On Intuitionistic Fuzzy Extended Modal Operators for Negation in Medical Diagnosis

A. Edward Samuel\textsuperscript{1} and S. Rajakumar\textsuperscript{2}

\textsuperscript{1,2}Ramanujan Research Centre, P.G. & Research Department of Mathematics, Government Arts College (Autonomous), Kumbakonam, Tamil Nadu, S. India. Pin Code 612 002.

Received: June 02, 2018 \hspace{0.2in} Accepted: July 23, 2018

ABSTRACT

This paper we propose a new approach for Medical diagnosis with the symptoms of disease using IFS with Extended Modal Operators For Negation. This operators apply to identified the disease of the patient with symptoms in the data. The membership and non-membership values are not always possible upto our satisfaction, but in deterministic (hesitation) part has more important role here, the fact that in decision making, particularly in case of medical diagnosis, there is a fair chance of the existence of a non-zero hesitation part at each moment of valuation.

Keywords: Intuitionistic Fuzzy set(IFS), Intuitionistic Fuzzy Relation (IFR), Extended Modal Operators For Negation (EMON), Intuitionistic Medical Diagnosis(IMD).

1. Introduction

The field of medicine is one of the best areas of application of fuzzy set theory. In the discrimination analysis, the symptoms are ranked according to the grade of discrimination of each disease by a particular symptom. In real world, we frequently deal with vague or imprecise information. Information available is sometimes vague, sometimes inexact or sometimes insufficient. Out of several higher order fuzzy sets, intuitionistic fuzzy sets\cite{1,2} have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Due to the same reason, evaluation of non-membership values is not always possible and consequently there remains a part indeterministic on which hesitation survives.

Certainly fuzzy set theory is not appropriate to deal with such problem, rather intuitionistic fuzzy set theory is more suitable. Out of several generalizations of fuzzy set theory for various objectives, the notion introduced by Atanassov\cite{1} in defining intuitionistic fuzzy sets is interesting and useful. Fuzzy sets are intuitionistic fuzzy sets but the converse is not necessarily true\cite{1}. In fact, there are situations where intuitionistic fuzzy set theory is more appropriate to deal with\cite{5}. Besides, it has been cultured in\cite{6} that vague sets\cite{8} are nothing but intuitionistic fuzzy sets.

In the present paper we study Sanchez’s method\cite{9} for medical diagnosis using intuitionistic fuzzy extended modal operators for negation [EMON] in\cite{3}. The method of intuitionistic medical diagnosis[IMD] involves intuitionistic fuzzy relations [IFR] as defined in\cite{4}.

2. Preliminaries

We give here some basic definitions, which are used in our next section.

2.1 Definition

Let a set E be fixed. An intuitionistic fuzzy set (IFS) A in E is an object having the form. $\tilde{A} = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\}$ where the function $\mu_A : E \to [0,1]$ and $\gamma_A : E \to [0,1]$ define the degree of membership and degree of non-membership respectively of the element $x \in E$ to the set A which is a subset of E and for every $x \in E$, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

The amount $\pi_A(x) = 1 - (\mu_A(x) + \gamma_A(x))$ is called the hesitation part which may cater to either membership value or non-membership value or both.

3. Methodology

3.1 Definition

If A and B are two IFS of the set E, then let $A = \{(x, \mu_A(x), \gamma_A(x)) / x \in E\}$ and $B = \{(x, \mu_B(x), \gamma_B(x)) / x \in E\}$ be two intuitionistic fuzzy set the cartesian product of these two intuitionistic fuzzy sets the set $A \times B = \{(\mu_A(x) \times \mu_B(x), \gamma_A(x) \times \gamma_B(x)) / x \in E\}$

3.2 Definition

If A and B are two IFS of the set E, then
3.3 Definition
An operator over an intuitionistic fuzzy set $A$ (IFS $A$), given the fixed numbers $\alpha$, $\alpha \in [0,1]$, as
\[
d_a(A) = \{x \in E : (\gamma_A(x) + \alpha \cdot \pi_A(x), \mu_A(x) + (1 - \alpha) \cdot \pi_A(x)) / x \in E\}.
\]

