SOME STUDY ON STRING COSMOLOGICAL MODEL WITH BULK VISCOSITY IN GENERAL RELATIVITY

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ABSTRACT *In this paper, we study the string cosmological model with bulk viscosity and discuss that the bulk viscous play a significant role in the evolution of universe. To obtain a determinate solution, we assume that the coefficient of the bulk viscosity is a power function of the scalar of expansion* $\xi = k\theta^{a+bm}$ *and the shear scalar is proportional to scalar of expansion* $\sigma \propto \theta$, *which leads to the relation between metric potentials* $\beta = a + b\gamma^{\mu+1}$. *The physical and geometric features of the model are also discussed. It is found that the power index m has significant influence on the string model. There is a big-bang start in the model when* $m \leq 1$ *but there is no any big-bang in the start when* $m > 1$. *In particular when* $m = 0$ *the model reduces to the string model of constant coefficient of bulk viscosity, which was previously given by many researcher.*

Keywords: *String cosmological models, Phase transition, Big-bang, Bulk viscosity, and Scalar of expansion.*

1. Introduction

We review recent progress in string cosmology where string dualities are applied so as to obtain complete cosmological evolution. String theory has several potential applications to cosmology, covering the entire history of the cosmos. One line of inquiry focus on the initial singularity, what happen when the universe was squeezed to a size close to the Plank scale [15,32]. In recent years, there has been considerable interest is string cosmology, as string are believed to have played an important role during early stages of the universe [30, 16] and can generate density fluctuations which lead to galaxy formation [11, 40]. It is still a problem to know the exact physical situation at the very early stages of the formation of the universe. It appears that after the big bang the universe may have undergone a series of phase transitions as its temperature lowered down. During the phase transition, the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects, vacuum domain walls, strings and monopoles [11]. Out of these three topological characteristics, only strings can lead to very interesting cosmological consequences. They are believed to give rise to density perturbation leading to the formation of galaxies [34, 40].As these strings possess stress-energy and are coupled to the gravitational field, that arise from strings. In fact, the general relativistic treatment of strings was initiated by Letelier [14] and Stachel [26].This model has been used as a source for Bianchi type-I and Kantowski-Sachs cosmologies by Letelier [14]. After wards, Krori et.al. [12, 13] and Wang [31,35-38] have discussed the solutions cosmological models for a cloud string. Singh et.al.[25], Turyshev[29] and Vilenkein[30] have presented the exact solutions of Bianchi type –III and spherically symmetric cosmology respectively for a cloud string.

On the other hand, the matter distribution is satisfactorily described by a perfect fluid due to large-scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid [7,10,13]. It is well known that when neutrino decoupling occurs, the matter behaves as a viscous fluid in an early stage of the universe. Recently Bali and Dave [4] have presented Bianchi type-III string cosmological model with bulk viscosity, where the constant coefficient of bulk viscosity is considered. However, it is known that the coefficient of bulk viscosity is not constant but decreases as the universe expands [1, 3, 8]. Arbab [2], Bali and Tinker [6], Pradhan et.al.[17-19], Ray and Mukhopadhay [20], Singh and Singh [21], Singh and Pradhan [22], Singh and Kumar [23-24], Yadav and Pradhan [39] are some of the authors who have studied various aspects of interacting fields in the framework of string cosmological model with bulk viscosity in general relativity.

In this paper, we study the string cosmological model with bulk viscosity. To obtain a determinate solution, we assume that the coefficient of the bulk viscosity is a power function of the scalar of expansion $\xi = k\theta^{a+bm}$ and the shear scalar is proportional to scalar of expansion $\sigma \propto \theta$, which leads to the relation between metric potentials $\beta = a + b\gamma^{\mu+1}$. The physical and geometric features of the model are also discussed.

2. Field Equations and their solutions

The space-time metric we considered here is [5]

(2.1) $$ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{2x} dy^2 + \gamma^2 dz^2$$

where α, β, and γ are only the functions of time t.

The energy-momentum tensor for a cloud of string with bulk viscosity is [5].

