STUDY OF SQUEEZE FILM PERFORMANCE WITH MHD AND COUPLE STRESS BETWEEN CURVED ANNULAR PLATES

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ABSTRACT *The squeeze film performance of an electrically conducting fluid in the presence of applied magnetic field and couple stress for curved annular plates is investigated in the present study. A modified Reynolds-type equation is mathematically formulated and expressions for film pressure, load supporting capacity and response time are obtained for various parameters. As per the results, in the presence of applied magnetic field there is increase in pressure, load supporting capacity and response time. The effect of couple stress is to enhance pressure, load supporting capacity and response time. Also for increasing values of curvature parameter, the pressure, load supporting capacity and response time increases but load supporting capacity and response time decrease for increasing values of radius ratio of the curved annular plates.*

Keywords: *Curved annular plates, Couple stress, Squeeze film, electrically conducting fluid, MHD.*

I. INTRODUCTION

Squeeze film performance play a vital role in the field of engineering, science and industrial application such as clutch plates in automotive transmission, impact film in bio-lubricated joints, and squeezing cushion in machine elements. Thus the investigation of squeeze film performance focuses upon the use of a non-conducting viscous lubricant. In view of their wide range of applications, numerous theoretical and experimental studies have been conducted such as the squeeze film between two rectangular plates by Hays[1] , parallel plates by Gould [2] , curved annular plates by Gupta and Vora [3]. The unexpected difference of lubricant viscosity due to high temperature can be controlled by using electrically conducting fluid as the lubricant. Thus the behaviour of squeeze film in the presence of external magnetic field have been discussed by several authors [4-9] for various bearing configurations.

Kuzma [10], analysed the characteristics of squeeze film for circular plates and infinitely long rectangular plates with transverse magnetic field and shown that the application of a magnetic field improves the squeeze film action. The enhancement in load-carrying capacity and squeeze film time due to applied magnetic field is studied by Lin, et al. [11], Naduvinamani, et al.[12], and SyedaTasneem, et al. [13].

The concept of a couple stress fluids is very important to understand different physical problems. The classical Newtonian theory cannot explain the rheological properties of lubricants blended with various additives, several micro continuum theories are proposed to model the flow rheology, but micro continuum theory derived by Stokes[14]is the simplest theory which describes the couple stress concepts. The MHD couple stress squeeze-film characteristics between a sphere and a plane surface is investigated by Naduvinamani and Rajashekar[15] and found that for increasing values of Hartmann number, there is significant raise in the pressure, load-carrying capacity and response time. Recently, the effect of MHD and couple stress squeeze film for various configuration is analysed by Hanumagowda.et.al [16-17] and found that the Non-Newtonian properties of dilatants fluids enhances the pressure, load carrying capacity and squeeze film time as compare to Newtonian and non-magnetic case. The MHD squeeze film performance with couple stress between curved annular plates is still not studied. Therefore, a further analysis is motivated in the present study. The modified Reynolds equation is derived with the help of Stokes model and MHD flow model and using these the expressions for pressure, load supporting capacity and response time are evaluated and the same is used to draw the graph for distinct values of Hartmann number, couple stress parameter, radius ratio and curved shape parameter.

II. MATHEMATICAL ANALYSIS

The geometry of squeeze film curved annular plates with *b* as inner radius and *a* as outer radius is shown in Fig: 1, where upper plate is moving towards lower plate with a squeeze velocity $\left(V = -dh_m / dt\right)$.

