Posterior Risk Analysis of Survival Function of Weibull Distribution

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ABSTRACT *In this paper, two parameter Weibull distribution is considered and posterior risk of survival function is obtained by assuming non-informative and informative priors such as Extension of Jeffrey's, Lognormal-Inverted Gamma and Exponential-Gamma using with Quadratic loss function (QLF), Weighted loss function (WLF), Squared logarithmic loss function (SLLF) and Precautionary loss function (PLF). To illustrate the methodology, simulation study is carried out and done the parametric analysis.*

Keywords: *Bayesian analysis, Maximum likelihood estimation, Posterior risk, Precautionary loss function, Quadratic loss function, Squared logarithmic loss function, Weibull distribution, Weighted loss function.*

1. Introduction

In real life situation, Weibull model is widely used in reliability and life testing problems. The primary advantage of Weibull analysis is the ability to provide precise failure measures and forecasts with small samples. Samples with small size is also allowed in cost effective component testing. Sinha, S.K, (1986) obtained the Bayes estimate of reliability function and the hazard rate of the Weibull distribution by assigning squared error loss function. Al Omari Mohammed Ahmed and et al., (2010) compared the Bayesian and maximum likelihood estimation for Weibull distribution using Jeffrey's and Extension of Jeffrey's priors. Al Omari Mohammed Ahmed and et al., (2011) studied the Bayesian survival estimator with censored data using Jeffrey's prior and Extension of Jeffrey's prior information. Pandey, B.N and et al., (2011) compared the Bayesian and ML estimate of scale parameter of the Weibull distribution under Linex loss function. Chris Bambay Guure and et al., (2014) have studied the performance of Bayes and frequentist estimators for the two parameter Weibull failure time distribution by using non-informative prior and also the reliability and hazard functions were derived under Squared error loss, General entropy loss and Linex loss functions. Lavanya, A. and T. Leo Alexander (2016) have studied the Bayesian estimation of survival function under the Constant Shape Bi-Weibull distribution by using Extension of Jeffrey's prior under the losses such as Squared error loss, General entropy loss and Linex loss functions. Venkatesan,G and P.Saranya (2018a) have studied the performance of maximum likelihood estimation and Bayesian estimation of survival function of two parameter Weibull distribution assuming informative prior under Squared error loss, General entropy loss and Linex loss functions. Venkatesan, G and P.Saranya (2018b) carried out the Bayes risk of survival analysis through Weibull distribution using non-informative and informative priors as Jeffrey's, Extension of Jeffrey's, Lognormal-Inverted Gamma priors under Squared error loss, General entropy loss, Quadratic loss, Weighted loss and Squared logarithmic loss functions.

In our study, we proposed to obtain Bayes risk of survival function for the two parameter Weibull distribution using non-informative and informative priors with different loss functions. As mentioned above, the cases of Extension of Jeffrey's prior with Precautionary loss function, Lognormal-Inverted gamma prior with precautionary loss function and Exponential-Gamma prior with Quadratic loss, Weighted loss, Squared logarithmic loss, Precautionary loss functions are studied and done the posterior analysis.

2. Maximum Likelihood Estimation

Let $t_1, t_2, .., t_n$ be a random sample of size n from Weibull distribution with scale parameter (β) and shape parameter (α). The probability density function is

$$f(t;\alpha,\beta) = \frac{\alpha}{\beta} t^{\alpha-1} exp\left(\frac{-t^{\alpha}}{\beta}\right),\ t > 0, \alpha > 0, \beta > 0 \quad \text{... (2.1)}$$

The cumulative distribution function is

$$F(t;\alpha,\beta) = 1 - exp\left(\frac{-t^{\alpha}}{\beta}\right),\ t > 0, \alpha > 0, \beta > 0 \quad \text{...(2.2)}$$

The survival function of t is

$$S(t;\alpha,\beta) = exp\left(\frac{-t^{\alpha}}{\beta}\right) \quad \text{...(2.3)}$$

The likelihood function of t is

$$L(t;\alpha,\beta) = \prod_{i=1}^{n}\left\{\frac{\alpha}{\beta}\, t_i^{\alpha-1} exp\left(\frac{-t_i^{\alpha}}{\beta}\right)\right\} \qquad ...(2.4)$$

Using the principle of MLE, we get

$$\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n} t_i^{\hat{\alpha}} \qquad ...(2.5)$$

where $\hat{\alpha}$ can be determined by using Newton-Raphson method and taking the initial value ofα as $\alpha_i$as per our convenience and iterating the process till it converges.

$$\alpha_{i+1} = \alpha_i - \frac{\frac{n}{\alpha}+\sum_{i=1}^{n} log t_i - \frac{\sum_{i=1}^{n} t_i^{\alpha} log t_i}{\frac{1}{n}\sum_{i=1}^{n} t_i^{\alpha}}}{-\left\{\frac{n}{\alpha^2}+\frac{\sum_{i=1}^{n} t_i^{\alpha}(log t_i)^2}{\frac{1}{n}\sum_{i=1}^{n} t_i^{\alpha}}\right\}} \qquad ... (2.6)$$

The estimate of survival function of Weibull distribution under the MLE is

$$\hat{S}(t_i) = exp\left[-\left(\frac{t_i^{\hat{\alpha}}}{\hat{\beta}}\right)\right] \qquad ... (2.7)$$

3.Bayesian Estimation

The posterior distribution of the parameter can be obtained by multiplying likelihood function and prior of the parameters. Bayesian estimation of the parameter can be obtained by using posterior distribution instead of likelihood function. Bayesian estimation approach has received a lot of attention for analyzing failure time data. When prior knowledge about the parameter is not available, it is possible to make use of the non-informative prior. When we have knowledge on the parameter, the informative prior is preferred to use for the analysis.