(2.2) $$\mathcal{T}_{ij} = \rho u_i u_j - \lambda \mathcal{X}_i \mathcal{X}_j - \xi\theta(u_i u_j + g_{ij})$$

where $\rho = \rho_p + \lambda$, is the rest energy density of the cloud of strings with particles attached to them, ρ_p is the rest energy density of particles, λ is the tension density of the cloud of strings, $\theta = u^i_{;i}$, is the scalar of expansion, and ξ is the coefficient of bulk viscosity. According to Letelier [14] the energy density for the coupled system ρ and ρ_p is restricted to be positive, while the tension density λ may be positive or negative. The vector u^i describes the cloud four-velocity and $\mathcal{X}^i$ represents a direction of anisotropy, i.e. the direction of string. They satisfy the standard relations [14].

(2.3) $$u^i u_j = -\mathcal{X}^i \mathcal{X}_j = -1, \quad u^i \mathcal{X}_i = 0$$

The expressions for scalar of expansion and shear scalar are (kinematical parameters)

(2.4) $$\theta = u^i_{;i} = \frac{\dot\alpha}{\alpha} + \frac{\dot\beta}{\beta} + \frac{\dot\gamma}{\gamma}$$

(2.5) $$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$
$$= \frac{1}{3}\left(\frac{\dot\alpha^2}{\alpha^2} + \frac{\dot\beta^2}{\beta^2} + \frac{\dot\gamma^2}{\gamma^2} - \frac{\dot\alpha\dot\beta}{\alpha\beta} - \frac{\dot\beta\dot\gamma}{\beta\gamma} - \frac{\dot\alpha\dot\gamma}{\alpha\gamma}\right)$$

Einstein's equation we consider here is

(2.6) $$R_{ij} - \frac{1}{2}Rg_{ij} = \mathcal{T}_{ij}$$

where we have choose the units such that c = 1 and 8πG = 1. In the co-moving coordinates $u^i = \delta^i_0$ and $u^i = -\delta^0_i$, and with the help of Eqs. (2.1)- (2.3), the Einstein equation (2.6) can be written as [5]

(2.7) $$\frac{\ddot\beta}{\beta} + \frac{\ddot\gamma}{\gamma} + \frac{\dot\beta\dot\gamma}{\beta\gamma} = \xi\theta$$

(2.8) $$\frac{\ddot\alpha}{\alpha} + \frac{\ddot\gamma}{\gamma} + \frac{\dot\alpha\dot\gamma}{\alpha\gamma} = \xi\theta$$

(2.9) $$\frac{\ddot\alpha}{\alpha} + \frac{\ddot\beta}{\beta} + \frac{\dot\alpha\dot\beta}{\alpha\beta} - \frac{1}{\alpha^2} = \lambda + \xi\theta$$

(2.10) $$\frac{\dot\alpha\dot\beta}{\alpha\beta} + \frac{\dot\beta\dot\gamma}{\beta\gamma} + \frac{\dot\alpha\dot\gamma}{\alpha\gamma} - \frac{1}{\alpha^2} = \rho$$

(2.11) $$\frac{\dot\alpha}{\alpha} - \frac{\dot\beta}{\beta} = 0$$

where the dot denotes the differentiation with respect to time t. From Eq. (2.11), we have

(2.12) $$\boldsymbol{\alpha = H\beta}$$

where H is the constant of integration. In order to obtain a more general solution, we assume

(2.13) $$\boldsymbol{\xi = k\theta^{a+bm}}$$

where a, b, k and m are the positive constants.

We note that the five independent equations (2.8)-(2.11) and (2.13) connect six unknown variables ($\alpha, \beta, \gamma, \lambda, \rho, \xi$). Thus, one more relation connecting these variables is needed to solve these equations. In order to obtain explicit solutions, one additional relation is needed and we adopt an assumption that the shear scalar is proportional to the scalar of expansion $\sigma \propto \theta$, which leads to

(2.14) $$\boldsymbol{\beta = a + b\gamma^{\mu+1}}$$

where μ is a constant.