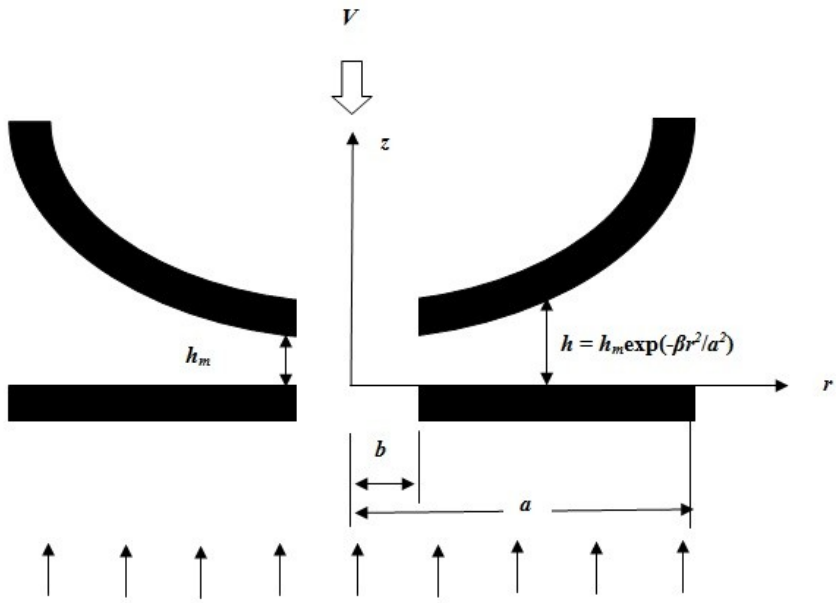

Figure 1: Configuration of curved annular plates.

The film shape h is taken to be an exponential type as Jaw-Rein Lin.et.al[18]

$$h = h_m \exp(-\beta r^2 / a^2), \qquad b \le r \le a \qquad (1)$$

Where β is curvature parameter and h_m is the minimum film thickness.

The Reynolds equation in the modified form to discuss the MHD couple stress squeeze film between curved circular plates was derived by Hanumagowda.et.al [16] and is,

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{rS(h,l,M_0)\frac{\partial p}{\partial r}\right\} = \mu V \quad (2)$$

Where,

$$S(h,l,M_0) = \begin{cases} \dfrac{h_0^2}{M_0^2}\left\{\dfrac{2l}{(A^2-B^2)}\left(\dfrac{B^2}{A}\tanh\dfrac{Ah}{2l} - \dfrac{A^2}{B}\tanh\dfrac{Bh}{2l}\right)+h\right\}, & for\ \ M_0^2 l^2/h_0^2 < 1 \\ \dfrac{h_0^2}{M_0^2}\left\{\dfrac{h}{2}\sec h^2\left(\dfrac{h}{2\sqrt{2l}}\right) - 3\sqrt{2l}\tanh\left(\dfrac{h}{2\sqrt{2l}}\right)+h\right\}, & for\ M_0^2 l^2/h_0^2 = 1 \\ \dfrac{h_0^2}{M_0^2}\left\{\dfrac{2lh_0}{M_0}\left(\dfrac{(A_2\cot\theta - B_2)\sin B_2 h - (B_2\cot\theta + A_2)\sin A_2 h}{\cos B_2 h + \cosh A_2 h}\right)+h\right\}, & for\ M_0^2 l^2/h_0^2 > 1 \end{cases}$$

Introducing the following non-dimensional quantities in equation (2),

$$\bar{r} = \frac{r}{a}, \bar{h}_m = \frac{h_m}{h_0}, \bar{h} = \frac{h}{h_0}\ \bar{l} = \frac{2l}{h_0}, \bar{P} = -\frac{h_m{}^3 p}{\mu a^2 V}$$

The Reynolds equation for film pressure is given by

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left\{\bar{r}\bar{S}\left(\bar{h}_m, \bar{l}, M_0\right)\frac{\partial \bar{P}}{\partial \bar{r}}\right\} = -1 \qquad (3)$$

Where,

$$\bar{S}(\bar{h}_m,\bar{l},M_0)=\begin{cases}\dfrac{1}{M_0^2}\left\{\dfrac{\bar{l}}{(\bar{A}^2-\bar{B}^2)}\left(\dfrac{\bar{B}^2}{\bar{A}}\tanh\dfrac{\bar{A}\bar{h}_m}{\bar{l}}-\dfrac{\bar{A}^2}{\bar{B}}\tanh\dfrac{\bar{B}\bar{h}_m}{\bar{l}}\right)+\bar{h}_m\right\}, & \text{for } M_0^2\bar{l}^2<1\\ \dfrac{1}{M_0^2}\left\{\dfrac{\bar{h}_m}{2}\sec h^2\left(\dfrac{\bar{h}_m}{\sqrt{2\bar{l}}}\right)-\dfrac{3\bar{l}}{\sqrt{2}}\tanh\left(\dfrac{\bar{h}_m}{\sqrt{2\bar{l}}}\right)+\bar{h}_m\right\}, & \text{for } M_0^2\bar{l}^2=1\\ \dfrac{1}{M_0^2}\left\{\left(\dfrac{\bar{l}\left(\bar{A}_2\cot\bar{\theta}-\bar{B}_2\right)\sin\bar{B}_2\bar{h}_m-\bar{l}\left(\bar{B}_2\cot\bar{\theta}+\bar{A}_2\right)\sinh\bar{A}_2\bar{h}_m}{M_0(\cos\bar{B}_2\bar{h}_m+\cosh\bar{A}_2\bar{h}_m)}\right)+\bar{h}_m\right\}, & \text{for } M_0^2\bar{l}^2>1\end{cases}$$