3.1.Extension of Jeffrey's prior

The Extension of Jeffrey's prior for the Weibull distribution is

$$v_1(\alpha,\beta) \propto \left(\frac{1}{\alpha\beta}\right)^{2m}, m \in R^+ \qquad ...(3.1)$$

The posterior distribution of the parameters α and β is obtained by multiplying the equations (2.4) and (3.1) as

$$\pi_1(\alpha,\beta) \propto \left(\frac{1}{\alpha\beta}\right)^{2m} \prod_{i=1}^{n}\left\{\frac{\alpha}{\beta}\, t^{\alpha-1} exp\left(\frac{-t^{\alpha}}{\beta}\right)\right\} \qquad ... (3.2)$$

where α, β and m are described in equations (2.4) and (3.1).

3.2.Lognormal-Inverted Gamma prior

The shape parameter α follows Lognormal distribution with hyperparameter c and scale parameter β follows Inverted gamma distribution with hyperparameters a and b. The joint prior distribution of α and β is

$$v_2(\alpha,\beta) \propto \frac{1}{\alpha} exp\left\{\frac{-(log\, \alpha)^2}{2c^2}\right\}\left(\frac{1}{\beta}\right)^{a+1} exp\left(\frac{-b}{\beta}\right), \quad \alpha,\beta > 0; a,b,c > 0 \qquad ... (3.3)$$

The posterior distribution of the parameters α and β is obtained by multiplying the equations (2.4) and (3.3) as

$$\pi_2(\alpha,\beta) \propto \prod_{i=1}^{n}\left\{\frac{\alpha}{\beta}\, t^{\alpha-1} exp\left(\frac{-t^{\alpha}}{\beta}\right)\right\}\left(\frac{1}{\alpha}\right) exp\left\{\frac{-(log\, \alpha)^2}{2c^2}\right\}\left(\frac{1}{\beta}\right)^{a+1} exp\left(\frac{-b}{\beta}\right) \qquad ... (3.4)$$

where α, β, a, b and c are described in equations (2.4) and (3.3).

3.3.Exponential-Gamma prior

The shape parameter α follows Gamma distribution with hyperparameters a_1 and b_1 and scale parameter β follows Inverted Gamma distribution with hyperparameter c_1. The joint prior distribution of α and β is

$$v_3(\alpha,\beta) \propto exp(-\beta c_1)\, \alpha^{a_1-1}\, exp(-b_1\alpha), \quad \alpha,\beta > 0;\ a_1, b_1, c_1 > 0 \qquad ... (3.5)$$

The posterior distribution of the parameters α and β is obtained by multiplying the equation (2.4) and (3.5) is

$$\pi_3(\alpha,\beta) \propto \prod_{i=1}^{n}\left\{\frac{\alpha}{\beta}\, t^{\alpha-1} exp\left(\frac{-t^{\alpha}}{\beta}\right)\right\} exp(-\beta c_1)\, \alpha^{a_1-1}\, exp(-b_1\alpha) \qquad ... (3.6)$$

where α, β, a_1, b_1 and c_1 are described in equations (2.4) and (3.5).

4.Posterior Risk of Survival Function under Different Loss Function

We consider the Quadratic loss, Weighted loss, Precautionary loss and Squared logarithmic loss functions. In the following table shown the expression of Bayesian estimator and Bayes risk with different loss functions for the parameter.

Loss Function	Expression	Bayes estimator	Posterior Risk
Quadratic	$L(\theta,\hat{\theta})=\left(1-\frac{\hat{\theta}}{\theta}\right)^2$	$\frac{E(\theta^{-1}/t)}{E(\theta^{-2}/t)}$	$1-\frac{[E(\theta^{-1})]^2}{E(\theta^{-2})}$
Weighted	$L(\hat{\theta},\theta)=\frac{(\hat{\theta}-\theta)^2}{\theta}$	$[E(\theta^{-1}/t)]^{-1}$	$E(\theta)-[E(\theta^{-1})]^{-1}$
Precautionary	$L(\hat{\theta},\theta)=\frac{(\hat{\theta}-\theta)^2}{\hat{\theta}}$	$\sqrt{E(\theta^2/t)}$	$2\left\{\sqrt{E(\theta^2/t)}-E(\theta/t)\right\}$
Squared Logarithmic	$L(\theta,\hat{\theta})=(log\hat{\theta}-log\theta)^2$	$exp[E(log\ \theta/t)]$	$E[(log\theta/t)^2]-[E(log\theta/t)]^2$

4.1. Extension of Jeffrey's prior under Precautionary loss function

The Bayes risk of survival function using Extension of Jeffrey's prior under Precautionary loss function is

$$R[\hat{S}(t_i)]_{BP}=2\left\{\left\{\sqrt{E\left\{\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}^2\right\}}\right\}-E\left\{\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}\right\}\right\}=2\left\{\left\{\sqrt{\frac{\iint\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]^2\pi_1(\alpha,\beta)d\beta d\alpha}{\iint\pi_1(\alpha,\beta)d\beta d\alpha}}\right\}-\frac{\iint exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\pi_1(\alpha,\beta)d\beta d\alpha}{\iint\pi_1(\alpha,\beta)d\beta d\alpha}\right\} \quad ...(4.1)$$

The above equation contains a ratio of two integrals which cannot be solved analytically, so we use Lindley's approximation procedure to estimate the survival function.

Using Lindley's approximation, the expansion of $\frac{\int u(\theta)v(\theta)exp[L(\theta)]d\theta}{\int v(\theta)exp[L(\theta)]d\theta}$can be performed as

$$\hat{\theta}=u+\frac{1}{2}[u_{11}\sigma_{11}+u_{22}\sigma_{22}]+u_1\rho_1\sigma_{11}+u_2\rho_2\sigma_{22}+\frac{1}{2}[L_{30}u_1\sigma^2{}_{11}+L_{03}u_2\sigma^2{}_{22}] \quad ...(4.2)$$

where L is the log likelihood function and ρ is the logarithmic of prior distribution.