Now we consider $a = 0$ and $b = 1$, then Equation (2.13) and (2.14) reduces to-

(2.15) $$\xi = k\theta^m$$

(2.16) $$\beta = \gamma^{\mu+1}$$

Substituting Eq. (2.16) into Eq. (2.4) and using Eq. (2.15) we have

(2.17) $$\theta = (2\mu + 3)\frac{\dot{\gamma}}{\gamma}$$

(2.18) $$\xi\theta = \mathcal{K}\frac{\dot{\gamma}^{m+1}}{\gamma^{m+1}}$$

where

(2.19) $$\mathcal{K} = k(2\mu + 3)^{m+1}$$

with the help of Equations (2.16) and (2.18), Eq. (2.7) reduces to

(2.20) $$\frac{\ddot{\gamma}}{\gamma} + \frac{(\mu+1)^2}{\mu+2}\frac{\dot{\gamma}^2}{\gamma^2} = \frac{\mathcal{K}}{\mu+2}\frac{\dot{\gamma}^{m+1}}{\gamma^{m+1}}$$

To solve Eq. (2.20), we denote $\dot{\gamma} = \vartheta$, then $\ddot{\gamma} = \vartheta\frac{d\vartheta}{d\gamma}$, and the Eq. (2.20) can be reduced to the first-order differential equation in the following form:

(2.21) $$\frac{d\vartheta}{d\gamma} + l\frac{\vartheta}{\gamma} = \frac{\mathcal{K}}{\mu+2}\frac{\vartheta^m}{\gamma^m}$$

where

(2.22) $$l = \frac{(\mu+1)^2}{\mu+2}$$

Equation (2.21) can be written as (for m ≠ 1)

(2.23) $$\frac{d}{d\gamma}\left(\vartheta^{1-m}\gamma^{(1-m)l}\right) = \frac{(1-m)\mathcal{K}}{\mu+2}\gamma^{(1-m)l-m}$$

Thus the solution of Esq. (2.21) can easily be obtained:

(2.24) $$\vartheta = \left[\frac{\mathcal{K}\gamma^{1-m}}{(\mu+2)(l+1)} + D\gamma^{(m-1)l}\right]^{\frac{1}{1-m}}$$

where D is the constant of integration. With the help of Esq. (2.24), the line-element (2.1) reduces to

(2.25) $$ds^2 = -\left[\frac{\mathcal{K}\gamma^{1-m}}{(\mu^2+3\mu+3)} + D\gamma^{(m-1)l}\right]^{\frac{-2}{1-m}}d\gamma^2$$

$$+ H^2\gamma^{2\mu+2}dx^2 + \gamma^{2\mu+2}e^{2x}dy^2 + \gamma^2dz^2$$

Under suitable transformation of coordinates, Eq. (2.25) reduces to

(2.26) $$ds^2 = -\left[\frac{\mathcal{K}\mathcal{T}^{1-m}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)l}\right]^{\frac{-2}{1-m}}d\mathcal{T}^2$$

$$+ H^2\mathcal{T}^{2\mu+2}dx^2 + \mathcal{T}^{2\mu+2}e^{2x}dy^2 + \mathcal{T}^2dz^2$$

For the model of Eq. (2.26), the other physical and geometrical parameters can easily be obtained. The expressions for the energy density ρ, the string tension density λ, the particle density ρ_p, the coefficient of bulk viscosity ξ, the scalar of expression θ and the shear scalar σ^2 are, respectively, given by

(2.27) $$\rho = (\mu + 1)(\mu + 3).\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)(l+1)}\right]^{\frac{2}{1-m}} - \frac{\mathcal{T}^{-(2\mu+2)}}{H^2}$$

(2.28) $$\lambda = \frac{\mu}{1-m}.\left[\frac{\mathcal{K}\mathcal{T}^{1-m}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)l}\right]^{\frac{1+m}{1-m}}$$

$$.\left[\frac{(1-m)\mathcal{K}\mathcal{T}^{-(1+m)}}{(\mu^2+3\mu+3)} + D(m-1)l\mathcal{T}^{(m-1)l-2}\right]$$

$$+ \mu(2\mu+2).\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)(l+1)}\right]^{\frac{2}{1-m}} - \frac{\mathcal{T}^{-(2\mu+2)}}{H^2}.$$