The boundary conditions to present the squeeze problem is,

$\bar{P}=0$ at $\bar{r}=\delta=b/a$ (4a)

$\bar{P}=0$ at $\bar{r}=1$ (4b)

The relation for non-dimensional pressure $\bar{P}$ is found by solving equation (3) using the boundary conditions 4a and 4b is

$$\bar{P}=\frac{f_2(\bar{r})f_1(1)-f_1(\bar{r})f_2(1)}{2f_2(1)} \qquad (5)$$

Where, $f_1(\bar{r})=\int_{\bar{r}=\delta}^{\bar{r}}\frac{\bar{r}}{\bar{S}}d\bar{r}$ and $f_2(\bar{r})=\int_{\bar{r}=\delta}^{\bar{r}}\frac{1}{\bar{r}\,\bar{S}}d\bar{r}$,(6)

The load supporting capacity is derived by solving pressure field over the film region by integration and is,

$$W=\int_{r=b}^{a}2\pi rp\,dr \qquad (7)$$

The non-dimensional load supporting capacity $\bar{W}$ is given by

$$\bar{W}=\frac{Wh_m^3}{2\pi\mu a^4\left(-dh_m/dt\right)}=\frac{-1}{2}\int_{\bar{r}=\delta}^{1}f_1(\bar{r})\bar{r}\,d\bar{r}+\frac{1}{2}\frac{f_1(1)}{f_2(1)}\int_{\bar{r}=\delta}^{1}f_2(\bar{r})\bar{r}\,d\bar{r} \qquad (8)$$

The non – dimensional response time for the film thickness is

$$\bar{T}=\frac{Wh_m^2}{\pi\mu a^4}t=\int_{\bar{h}_m}^{1}\left\{\frac{2f_2(1)}{f_2(1)\int_{\bar{r}=\delta}^{1}f_1(\bar{r})\bar{r}\,d\bar{r}-f_1(1)\int_{\bar{r}=\delta}^{1}f_2(\bar{r})\bar{r}\,d\bar{r}}\right\}d\bar{h}_m \qquad (9)$$

III. RESULTS AND DISCUSSION.

The MHD squeeze-film behaviour of a curved annular plates lubricated with an electrically conducting fluid in the presence of external magnetic field is discussed in the present investigation. The results for non-dimensional pressure, load supporting capacity and response time is presented for various values of Hartmann number, couple stress parameter, radius ratio and curved shape parameter. Distinct numerical values are chosen to discuss the squeeze-film behaviour as follows, $M_0=0-4$, $\bar{l}=0.0-0.4$ $\delta=b/a=0.2-0.6$ and $\beta=-0.25,0,0.25$.

3.1 Squeeze film Pressure.

The variation of dimensionless pressure $\bar{P}$ versus dimensionless radial coordinate $\bar{r}$ for distinct values of β and M_0 with δ = 0.4 and $\bar{l}$ = 0.4 is presented in Fig: 2 in which solid lines represent Non-magnetic case M_0= 0 and dotted lines represents magnetic case M_0= 4 and it is observed that $\bar{P}$ increases for increasing values

of Hartmann number M_0ascompared to Non-magnetic case (M_0= 0). Further, it is found that $\bar{P}$ increases for increasing values of β. Fig: 3 depicts the $\bar{P}$ versus $\bar{r}$ for distinct values of M_0 and $\bar{l}$ with δ = 0.4 and β = 0.5. It is seen that there is enhancement in pressure $\bar{P}$ for increasing values couple stress parameter $\bar{l}$ when compared to Newtonian case ($\bar{l}$ =0).