3.4 Definition
An operator over an intuitionistic fuzzy set $A$ (IFS $A$), given the fixed numbers $\alpha, \beta$, $\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$, as
\[
f_{\alpha,\beta}(A) = \{x \in E : (\gamma_A(x) + \alpha \cdot \pi_A(x), \mu_A(x) + \beta \cdot \pi_A(x)) / x \in E\}.
\]

4. Medical Diagnosis
Suppose S is a set of symptoms, D is a set of Disease and P is a set of patient. Let $E_1$ be an intuitionistic fuzzy relations [IFR] $E_1 (P \rightarrow S)$ and $E_2$ be an intuitionistic fuzzy relations [IFR] $E_2 (S \rightarrow D)$. Then
\[
E_3 = A \times B = \{(\mu_A(x) \times \mu_B(x), \gamma_A(x) \times \gamma_B(x)) / x \in E\}
\]
\[
E_4 = A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))) / x \in E\}
\]
\[
E_5 = d_\alpha(A) = \{(x, (\gamma_A(x) + \alpha \cdot \pi_A(x), \mu_A(x) + (1 - \alpha) \cdot \pi_A(x)) / x \in E\}.
\]
Here $\alpha = 0.5$
\[
E_6 = f_{\alpha,\beta}(A) = \{(x, (\gamma_A(x) + \alpha \cdot \pi_A(x), \mu_A(x) + \beta \cdot \pi_A(x)) / x \in E\}.
\]
Here $\alpha, \beta = 0.5$
\[
E_7 = \mu_A(x) \lor \gamma_A(x) = \max(\mu_A(x), \gamma_A(x))
\]

4.1 Algorithm
Step 1:
The formula $E_3$ and $E_4$ are applied in Table 1 and Table 2, and get the results is named Table 3.

Step 2:
The Table 3 values are applied in the formula $E_5$ and $E_6$ individually, we get the results is named Table 4.

Step 3:
The Table 4 values applied in $E_7$ and get the result is named Table 5.

Step 4:
Finally, we select the maximum value from (Table 5) each row, and then we conclude that the Patients $P_i (i = 1, 2, 3, 4)$ is suffering from the Disease $D_j (j = 1, 2, 3, 4, 5)$

4.2 Case Study
Let there be four Patients $P = \{P_1, P_2, P_3, P_4\}$ and the set of symptoms $S=\{\text{Temperature, Headache, Stomach pain, Cough, Chest pain}\}$ and the set of Diseases $D=\{\text{Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}\}$