Let $u=\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}^2$ and $e=-\left(\frac{t_i{}^{\alpha}}{\beta}\right)$, where α and β is given in equation (2.6) and (2.5) respectively.

$$u_1=\frac{du}{d\beta}=\frac{-2ue}{\beta}\ ;\ u_{11}=\frac{du}{d\beta^2}=\frac{4ue}{\beta^2}(e+1);\ \ u_2=\frac{du}{d\alpha}=2ue(logt_i)\ ;\ u_{22}=\frac{du}{d\alpha^2}=2ue(logt_i)^2(1+2e) \quad ...(4.3)$$

$$\rho(\alpha,\beta)=-log(\alpha^{2m})-log(\beta^{2m})\ ;\ \rho_1=\frac{du}{d\beta}=\frac{-1}{\beta^{2m}}\ ;\quad \rho_2=\frac{du}{d\alpha}=-\frac{1}{\alpha^{2m}} \quad ...(4.4)$$

$$\sigma_{11}=(-L_{20})^{-1}\ ;\ \ \sigma_{22}=(-L_{02})^{-1}$$

$$L_{02}=\frac{d^2L}{d\alpha^2}=-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\ \ ;\ L_{03}=\frac{d^3L}{d\alpha^3}=\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^3 \quad ...(4.5)$$

$$L_{20}=\frac{d^2L}{d\beta^2}=\frac{n}{\beta^2}-\frac{2}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}\ \ ;\ L_{30}=\frac{d^3L}{d\beta^3}=-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i{}^{\alpha} \quad ...(4.6)$$

Let $u=exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]$

$$u_1=\frac{du}{d\beta}=\frac{-ue}{\beta}\ \ ;u_{11}=\frac{du}{d\beta^2}=\frac{ue}{\beta^2}(e+2)\ ;\ \ u_2=\frac{du}{d\alpha}=ue(logt_i);\ u_{22}=\frac{du}{d\alpha^2}=2ue(logt_i)^2(1+e) \quad ...(4.7)$$

The Bayes risk of survival function using Extension of Jeffrey's prior under Precautionary loss function $R[\hat{S}(t_i)]_{BP}$ is

$$2\left\{\sqrt{\begin{Bmatrix}\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2+\frac{1}{2}\left\{\frac{\left(\frac{4ue}{\beta^2}\right)(e+1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{2ue(logt_i)^2(1+2e)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-2ue}{\beta}\right)\left(\frac{-1}{\beta^{2m}}\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{(2ue(logt_i))\left(\frac{-1}{\alpha^{2m}}\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\\+\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-2ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][2ue(logt_i)]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\end{Bmatrix}}-\begin{Bmatrix}exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}\right)(e+2)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{[ue(logt_i)^2(1+e)]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-ue}{\beta}\right)\left(\frac{-1}{\beta^{2m}}\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{(ue(logt_i))\left(\frac{-1}{\alpha^{2m}}\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\\+\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][ue(logt_i)]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\end{Bmatrix}\right\} \quad ...(4.8)$$

4.2.Lognormal-Inverted Gamma prior under Precautionary loss function

The Bayes risk of survival function using Lognormal-Inverted Gamma prior under Precautionary loss function is

$$R[\hat{S}(t_i)]_{BP}=2\left\{\left\{\sqrt{E\left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2\right\}}\right\}-E\left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}\right\}=2\left\{\left\{\sqrt{\frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^2\pi_2(\alpha,\beta)d\beta d\alpha}{\iint\pi_2(\alpha,\beta)d\beta d\alpha}}\right\}-\frac{\iint exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\pi_2(\alpha,\beta)d\beta d\alpha}{\iint\pi_2(\alpha,\beta)d\beta d\alpha}\right\} \quad ...(4.9)$$

$$\rho(\alpha,\beta)=-log\,\alpha-\frac{(log\alpha)^2}{2c^2}-(a+1)log\beta-\frac{b}{\beta};\ \rho_1=\frac{du}{d\beta}=\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\ ;\ \rho_2=\frac{du}{d\alpha}=-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2} \quad ...(4.10)$$

The procedure of Lindley's approximation used in 4.1 to obtained the Bayes risk of survival function using Lognormal-Inverted Gamma prior under Precautionary loss function$R[\hat{S}(t_i)]_{BP}$ is

$$2\left\{\sqrt{\begin{Bmatrix}\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2+\frac{1}{2}\left\{\frac{\left(\frac{4ue}{\beta^2}\right)(e+1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{[2ue(logt_i)^2(1+2e)]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-2ue}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{(2ue(logt_i))\left(-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2}\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\\+\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-2ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][2ue(logt_i)]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\end{Bmatrix}}-\begin{Bmatrix}exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}\right)(e+2)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{[ue(logt_i)^2(1+e)]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-ue}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{(ue(logt_i))\left(-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2}\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\\+\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][ue(logt_i)]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\end{Bmatrix}\right\} \quad ...(4.11)$$

4.3.Exponential-Gamma prior under Precautionary loss function

The Bayes risk of survival function using Exponential-Gamma prior under Precautionary loss function is

$$R[\hat{S}(t_i)]_{BP}=2\left\{\left\{\sqrt{E\left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2\right\}}\right\}-E\left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}\right\}=2\left\{\left\{\sqrt{\frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^2\pi_3(\alpha,\beta)d\beta d\alpha}{\iint\pi_3(\alpha,\beta)d\beta d\alpha}}\right\}-\frac{\iint exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\pi_3(\alpha,\beta)d\beta d\alpha}{\iint\pi_3(\alpha,\beta)d\beta d\alpha}\right\} \quad ...(4.12)$$

$$\rho(\alpha,\beta)=-\beta c_1+(a_1-1)log\alpha-b_1\alpha;\ \rho_1=\frac{du}{d\beta}=-c_1\ ;\ \rho_2=\frac{du}{d\alpha}=\frac{(a_1-1)}{\alpha}-b_1 \quad ...(4.13)$$