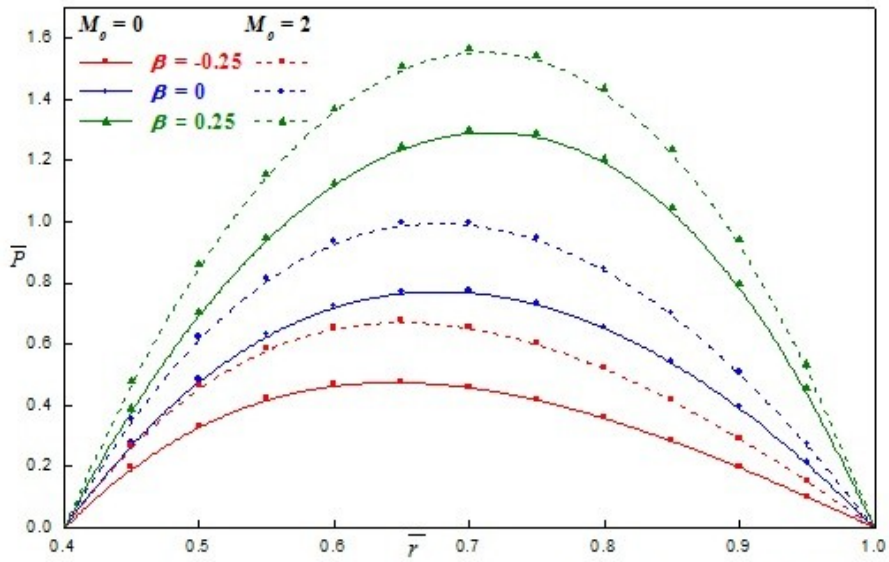

Figure 2:Outline of $\bar{P}$ against $\bar{r}$ for distinct values of M_0 & β fixed $\bar{l} = 0.4$

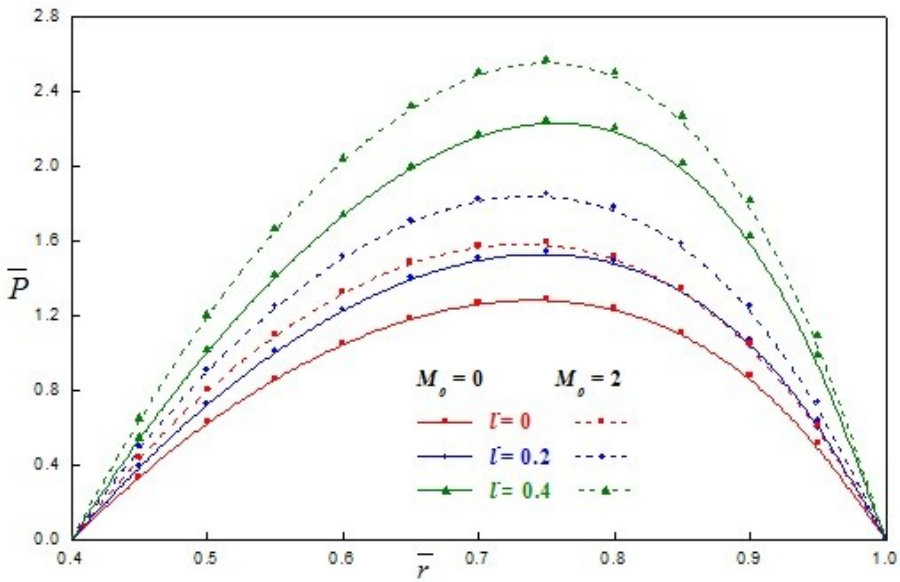

Figure 3: Outline of $\bar{P}$ against $\bar{r}$ for distinct values of M_0 & $\bar{l}$ fixed $\beta = 0.5$.

3.2 Load supporting capacity.

Figure: 4 describes the deviation of non-dimensional load supporting capacity $\bar{W}$ versus curvature parameter β for various values of Hartmann Number $M_0$and radius ratio δ with fixed $\bar{l}$ = 0.4. Here solid lines represent Non-magnetic case M_0= 0 and dotted lines represents magnetic case M_0= 4 and observed that $\bar{W}$ increases for increasing values of M_0 and also it is seen that there is a decrease in $\bar{W}$ for increasing values of δ . The non-dimensional load supporting capacity $\bar{W}$ versus β for distinct values of $M_0$and $\bar{l}$

keeping δ =0.4 as fixed is plotted in Fig: 5 and it is found that $\overline{W}$ significantly increases for increasing values of $\overline{l}$ .From both figures it is seen that there is enhancement in $\overline{W}$ for increasing values of β.