<table>
<thead>
<tr>
<th>$E_1$</th>
<th>Temperature</th>
<th>Headache</th>
<th>Stomach pain</th>
<th>Cough</th>
<th>Chest pain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>(0.8,0.1)</td>
<td>(0.6,0.1)</td>
<td>(0.2,0.8)</td>
<td>(0.6,0.1)</td>
<td>(0.1,0.6)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>(0.0,0.8)</td>
<td>(0.4,0.4)</td>
<td>(0.6,0.1)</td>
<td>(0.1,0.7)</td>
<td>(0.1,0.8)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>(0.8,0.1)</td>
<td>(0.8,0.1)</td>
<td>(0.0,0.6)</td>
<td>(0.2,0.7)</td>
<td>(0.0,0.5)</td>
</tr>
<tr>
<td>$P_4$</td>
<td>(0.6,0.1)</td>
<td>(0.5,0.4)</td>
<td>(0.3,0.4)</td>
<td>(0.7,0.2)</td>
<td>(0.3,0.4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$E_2$</th>
<th>Viral fever</th>
<th>Malaria</th>
<th>Typhoid</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>(0.4,0.0)</td>
<td>(0.7,0.0)</td>
<td>(0.3,0.3)</td>
<td>(0.1,0.7)</td>
<td>(0.1,0.8)</td>
</tr>
<tr>
<td>Headache</td>
<td>(0.3,0.5)</td>
<td>(0.2,0.6)</td>
<td>(0.6,0.1)</td>
<td>(0.2,0.4)</td>
<td>(0.0,0.8)</td>
</tr>
<tr>
<td>Stomach pain</td>
<td>(0.1,0.7)</td>
<td>(0.0,0.9)</td>
<td>(0.2,0.7)</td>
<td>(0.8,0.0)</td>
<td>(0.2,0.8)</td>
</tr>
<tr>
<td>Cough</td>
<td>(0.4,0.3)</td>
<td>(0.7,0.0)</td>
<td>(0.2,0.6)</td>
<td>(0.2,0.7)</td>
<td>(0.2,0.8)</td>
</tr>
<tr>
<td>Chest pain</td>
<td>(0.1,0.7)</td>
<td>(0.1,0.8)</td>
<td>(0.1,0.9)</td>
<td>(0.2,0.7)</td>
<td>(0.8,0.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P_i$</th>
<th>Viral fever</th>
<th>Malaria</th>
<th>Typhoid</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>(0.32,0.00)</td>
<td>(0.56,0.00)</td>
<td>(0.42,0.00)</td>
<td>(0.16,0.00)</td>
<td>(0.12,0.06)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>(0.12,0.00)</td>
<td>(0.42,0.00)</td>
<td>(0.24,0.04)</td>
<td>(0.48,0.00)</td>
<td>(0.12,0.08)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>(0.32,0.00)</td>
<td>(0.56,0.00)</td>
<td>(0.48,0.00)</td>
<td>(0.16,0.00)</td>
<td>(0.08,0.05)</td>
</tr>
</tbody>
</table>
Table 4

<table>
<thead>
<tr>
<th></th>
<th>Viral fever</th>
<th>Malaria</th>
<th>Typhoid</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>(0.66,0.34)</td>
<td>(0.78,0.22)</td>
<td>(0.71,0.29)</td>
<td>(0.58,0.42)</td>
<td>(0.53,0.47)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>(0.56,0.44)</td>
<td>(0.71,0.29)</td>
<td>(0.60,0.40)</td>
<td>(0.74,0.26)</td>
<td>(0.52,0.48)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>(0.66,0.34)</td>
<td>(0.78,0.22)</td>
<td>(0.73,0.26)</td>
<td>(0.74,0.26)</td>
<td>(0.51,0.48)</td>
</tr>
<tr>
<td>$P_4$</td>
<td>(0.64,0.36)</td>
<td>(0.74,0.25)</td>
<td>(0.63,0.36)</td>
<td>(0.62,0.38)</td>
<td>(0.62,0.38)</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th></th>
<th>Viral fever</th>
<th>Malaria</th>
<th>Typhoid</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.66</td>
<td>\textbf{0.78}</td>
<td>0.71</td>
<td>0.58</td>
<td>0.53</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.56</td>
<td>0.71</td>
<td>0.60</td>
<td>\textbf{0.74}</td>
<td>0.52</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.66</td>
<td>\textbf{0.78}</td>
<td>0.73</td>
<td>0.74</td>
<td>0.51</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.64</td>
<td>\textbf{0.74}</td>
<td>0.63</td>
<td>0.62</td>
<td>0.62</td>
</tr>
</tbody>
</table>

From Table 5, we see that the maximum value of $P_1$, $P_3$, and $P_4$ is 0.78 and 0.74 respectively, therefore they suffer from Malaria. Whereas the maximum value of $P_2$ is 0.74 and therefore $P_2$ faces a Stomach problem.

5. Conclusion

In this paper, a new technique to diagnose the symptom of the disease using intuitionistic fuzzy set is proposed and it is successful and effective to solve many problems faced by patients.

References

2. Atanassov, K., New operations defined over intuitionistic fuzzy sets, Fuzzy sets and systems, 61, 137-142 (1994).