The procedure of Lindley's approximation used in 4.1 to obtained the Bayes risk of survival function using Exponential-Gamma prior under Precautionary loss function$R[\hat{S}(t_i)]_{BP}$ is

$$2\left\{\sqrt{\begin{array}{l}\left\{\begin{array}{l}\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2+\frac{1}{2}\left\{\frac{\left(\frac{4ue}{\beta^2}\right)(e+1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{\left[2ue(\log t_i)^2(1+2e)\right]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right)}\right\}+\frac{\left(\frac{-2ue}{\beta}\right)(-c_1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{(2ue(\log t_i))\left(\frac{(a_1-1)}{\alpha}-b_1\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right)}\\ +\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-2ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^3\right]\left[2ue(\log t_i)\right]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right]^2}\right)\end{array}\right\}\\ -\left\{\begin{array}{l}\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}\right)(e+2)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{\left[ue(\log t_i)^2(1+e)\right]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right)}\right\}+\frac{\left(\frac{-ue}{\beta}\right)(-c_1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{(ue(\log t_i))\left(\frac{(a_1-1)}{\alpha}-b_1\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right)}\\ +\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^3\right]\left[ue(\log t_i)\right]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right]^2}\right)\end{array}\right\}\end{array}}\right\} \quad \ldots(4.14)$$

4.4. Exponential-Gamma prior under Quadratic loss function

The Bayes risk of survival function using Exponential-Gamma prior under Quadratic loss function is

$$R\left[\hat{S}(t_i)\right]_{BQ}=1-\frac{\left\{E\left\{\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\right\}^2}{E\left\{\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\right\}}=1-\frac{\left\{\frac{\iint\left\{\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\pi_3(\alpha,\beta)d\beta d\alpha}{\iint\pi_3(\alpha,\beta)d\beta d\alpha}\right\}^2}{\left\{\frac{\iint\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\pi_3(\alpha,\beta)d\beta d\alpha}{\iint\pi_3(\alpha,\beta)d\beta d\alpha}\right\}} \quad \ldots(4.15)$$

Let $u=\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}$; $u_1=\frac{du}{d\beta}=\frac{ue}{\beta}$; $u_{11}=\frac{du}{d\beta^2}=\frac{ue}{\beta^2}(e-2)$;

$u_2=\frac{du}{d\alpha}=-ue(\log t_i)$; $u_{22}=\frac{du}{d\alpha^2}=-ue(\log t_i)^2(1-e)$.

Let $u=\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}$; $u_1=\frac{du}{d\beta}=\frac{2ue}{\beta}$; $u_{11}=\frac{du}{d\beta^2}=\frac{4ue}{\beta^2}(e-1)$

$u_2=\frac{du}{d\alpha}=-2ue(\log t_i)$; $u_{22}=\frac{du}{d\alpha^2}=2ue(\log t_i)^2(2e-1)$.

The procedure of Lindley's approximation used in 4.1 to obtained the Bayes risk of survival function using Exponential-Gamma prior under Quadratic loss function $R\left[\hat{S}(t_i)\right]_{BQ}$ is

$$1-\frac{\left\{\begin{array}{l}\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}+\frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}(e-2)\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{\left[ue(\log t_i)^2(e-1)\right]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)(-c_1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{(-ue(\log t_i))\left(\frac{(a_1-1)}{\alpha}-b_1\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right)}\\ +\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^3\right]\left[2ue(\log t_i)\right]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right]^2}\right)\end{array}\right\}^2}{\left\{\begin{array}{l}\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}+\frac{1}{2}\left\{\frac{\left(\frac{4ue}{\beta^2}(e-1)\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{\left[2ue(\log t_i)^2(2e-1)\right]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right)}\right\}+\frac{\left(\frac{2ue}{\beta}\right)(-c_1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)}+\frac{(-2ue(\log t_i))\left(\frac{(a_1-1)}{\alpha}-b_1\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right)}\\ +\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{2ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^3\right]\left[-2ue(\log t_i)\right]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(\log t_i)^2\right]^2}\right)\end{array}\right\}} \quad \ldots(4.16)$$

4.5. Exponential-Gamma prior under Weighted loss function

The Bayes risk of survival function using Exponential-Gamma prior under Weighted loss function is

$$R[\hat{S}(t_i)]_{BW} = E\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\} - \left[E\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\right]^{-1} = \frac{\iint exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha} - \left[\frac{\iint \left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha}\right]^{-1} \quad \ldots (4.17)$$

The procedure of Lindley's approximation used in 4.1 to obtained the Bayes risk of survival function using Exponential-Gamma prior under Weighted loss function $R[\hat{S}(t_i)]_{BW}$is

$$\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right] + \frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}(e+2)\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)} + \frac{[2ue(logt_i)^2(1+e)]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\} + \frac{\left(\frac{-ue}{\beta}\right)(-c_1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)} + \frac{(ue(logt_i))\left(\frac{(a_1-1)}{\alpha}-b_1\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)} + \frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][ue(logt_i)]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\} - \left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^{-1} + \frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}(e-2)\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)} + \frac{[-ue(logt_i)^2(1-e)]}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\} + \frac{\left(\frac{ue}{\beta}\right)(-c_1)}{\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}} + \frac{(-ue(logt_i))\left(\frac{(a_1-1)}{\alpha}-b_1\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)} + \frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{ue}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][-ue(logt_i)]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^{-1} \quad \ldots (4.18)$$

4.6. Exponential-Gamma prior under Squared logarithmic loss function

The Bayes risk of survival function using Exponential-Gamma prior under Squared logarithmic loss function is

$$R[\hat{S}(t_i)]_{BSL} = E\left\{\left\{log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}^2\right\} - \left\{E\left\{log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}\right\}^2 = \frac{\iint\left\{log\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]\right\}^2\pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha} - \left\{\frac{\iint log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha}\right\}^2 \quad \ldots (4.19)$$

Let $u = -\left(\frac{t_i^{\alpha}}{\beta}\right)$; $u_1 = \frac{du}{d\beta} = \frac{-u}{\beta}$; $u_{11} = \frac{du}{d\beta^2} = \frac{2u}{\beta^2}$; $u_2 = \frac{du}{d\alpha} = u(logt_i)$; $u_{22} = \frac{du}{d\alpha^2} = u(logt_i)^2$

Let $u = \left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]^2$; $u_1 = \frac{du}{d\beta} = \frac{-2u}{\beta}$; $u_{11} = \frac{du}{d\beta^2} = \frac{6u}{\beta^2}$; $u_2 = \frac{du}{d\alpha} = 2u(logt_i)$; $u_{22} = \frac{du}{d\alpha^2} = 4u(logt_i)^2$