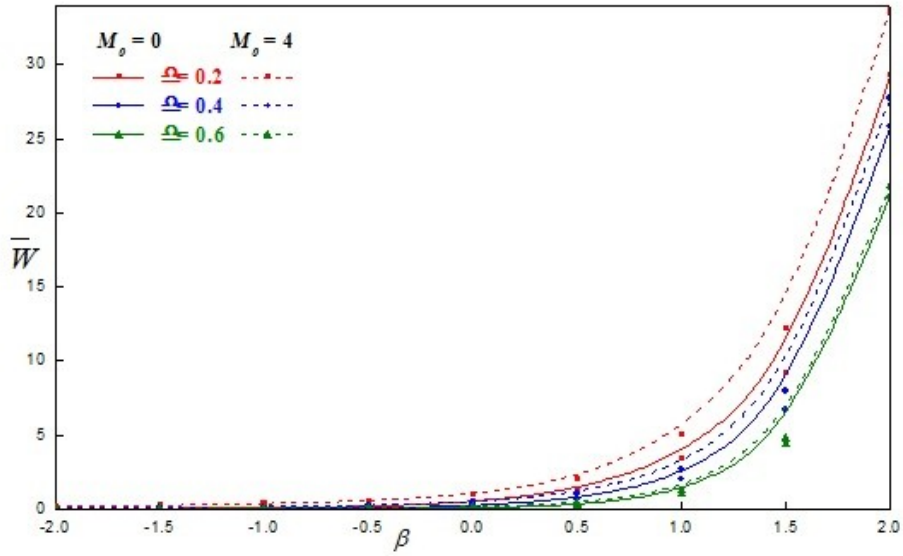

Figure 4: Outline of $\overline{W}$ against β for distinct values of M_0 & δ fixed $\overline{l} = 0.4$

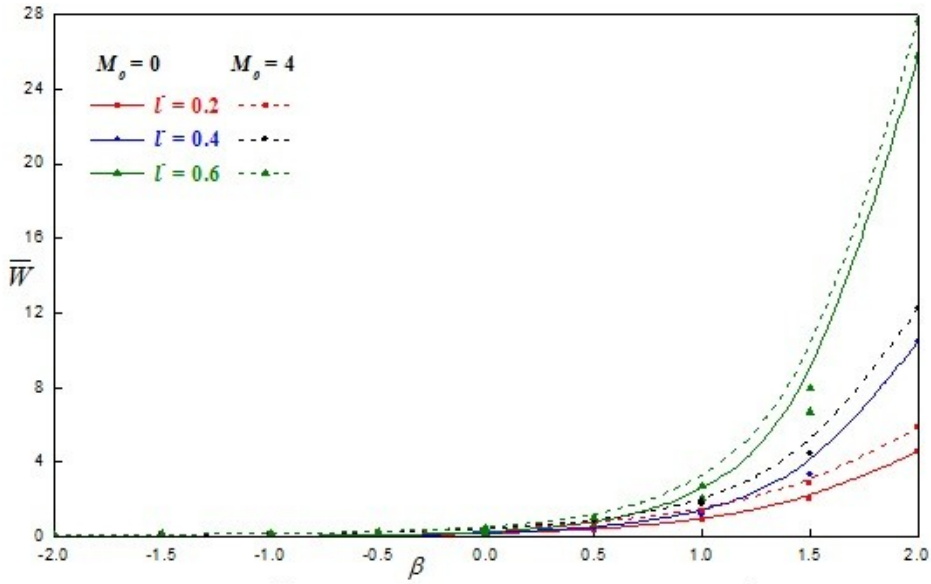

Figure 5: Outline of $\overline{W}$ against β for distinct values of M_0 & $\overline{l}$ fixed $\delta = 0.4$.

