145	-0.94509	0.834894
60	69	-1.2	5.2256	-0.07961	5.017594	5.433606	-0.41601	0.923437
61	70	2.1	5.3108	0.132957	5.663854	4.957746	0.706108	1.142425
62	71	2.2323	5.37	0.136287	5.735932	5.004068	0.731863	1.146254

63	72	-2	5.347	-0.12145	5.022311	5.671689	-0.64938	0.885505
64	73	3.17	5.34	0.190355	5.848247	4.831753	1.016495	1.210378
65	74	-1.9	5.3658	-0.11147	5.066734	5.664866	-0.59813	0.894414
66	75	2.18	5.3596	0.126485	5.698556	5.020644	0.677911	1.135025
67	76	-0.96	5.4126	-0.0539	5.266742	5.558458	-0.29172	0.947518
68	75	2.18	5.3596	0.126485	5.698556	5.020644	0.677911	1.135025
69	77	0.751	5.4032	0.04176	5.516018	5.290382	0.225636	1.04265
70	78	1.461	5.427	0.079496	5.642713	5.211287	0.431425	1.082787
71	80	0.87	5.4648	0.045519	5.589176	5.340424	0.248751	1.046579

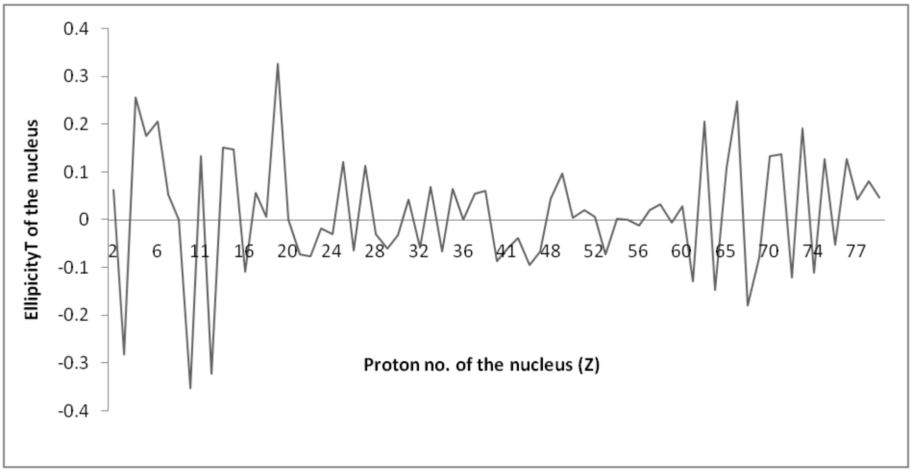

Fig -1

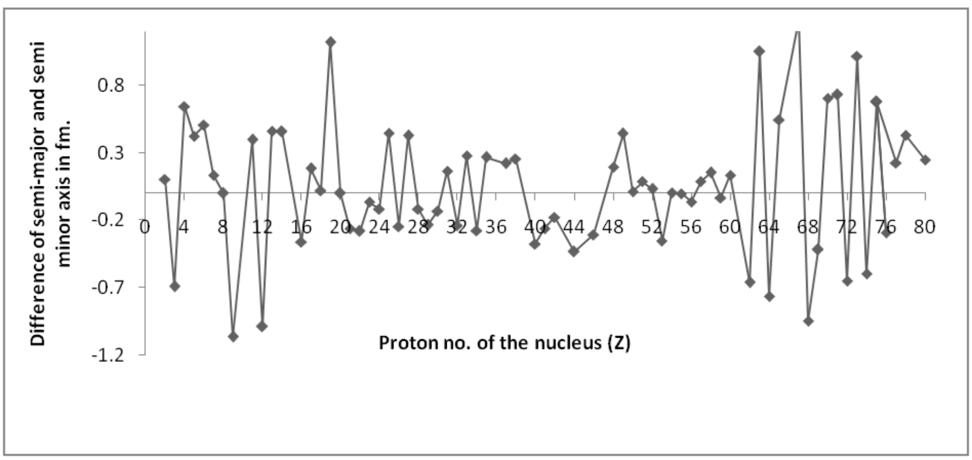

Fig -2

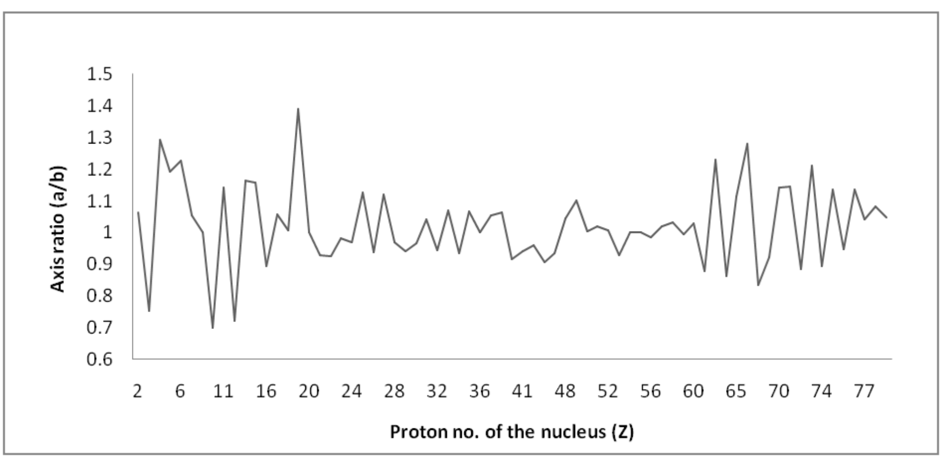

Fig -3

IV. Discussions and conclusion

It is observed from the table that ellipticity varies with proton number of the nucleus and lengths of semi-major, semi-minor axis. The value of ε varies from some positive values to some negative values including zero. The value is zero or very small for magic number nuclei i.e. nuclei having proton number 2,8,20,36,50 etc. in addition to that such values are observed for other nuclei with proton number 18,52,54,55 etc. For these nuclei as well as for magic no. nuclei, differences between two axis (a-b) are either zero or very small and axis ratio(a/b) are one or very near to one. Just after the magic number nuclei (considering proton number) the value of ε is negative and then becomes positive and increases with the increase of proton number. This increment continues until it reaches the maximum value and then decreases to zero at the next magic proton number. Some way the variation of ε takes place from one magic proton number to the nextas seen in the fig.1.

The after the magic proton number the difference between the semi-major and semi minor axis (a-b) is negative and then becomes positive and decreases with the increase of proton number. This de increment continues until it reaches zero at the next magic proton number. Zigzag way, first zero to negative and then positive high to zero, thisvariation takes place from one magic proton number to the nextas seen in the fig.2.

The axis ratio (a/b) is one at magic number proton, then decreases with increase of proton number and becomes the least value. After that the ratio increases and reached the maximum value (greater than one) and then decreases to one at next proton magic number. The variation of the ratio takes place almost same way from one magic proton nucleus to the next proton magic nucleus as observe from the fig.3.

All the variations are periodic and almost symmetric (through value not the same). From the variations of ε ,(a-b) and (a/b) ,it may concludes the existence of proton shell in the nucleus that agrees single particle nuclear shell model.

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