The procedure of Lindley' approximation used in 4.1 to obtained the Bayes risk of survival function using Exponential-Gamma prior under Squared logarithmic loss function $R[\hat{S}(t_i)]_{BSL}$ is

$$\left\{\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]^2 + \frac{1}{2}\left\{\frac{\left(\frac{6u}{\beta^2}\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)} + \frac{4u(logt_i)^2}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\} + \frac{\left(\frac{-2u}{\beta}\right)(-c_1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)} + \frac{(2u(logt_i))\left(\frac{(a_1-1)}{\alpha}-b_1\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)} + \frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-2u}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][2u(logt_i)]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\} - \left\{\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right] + \frac{1}{2}\left\{\frac{\left(\frac{2u}{\beta^2}\right)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)} + \frac{u(logt_i)^2}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\} + \frac{\left(\frac{-u}{\beta}\right)(-c_1)}{\left(\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right)} + \frac{(u(logt_i))\left(\frac{(a_1-1)}{\alpha}-b_1\right)}{\left(\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)} + \frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right)\left(\frac{-u}{\beta}\right)}{\left[\frac{-n}{\beta^2}+\frac{2}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][u(logt_i)]}{\left[\frac{n}{\alpha^2}+\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^2 \quad \ldots (4.20)$$

5.Simulation study

In this study, we chose a sample size of n=25, 50 and 100 to represent small, medium and large dataset. The posterior risk of survival function is estimated for Weibull distribution using non-informative and informative priors under different loss functions. The values of the parameters chosen as α=0.8, 1.2, 3 and β=0.5, 1.5, 5. The hyperparameter values of Extension of Jeffrey's prior is m=0.4, 1.4 and for the informative priors are chosen as a=0.4, 1.4; b=0.6, 1.6 and c=0.9, 1.9; a_1=2.5, 5; b_1=1.5, 3 and c_1=0.5, 1.5. The results of the simulation study are obtained, discussed and reported as follows:

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Extension of Jeffrey's prior m=0.4) under various loss functions is obtained and presented in Table-1.

Table-1: Estimation of Bayes risk of Survival Function under Extension of Jeffrey's prior m=0.4.

n	β	α	$R(\hat{S}(t_i))_{BQ}$	$R(\hat{S}(t_i))_{BW}$	$R(\hat{S}(t_i))_{BSL}$	$R(\hat{S}(t_i))_{BP}$
25	0.5	0.8	5.208284e-04	9.570962e-05	6.036503e-04	9.480409e-05
		1.2	5.231287e-04	9.545640e-05	6.064596e-04	9.456887e-05
		3	5.252592e-04	9.521760e-05	6.072308e-04	9.439444e-05
	1.5	0.8	8.732044e-04	8.543423e-05	1.099159e-03	8.170706e-05
		1.2	8.784457e-04	8.539660e-05	1.104007e-03	8.151321e-05
		3	8.868703e-04	8.574622e-05	1.105862e-03	8.149815e-05
	5	0.8	8.594213e-04	9.964047e-05	1.096080e-03	9.592371e-05
		1.2	8.569570e-04	1.036488e-04	1.158626e-03	9.987858e-05
		3	8.626487e-04	1.003821e-04	1.095756e-03	9.618801e-05
50	0.5	0.8	2.730612e-04	4.891747e-05	2.970760e-04	4.866313e-05
		1.2	2.737793e-04	4.884043e-05	2.977854e-04	4.859014e-05
		3	2.747176e-04	4.875378e-05	2.983017e-04	4.851615e-05
	1.5	0.8	4.755941e-04	4.405626e-05	5.421455e-04	4.295617e-05
		1.2	4.765607e-04	4.406275e-05	5.425724e-04	4.291700e-05
		3	4.785085e-04	4.399796e-05	5.441630e-04	4.273555e-05
	5	0.8	4.619939e-04	5.192860e-05	5.245764e-04	5.089370e-05
		1.2	4.623854e-04	5.194909e-05	5.246041e-04	5.087556e-05
		3	4.641882e-04	5.177713e-05	5.268027e-04	5.060522e-05
100	0.5	0.8	1.450904e-04	2.443432e-05	1.530083e-04	2.436757e-05
		1.2	1.452686e-04	2.441760e-05	1.531291e-04	2.435266e-05
		3	1.458123e-04	2.437940e-05	1.535824e-04	2.432109e-05
	1.5	0.8	2.589652e-04	2.161150e-05	2.840584e-04	2.131748e-05
		1.2	2.596246e-04	2.158846e-05	2.845634e-04	2.128211e-05
		3	2.606887e-04	2.154931e-05	2.854063e-04	2.121440e-05
	5	0.8	2.517572e-04	2.558599e-05	2.768560e-04	2.529011e-05
		1.2	2.519143e-04	2.559196e-05	2.768818e-04	2.528538e-05
		3	2.527330e-04	2.557132e-05	2.774132e-04	2.523872e-05

From the table-1, it is observed that the Bayes risk of survival function is minimum under the QLF and SLLF when β<1 and α<1 than β<1 and α>1 and also minimum when β>1 and α<1 than β>1 and α>1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is maximum under the WLF and PLF when β<1 and α<1 than β<1 and α>1 and minimum when β>1 and α<1 than β>1 and α>1 for n=25. The values of scale and shape parameters of the Bayes risk of survival function is maximum when β<1 and α<1 than β<1 and α>1 and also minimum when β>1 and α<1 than β>1 and α>1 for n=50 and 100. The Bayes risk of survival function for Weibull distribution using non-informative prior (Extension of Jeffrey's) under PLF is better than using other loss functions (QLF, WLF and SLLF) proposed in this study.

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Extension of Jeffrey's prior m=1.4) under various loss functions is obtained and presented in Table-2.