3.3 Response Time.

The variation of non-dimensional response time $\overline{T}$ as function $\overline{h}_m$ for distinct values of M_0 and β keeping $\overline{l}$ =0.4 and $\delta = 0.4$ as fixed is elaborated in Fig: 6.Here solid lines represent Non-magnetic case M_0= 0 and dotted lines represents magnetic case M_0= 4 and it is noticed that response time increases for increasing value of M_0. Further it is seen that there is significant increases in the response time for increasing values of β. Fig: 7 describes non-dimensional response time $\overline{T}$ against $\overline{h}_m$ with fixed $\delta = 0.4$ and β =0.5 for various values of M_0 and $\overline{l}$, it is found that for increasing values of $\overline{l}$, the response time increases. The variation of

$\overline{T}$ versus $\overline{h_m}$ for distinct of $\overline{l}$ and δ for fixed M_0= 4 and β =0.5 is illustrated in Fig: 8 and it is seen that $\overline{T}$ decreases for increasing values of δ .

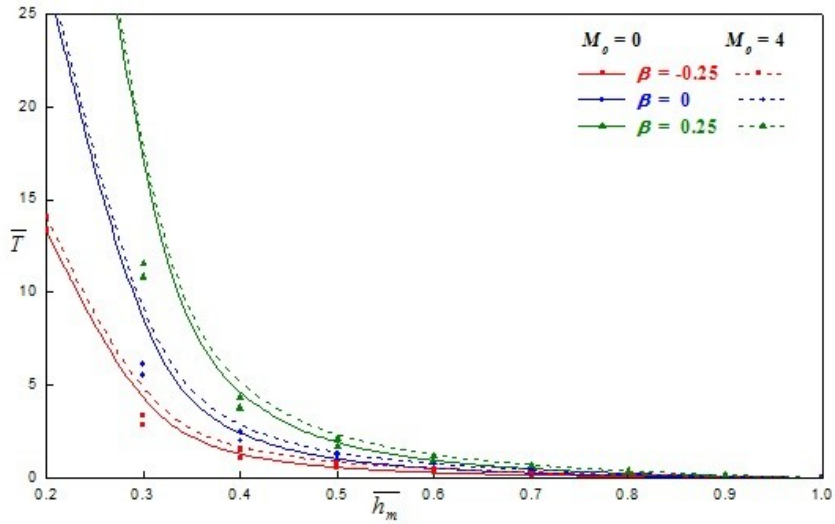

Figure 6: Outline of $\overline{T}$ against $\overline{h_m}$ for distinct values of M_0 & β fixed $\overline{l} = 0.4$ & $\delta = 0.4$

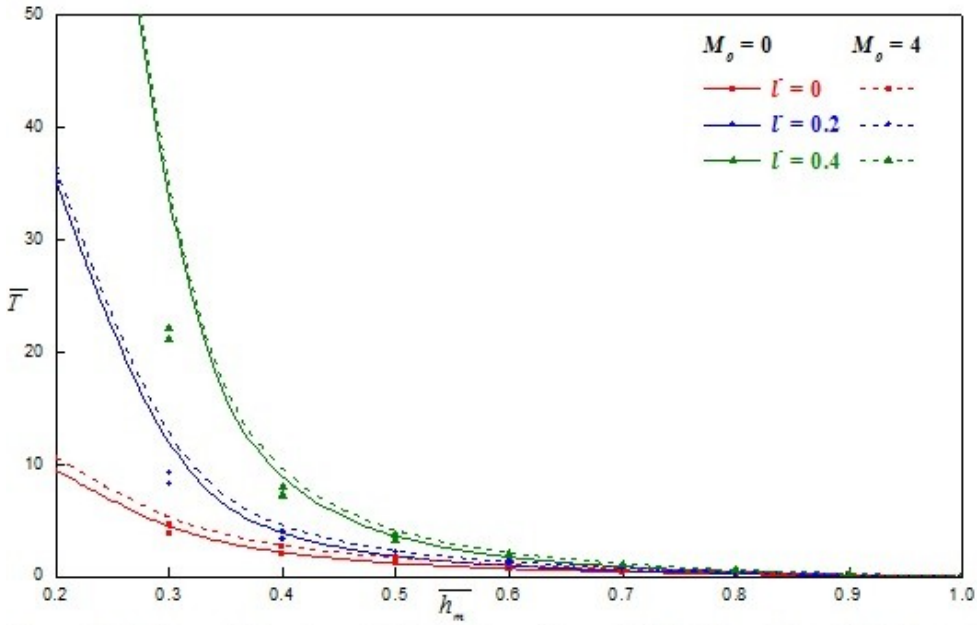

Figure 7: Outline of $\overline{T}$ against $\overline{h_m}$ for distinct values of M_0 & $\overline{l}$ fixed $\beta = 0.5$ & $\delta = 0.4$