(2.29) $$\rho_p = \frac{-\mu}{1-m}.\left[\frac{\mathcal{K}\mathcal{T}^{1-m}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)l}\right]^{\frac{1+m}{1-m}}$$

$$\times\left[\frac{(1-m)\mathcal{K}\mathcal{T}^{-(1+m)}}{(\mu^2+3\mu+3)} + D(m-1)l\mathcal{T}^{(m-1)l-2}\right]$$

$$+ (\mu+1)(3-\mu).\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)(l+1)}\right]^{\frac{2}{1-m}}$$

(2.30) $$\theta = (2\mu+3).\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)(l+1)}\right]^{\frac{1}{1-m}}$$

(2.31) $$\sigma^2 = \frac{\mu^2}{3}.\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)(l+1)}\right]^{\frac{2}{1-m}}$$

From Eq. (2.27) it is observed that the standard condition $\rho \geq 0$ is fulfilled when

(2.32) $$(\mu+1)(\mu+3)\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{(m-1)(l+1)}\right]^{\frac{2}{1-m}} \geq \frac{\mathcal{T}^{-(2\mu+2)}}{H^2}$$

It is seen that in the case m < 1, the scalar of expansion θ tends to infinitely large and the energy density $\rho \to \infty$ when $\mathcal{T} \to 0$, but θ tends to finite and ρ tends to finite when $\mathcal{T} \to \infty$ due to the presence of bulk viscosity (in the absence of bulk viscosity $\mathcal{K} = 0$, $\theta \to 0$ and $\rho \to 0$ when $\mathcal{T} \to \infty$). Hence the model represents the shearing and non-rotating expanding universe with the big-bang start. However, in the case m > 1, it is observed that $\theta \to 0$ when $\mathcal{T} \to \infty$, but θ tends to finite when $\mathcal{T} \to 0$ due to the presence of bulk viscosity (in the absence of bulk viscosity $\mathcal{K} = 0$, $\theta \to \infty$ when $\mathcal{T} \to 0$). Here $\rho \to \infty$ when $\mathcal{T} \to 0$ and $\rho \to 0$ when $\mathcal{T} \to \infty$. Therefore the model describes a shearing non rotating expanding universe without the big-bang start. We can see from the above discussion that the bulk viscosity plays a significant role in the evolution of universe [37, 38]. Furthermore, since $\lim_{\mathcal{T}\to\infty} \frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large values of $\mathcal{T}$. The shear scalar σ is zero when $\mu = 1$, hence $\mu = 1$ is the isotropy condition.

- ***Special Case*** :-

If we consider $m = 0$, then the model (2.26) reduces to the string model of constant coefficient of bulk viscosity. That is

(2.33) $$ds^2 = -\left[\frac{\mathcal{K}\mathcal{T}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{-l}\right]^{-2} d\mathcal{T}^2$$

$$+ H^2\mathcal{T}^{2\mu+2}dx^2 + \mathcal{T}^{2\mu+2}e^{2x}dy^2 + \mathcal{T}^2dz^2$$

(2.34) $$\rho = (\mu+1)(\mu+3).\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{-(l+1)}\right]^2 - \frac{\mathcal{T}^{-(2\mu+2)}}{H^2}$$

(2.35) $$\lambda = \left[\frac{(2\mu^2+3\mu)\mathcal{K}}{(\mu^2+3\mu+3)} + \frac{\mu(\mu+1)(\mu+3).D\mathcal{T}^{-(l+1)}}{\mu+2}\right].\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{-(l+1)}\right] - \frac{\mathcal{T}^{-(2\mu+2)}}{H^2}.$$

(2.36) $$\rho_p = \left[\frac{(\mu-\mu^2+3)\mathcal{K}}{(\mu^2+3\mu+3)} + \frac{2(\mu+1)(\mu+3).D\mathcal{T}^{-(l+1)}}{\mu+2}\right].\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{-(l+1)}\right]$$

(2.37) $$\theta = (2\mu+3).\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{-(l+1)}\right]$$

(2.38) $$\sigma^2 = \frac{\mu^2}{3}.\left[\frac{\mathcal{K}}{(\mu^2+3\mu+3)} + D\mathcal{T}^{-(l+1)}\right]^2$$

In fact, this model is the result previously given by Bali and Dave [5] and has already been discussed by them, but the proper time was used there.