Table-2: Estimation of Bayes risk of Survival Function under Extension of Jeffrey's prior m=1.4.

n	β	α	$R(\widehat{S}(t_i))_{BQ}$	$R(\widehat{S}(t_i))_{BW}$	$R(\widehat{S}(t_i))_{BSL}$	$R(\widehat{S}(t_i))_{BP}$
25	0.5	0.8	4.854044e-04	9.123676e-05	5.890452e-04	9.501293e-05
		1.2	4.816272e-04	9.202690e-05	5.900511e-04	9.561381e-05
		3	4.779920e-04	9.243430e-05	5.884622e-04	9.585903e-05
	1.5	0.8	9.004228e-04	8.624166e-05	1.093626e-03	7.934390e-05
		1.2	8.864641e-04	8.583559e-05	1.102853e-03	8.046320e-05
		3	8.766482e-04	8.571467e-05	1.106055e-03	8.157654e-05
	5	0.8	8.966749e-04	1.009054e-04	1.088200e-03	9.184748e-05
		1.2	8.855310e-04	1.007944e-04	1.091076e-03	9.291305e-05
		3	8.776419e-04	1.011292e-04	1.093373e-03	9.410572e-05
50	0.5	0.8	2.620660e-04	4.790535e-05	2.938076e-04	4.881408e-05
		1.2	2.609271e-04	4.803704e-05	2.941681e-04	4.890331e-05
		3	2.689402e-04	4.729754e-05	2.942211e-04	4.809671e-05
	1.5	0.8	4.842321e-04	4.427433e-05	5.409067e-04	4.226371e-05
		1.2	4.794382e-04	4.418670e-05	5.422383e-04	4.257955e-05
		3	4.758770e-04	4.400585e-05	5.441607e-04	4.269931e-05
	5	0.8	4.743538e-04	5.229715e-05	5.227940e-04	4.967015e-05
		1.2	4.712142e-04	5.225674e-05	5.236087e-04	4.996675e-05
		3	4.698256e-04	5.200628e-05	5.262044e-04	4.995091e-05
100	0.5	0.8	1.406651e-04	2.409439e-05	1.518015e-04	2.439446e-05
		1.2	1.401885e-04	2.414447e-05	1.518068e-04	2.442616e-05
		3	1.400565e-04	2.415083e-05	1.521149e-04	2.442023e-05
	1.5	0.8	2.619909e-04	2.166356e-05	2.837516e-04	2.115346e-05
		1.2	2.604401e-04	2.161462e-05	2.844978e-04	2.121255e-05
		3	2.593833e-04	2.154443e-05	2.854193e-04	2.122403e-05
	5	0.8	2.562304e-04	2.567326e-05	2.763601e-04	2.497771e-05
		1.2	2.550396e-04	2.566351e-05	2.766124e-04	2.505794e-05
		3	2.545767e-04	2.562132e-05	2.772582e-04	2.508360e-05

From the table-2, it is observed that the Bayes risk of survival function is minimum under the QLF and WLF when β<1 and α>1 than β<1 and α<1 and also minimum when β>1 and α>1 than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is maximum under the SLLF and PLF when β<1 and α>1than β<1 and α<1 and also maximum when β>1 and α>1than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using non-informative prior (Extension of Jeffrey's) under PLF is better than using other loss functions (QLF, WLF and SLLF) proposed in this study.

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Lognormal-Inverted Gamma prior a=0.4, b=0.6 and c=0.9) under various loss functions is obtained and presented in Table-3.

Table-3: Estimation of Bayes risk of Survival Function under Lognormal-Inverted Gamma prior with hyperparameters a=0.4, b=0.6 and c=0.9.

n	β	α	$R(\widehat{S}(t_i))_{BQ}$	$R(\widehat{S}(t_i))_{BW}$	$R(\widehat{S}(t_i))_{BSL}$	$R(\widehat{S}(t_i))_{BP}$
25	0.5	0.8	5.282939e-04	9.624001e-05	6.031299e-04	9.394479e-05
		1.2	5.335849e-04	9.587347e-05	6.055077e-04	9.373264e-05
		3	5.411442e-04	9.499880e-05	6.030666e-04	9.321342e-05
	1.5	0.8	8.691337e-04	8.534241e-05	1.099183e-03	8.191021e-05
		1.2	8.836994e-04	8.555882e-05	1.103624e-03	8.117023e-05
		3	9.115369e-04	8.627414e-05	1.100278e-03	7.980421e-05

	5	0.8	8.584955e-04	9.964109e-05	1.096056e-03	9.593320e-05
		1.2	8.649165e-04	9.999514e-05	1.095381e-03	9.549082e-05
		3	8.805897e-04	1.010661e-04	1.094178e-03	9.474787e-05
50	0.5	0.8	2.755083e-04	4.905101e-05	2.969631e-04	4.844867e-05
		1.2	2.771836e-04	4.893711e-05	2.975344e-04	4.836843e-05
		3	2.799200e-04	4.866861e-05	2.971998e-04	4.817584e-05
	1.5	0.8	4.745934e-04	4.403848e-05	5.421562e-04	4.299903e-05
		1.2	4.786114e-04	4.411566e-05	5.424595e-04	4.279685e-05
		3	4.869758e-04	4.415163e-05	5.427846e-04	4.220962e-05
	5	0.8	4.619720e-04	5.193960e-05	5.245875e-04	5.087757e-05
		1.2	4.642555e-04	5.202956e-05	5.246294e-04	5.073471e-05
		3	4.701926e-04	5.198188e-05	5.264628e-04	5.015653e-05
100	0.5	0.8	1.459791e-04	2.447051e-05	1.529589e-04	2.430167e-05
		1.2	1.464666e-04	2.444493e-05	1.530464e-04	2.428917e-05
		3	1.476104e-04	2.436196e-05	1.532860e-04	2.423555e-05
	1.5	0.8	2.585546e-04	2.160681e-05	2.840629e-04	2.132969e-05
		1.2	2.603468e-04	2.160087e-05	2.845320e-04	2.125238e-05
		3	2.637937e-04	2.158625e-05	2.850186e-04	2.108036e-05
	5	0.8	2.516691e-04	2.558608e-05	2.768564e-04	2.529047e-05
		1.2	2.525376e-04	2.560984e-05	2.768841e-04	2.525312e-05
		3	2.548820e-04	2.561930e-05	2.773052e-04	2.512664e-05

From the table-3, it is observed that the Bayes risk of survival function is maximum under the QLF when β<1 and α>1 than β<1 and α<1 and also maximum when β>1 and α>1 than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is minimum under the WLF when β<1 and α>1 than β<1 and α<1 and maximum when β>1 and α>1 than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is minimum under the SLLF when β<1 and α>1 than β<1 and α<1 and also minimum when β>1 and α>1 than β>1 and α<1 for n=25 and for n=50 and n=100, it is maximum when β<1 and α>1 than β<1 and α<1 and also maximum when β>1 and α>1 than β>1 and α<1. The Bayes risk of survival function is minimum under the PLF when β<1 and α>1than β<1 and α<1 and also minimum when β>1 and α>1than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using informative prior (Lognormal-Inverted Gamma) under PLF is better than using other loss functions (QLF, WLF and SLLF) proposed in this study.