IV. CONCLUSION.

The present study is proposed on the basis of the MHD flow theory along with Stokes micro-continuum theory, the annular curved circular squeeze-film plates are lubricated with non-Newtonian fluid in the presence of applied magnetic field. Analytical expressions of pressure, load supporting capacity and response time have been derived. The following conclusions are drawn,

- The effect of Hartmann number M_0 enhances the pressure, load carrying capacity and response time when compared to $M_0=0$ (Non-Magnetic case).
- The pressure, load carrying capacity and response time increases for increasing values of couple stress parameter $\overline{l}$ as compared to $\overline{l} = 0$ (Newtonian case).
- The load supporting capacity and response time decreases with increasing values of radius ratio δ .

- The pressure, load carrying capacity and response time increases for increasing values of curvature parameter β. On the whole, the squeeze film performance of curved annular plates is improved by the use of electrically conducting fluid in the presence of an applied magnetic field and couple stress fluid.

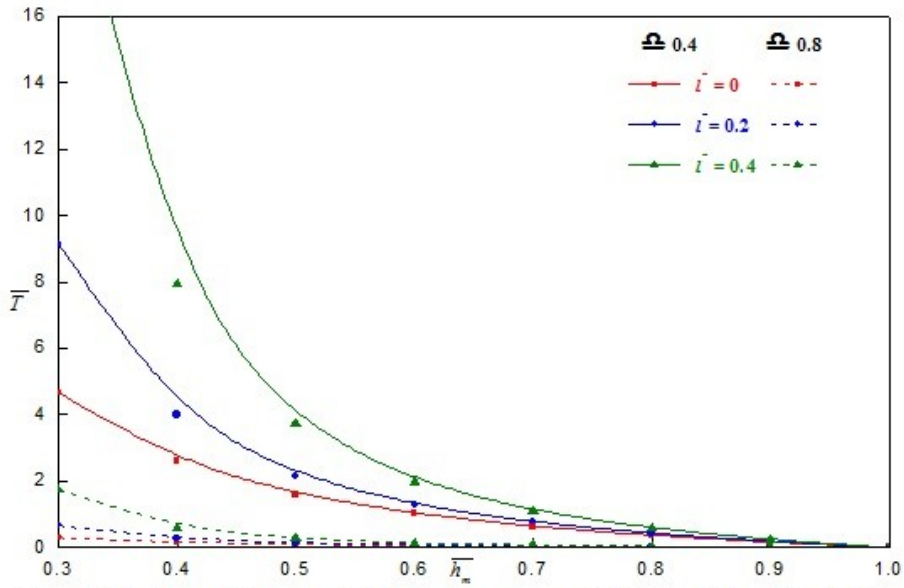

Figure8: Outline of $\bar{T}$ against $\bar{h}_m$ for distinct values of δ & $\bar{l}$ fixed $M_0 = 4$ & $\beta = 0.5$

V. NOMENCLATURE.

a Inner radius of the plate	r, z Radial and axial coordinates
b Outer radius of the plate	W Load carrying capacity
B_0 Transverse magnetic field	$\bar{W}$ Non-dimensional load carrying capacity
M_0 Hartmann number $(= B_0 h_0 (\sigma/\mu)^{1/2})$	T Response time
p Pressure in the film region	$\bar{T}$ Non-dimensional response time
$\bar{P}$ Non-dimensional pressure	β Curvature parameter
l Couple stress parameter $(\eta/\mu_0)^{1/2}$	η Material constant responsible for couple stresses
$\bar{l}$ Non-dimensional couple stress parameter $(2l/h_0)$	μ Lubricant viscosity
$\bar{h}_m$ Non-dimensional film thickness	δ Radius ratio
u, w Velocity components in r and z directions	

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