In the second special case, we assume that $\boldsymbol{m = 1}$, then Eq. (2.21) reduces to

(2.39) $$\frac{d\theta}{d\gamma} + f\frac{\theta}{\gamma} = 0$$

where

(2.40) $$f = \frac{(\mu+1)^2 - \mathcal{K}}{\mu+2}$$

Integration of Eq. (2.46) gives

(2.41) $$\vartheta = D\gamma^{-f}$$

where D is the constant of integration. In the same way as performed above, we can easily obtain

(2.42) $$ds^2 = -D^{-2}\mathcal{T}^{2f}\,d\mathcal{T}^2 + H^2\mathcal{T}^{2\mu+2}dx^2 + \mathcal{T}^{2\mu+2}e^{2x}dy^2 + \mathcal{T}^2dz^2$$

(2.43) $$\rho = (\mu+1)(\mu+3).D^2\mathcal{T}^{-2(f+1)} - \frac{\mathcal{T}^{-(2\mu+2)}}{H^2}$$

(2.44) $$\lambda = \mu(2\mu-f+2).D^2\mathcal{T}^{-2(f+1)} - \frac{\mathcal{T}^{-(2\mu+2)}}{H^2}$$

(2.45) $$\rho_p = [(\mu+1)(3-\mu) + \mu f].D^2\mathcal{T}^{-2(f+1)}$$

(2.46) $$\theta = (2\mu+3).D\mathcal{T}^{-(f+1)}$$

(2.47) $$\sigma^2 = \frac{\mu^2}{3}.D^2\mathcal{T}^{-2(f+1)}$$

From Eq. (2.50) it is observed that the standard condition $\rho \geq 0$ is fulfilled when

(2.48) $$(\mu+1)(\mu+3).D^2\mathcal{T}^{-2(f+1)} \geq \frac{\mathcal{T}^{-(2\mu+2)}}{H^2}.$$

3. ***Disscusion***

The scalar of expansion θ is infinitely large at $\mathcal{T} = 0$, and θ tends to zero when $\mathcal{T} \to \infty$ provided $f + 1 > 0$, hence the model represents the shearing and non-rotating expanding universe with the big-bang start. The energy density $\rho \to \infty$ when $\mathcal{T} \to 0$, ρ tends to zero when $\mathcal{T} \to \infty$. Furthermore, since $\lim_{\mathcal{T}\to\infty}\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large values of $\mathcal{T}$. In addition, we observe from Eqs. (2.15), (2.45), and (2.46) that the relation between the coefficient of bulk viscosity and the energy density of particles $\xi = \rho_p{}^{1/2}$ in this model, and such a relation is believed to be reasonable and has been considered in many papers [5,15].

4. *Summary*

In this chapter, we study the Bianchi type-III string cosmological model with bulk viscosity. To obtain a determinate solution, we assume that the coefficient of the bulk viscosity is a power function of the scalar of expansion $\xi = k\theta^{a+bm}$ and the shear scalar is proportional to scalar of expansion $\sigma \propto \theta$, which leads to the relation between metric potentials $\beta = a + b\gamma^{\mu+1}$. The physical and geometric features of the model are also discussed. It is found that the power index m has significant influence on the string model. There is a big-bang start in the model when m ≤ 1 but there is no any big-bang in the start when m > 1 [37, 38]. In particular when m = 0 the model reduces to the string model of constant coefficient of bulk viscosity, which was previously given by Bali and Dave [5].

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