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Lognormal-Inverted Gamma prior a=1.4, b=1.6 and c=1.9) under various loss functions is obtained and presented in Table-4.

Table-4: Estimation of Bayes risk of Survival Function under Lognormal-Inverted Gamma prior with hyperparameters a=1.4, b=1.6 and c=1.9.

n	β	α	$R(\hat{S}(t_i))_{BQ}$	$R(\hat{S}(t_i))_{BW}$	$R(\hat{S}(t_i))_{BSL}$	$R(\hat{S}(t_i))_{BP}$
25	0.5	0.8	5.420854e-04	9.635746e-05	5.982847e-04	9.240910e-05
		1.2	5.448577e-04	9.613147e-05	6.007912e-04	9.221268e-05
		3	5.473815e-04	9.595838e-05	6.010145e-04	9.212214e-05
	1.5	0.8	8.663036e-04	8.504459e-05	1.098819e-03	8.226439e-05
		1.2	8.722252e-04	8.503447e-05	1.103824e-03	8.203854e-05
		3	8.818817e-04	8.544112e-05	1.209283e-03	8.195829e-05
	5	0.8	8.427130e-04	9.850209e-05	1.093128e-03	9.721563e-05
		1.2	8.436748e-04	9.865009e-05	1.092669e-03	9.718416e-05
		3	8.471050e-04	9.934585e-05	1.093746e-03	9.746640e-05

50	0.5	0.8	2.794909e-04	4.909405e-05	2.958945e-04	4.805740e-05
		1.2	2.802919e-04	4.902223e-05	2.965493e-04	4.799285e-05
		3	2.813296e-04	4.894937e-05	2.969344e-04	4.793701e-05
	1.5	0.8	4.726937e-04	4.391935e-05	5.420197e-04	4.315856e-05
		1.2	4.738994e-04	4.393392e-05	5.424934e-04	4.311071e-05
		3	4.761496e-04	4.388361e-05	5.441664e-04	4.291920e-05
	5	0.8	4.562877e-04	5.159702e-05	5.238508e-04	5.127740e-05
		1.2	4.568454e-04	5.163016e-05	5.239518e-04	5.125648e-05
		3	4.588052e-04	5.147373e-05	5.262800e-04	5.098720e-05
100	0.5	0.8	1.474881e-04	2.447651e-05	1.525625e-04	2.417608e-05
		1.2	1.476904e-04	2.446177e-05	1.526667e-04	2.416387e-05
		3	1.482717e-04	2.442915e-05	1.530842e-04	2.413891e-05
	1.5	0.8	2.581480e-04	2.158601e-05	2.840390e-04	2.135749e-05
		1.2	2.588945e-04	2.156454e-05	2.845524e-04	2.131994e-05
		3	2.600769e-04	2.152890e-05	2.854163e-04	2.124858e-05
	5	0.8	2.496743e-04	2.550494e-05	2.766692e-04	2.538828e-05
		1.2	2.498936e-04	2.551373e-05	2.767174e-04	2.538290e-05
		3	2.508025e-04	2.549819e-05	2.772889e-04	2.533598e-05

From the table-4, it is observed that the Bayes risk of survival function is maximum under the QLF and SLLF when β<1 and α>1 than β<1 and α<1 and also maximum when β>1 and α>1 than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is minimum under the WLF and PLF when β<1 and α>1than β<1 and α<1 and also minimum when β>1 and α>1than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using informative prior (Lognormal-Inverted Gamma) under PLF is better than using other loss functions (QLF, WLF and SLLF) proposed in this study.

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Exponential-Gamma prior a_1=2.5, b_1=1.5 and c_1=0.5) under various loss functions is obtained and presented in Table-5.

Table-5: Estimation of Bayes risk of Survival Function under Exponential-Gamma prior with hyperparameters a_1=2.5, b_1=1.5 and c_1=0.5.

n	β	α	$R(\widehat{S}(t_i))_{BQ}$	$R(\widehat{S}(t_i))_{BW}$	$R(\widehat{S}(t_i))_{BSL}$	$R(\widehat{S}(t_i))_{BP}$
25	0.5	0.8	5.211552e-04	9.620724e-05	6.019231e-04	9.348989e-05
		1.2	5.273857e-04	9.606573e-05	5.925748e-03	9.353237e-05
		3	5.439872e-04	9.443907e-05	5.920144e-03	9.273856e-05
	1.5	0.8	8.508235e-04	8.485237e-05	6.417108e-03	8.272073e-05
		1.2	8.677717e-04	8.520040e-05	6.423641e-03	8.194561e-05
		3	9.275480e-04	8.647449e-05	6.412290e-03	7.844240e-05
	5	0.8	8.099268e-04	9.607257e-05	6.399008e-03	9.876921e-05
		1.2	8.181204e-04	9.671725e-05	6.402550e-03	9.849828e-05
		3	8.553700e-04	9.938874e-05	6.413488e-03	9.704632e-05
50	0.5	0.8	2.732493e-04	4.905791e-05	2.956745e-03	4.834683e-05
		1.2	2.753763e-04	4.900236e-05	2.957601e-03	4.833232e-05
		3	2.814574e-04	4.843906e-05	2.955948e-03	4.796124e-05
	1.5	0.8	4.682225e-04	4.387926e-05	3.201177e-03	4.327030e-05
		1.2	4.732930e-04	4.400481e-05	3.201969e-03	4.304697e-05
		3	4.932624e-04	4.420661e-05	3.199878e-03	4.171583e-05
	5	0.8	4.433519e-04	5.069016e-05	3.179187e-03	5.178684e-05
		1.2	4.464206e-04	5.088779e-05	3.180335e-03	5.170567e-05
		3	4.599562e-04	5.137487e-05	3.185550e-03	5.097059e-05
100	0.5	0.8	1.451463e-04	2.447029e-05	1.529198e-04	2.427462e-05
		1.2	1.457648e-04	2.445927e-05	1.530797e-04	2.427899e-05

		3	1.479620e-04	2.430839e-05	1.530586e-04	2.419924e-05
	1.5	0.8	2.563325e-04	2.157231e-05	2.839594e-04	2.139154e-05
		1.2	2.584637e-04	2.157693e-05	2.845654e-04	2.131058e-05
		3	2.660119e-04	2.160074e-05	2.844066e-04	2.095795e-05
	5	0.8	2.457410e-04	2.533264e-05	2.758394e-04	2.550382e-05
		1.2	2.468814e-04	2.537976e-05	2.761308e-04	2.547994e-05
		3	2.521427e-04	2.551528e-05	2.773062e-04	2.529174e-05

From the table-5, it is observed that the Bayes risk of survival function is maximum under the QLF and SLLF when β<1 and α>1 than β<1 and α<1 and also maximum when β>1 and α>1 than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. the Bayes risk of survival function is minimum under the WLF when β<1 and α>1 than β<1 and α<1 and maximum when β>1 and α>1 than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is minimum under the PLF when β<1 and α>1than β<1 and α<1 and also minimum when β>1 and α>1than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using informative prior (Exponential-Gamma) under PLF is better than using other loss functions (QLF, WLF and SLLF) proposed in this study.

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Exponential-Gamma prior a_1=5, b_1=3 and c_1=1.5) under various loss functions is obtained and presented in Table-6.

Table-6: Estimation of Bayes risk of Survival Function under Exponential-Gamma prior with hyperparameters a_1=5, b_1=3 and c_1=1.5.

n	β	α	$R(\widehat{S}(t_i))_{BQ}$	$R(\widehat{S}(t_i))_{BW}$	$R(\widehat{S}(t_i))_{BSL}$	$R(\widehat{S}(t_i))_{BP}$
25	0.5	0.8	5.000930e-04	9.536707e-05	5.915407e-03	9.359942e-05
		1.2	5.118432e-04	9.568133e-05	5.923994e-03	9.426304e-05
		3	5.407610e-04	9.069994e-05	5.909184e-03	9.074951e-05
	1.5	0.8	7.808193e-04	8.029911e-05	1.059929e-03	8.543198e-05
		1.2	8.133532e-04	8.168678e-05	1.083492e-03	8.466854e-05
		3	9.325714e-04	8.543900e-05	1.085775e-03	7.865817e-05
	5	0.8	6.396736e-04	7.283146e-05	8.427888e-04	8.575647e-05
		1.2	6.591121e-04	7.511627e-05	8.761237e-04	8.813808e-05
		3	7.400549e-04	8.402306e-05	9.913904e-04	9.421089e-05
50	0.5	0.8	2.665525e-04	4.885305e-05	2.954281e-04	4.843086e-05
		1.2	2.706481e-04	4.890583e-05	2.974157e-04	4.856353e-05
		3	2.809601e-04	4.717093e-05	2.921712e-04	4.716587e-05
	1.5	0.8	4.430120e-04	4.240674e-05	5.323114e-04	4.404673e-05
		1.2	4.538354e-04	4.289067e-05	5.373697e-04	4.385157e-05
		3	4.950138e-04	4.392062e-05	5.386811e-04	4.177640e-05
	5	0.8	3.766656e-04	4.224801e-05	4.522039e-04	4.770370e-05
		1.2	3.841073e-04	4.306558e-05	4.609921e-04	4.832630e-05
		3	4.140937e-04	4.581535e-05	4.931830e-04	4.982669e-05
100	0.5	0.8	1.429995e-04	2.442438e-05	1.526008e-04	2.428253e-05
		1.2	1.442475e-04	2.444084e-05	1.530363e-04	2.432936e-05
		3	1.479028e-04	2.400786e-05	1.522254e-04	2.403514e-05
	1.5	0.8	2.477180e-04	2.125069e-05	2.816973e-04	2.159450e-05
		1.2	2.519208e-04	2.133786e-05	2.833858e-04	2.151205e-05
		3	2.670463e-04	2.153575e-05	2.838040e-04	2.095023e-05
	5	0.8	2.236050e-04	2.346157e-05	2.758394e-04	2.494005e-05
		1.2	2.263789e-04	2.366681e-05	2.635146e-04	2.503640e-05
		3	2.383571e-04	2.441561e-05	2.714648e-04	2.524242e-05

From the table-6, it is observed that the Bayes risk of survival function is maximum under the QLF when β<1 and α>1 than β<1 and α<1 and also maximum when β>1 and α>1 than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is minimum under the WLF, SLLF and PLF when β<1 and α>1than β<1 and α<1 and maximum when β>1 and α>1than β>1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using non-informative prior (Exponential- Gamma) under PLF is better than using other loss functions (QLF, WLF and SLLF) proposed in this study.

6. Conclusion

In this study, we obtained the Bayes risk of survival function of the two parameter Weibull distribution using non-informative and informative priors as Extension of Jeffrey's, Lognormal-Inverted Gamma and Exponential-Gamma with different loss functions by applying Lindley's approximation and illustrated the methodology through simulation technique. By comparing the estimated values of survival function of Weibull distribution using various loss functions, the risk assuming under PLF is the least one among all the cases studied. It is observed that when the sample size as well as iteration process is increased the Bayes risk of survival function is decreased. Finally, all cases in our study the Weibull model with Exponential-Gamma prior under Precautionary loss function is performed well in posterior risk analysis.

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