A Collaborative Vendor – Buyer Deteriorating Inventory Model for Quadratic Demand under Trade Credit.

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Received: June 15, 2018 Accepted: July 28, 2018

ABSTRACT *This paper explores collaborative vendor – buyer inventory model when the deteriorating rate is constant and vendor offers trade credit to attract buyer's more order quantity. Here it is assumed that demand rate is a quadratic function of time. The maximization of the total profit per unit of time is taken as the objective function to study the retailer's optimal ordering policy. A model is described to find the best optimal strategy to determine buyer's order quantity and optimum cycle length of the inventory system. Numerical examples are given to validate the proposed inventory model. Sensitivity analysis is carried out to examine important model parameters.*

Keywords: *Collaborative vendor-buyer inventory model, deterioration, trade credit, quadratic demand.*

1. Introduction:

One of the important problems faced in inventory management is deterioration which is defined as a physical phenomena which hinders an item from being used for its original purpose. Spoilage, as in perishable food stuffs, fruits and vegetables; physical depletion, as in pilferage or evaporation of volatile liquids such as gasoline, perfumes, alcohol; decay as in radioactive substances, degradation, as in electronic components or loss of potency as in photographic films, pharmaceutical drugs, fertilizers etc are the example of deterioration. Inventory control of the items which deteriorate during the time is a research area with good scope. The analysis of decaying inventory problem began with Ghare and Schrader [1963], who proposed an inventory model having a constant rate of deterioration and constant rate of demand over a finite-planning horizon. It is now a well-established fact in literature and its effect cannot be ignored. One can refers article by Chang and Dye [2001],Chung [2000], Covert and Phillip [1973], Hwang and Shinn [1997], Rafaat [1991].

Further in conventional business transactions, it was implicitly assumed that the buyers must pay for the procured items as soon as they are received. However, in today's intensely competitive market, such an assumption would no longer be practical. Trade credit is now an invaluable promotional tool for the vendors to increase profit through stimulating more sales and a unique opportunity for the buyers to reduce demand uncertainty and its associated risks. In other words, when the vendor sends the ordered units to the buyer without being paid, he actually transfers the storage responsibility and costs to the buyers, while bearing the demand uncertainty risk. In recent years trade credit has been extensively studied. One can refer review article on trade credit by Kawale and Sanas (2017). Goyal (1985) was the first to develop an EOQ model with a constant demand rate under the condition of permissible delay in payments. The research article by Jaggi and Aggarwal (1994), Hwang and Shinn (1997), Jamal et al. (1997), Sarker et al. (2000), Liao et al. (2000, 2012), Singh and Pattanayak (2012), Guchhait et al. (2013) deals with offer of trade credit and deterioration.

Above stated studies neglected interaction and cooperation opportunity between the vendor and the buyer. However, it was found that cooperation between the players of the supply chain create a win win strategy to become partners and to resolve conflicting relationship. In a pioneering effort, Goyal (1976) developed a single-supplier single-retailer integrated inventory model. Banerjee (1986) extended Goyal's (1976) model and assumed that the supplier followed a lot-for-lot shipment policy with respect to a retailer. Later, Goyal (1988) relaxed the lot-for-lot policy and illustrated that the inventory cost could be significantly reduced if the supplier's economic production quantity (EPQ) was an integer multiple of the retailer's purchase quantity. Lu (1995) then generalized Goyal's (1988) model by relaxing the assumption that the supplier could supply the retailer only after completing the entire lot size. Several works have continued for integrated inventory models. Furthermore, Abad and Jaggi (2003) is the first who offered a supplier–buyer

integrated model following a lot-for-lot shipment policy under a permissible delay in payment. Ouyang et al. (2005), Sarmah et al. (2007) developed a stylized model to find the optimal strategy for the integrated vendor–buyer inventory model considering credit policy as coordination mechanism between buyer and vendor.

In this article, extending the previous studies we develop a collaborative vendor- buyer inventory model when demand rate is quadratic of the time. To maximize the joint total profit per unit time, two basic issues will be determined in this study. These issues are how large should the replenishment order be, and what should be the cycle time. An interactive procedure is developed to determine the optimal solution. Finally, numerical examples and sensitivity analysis are presented to illustrate the proposed model.

2. Notations

The following notations are used in the proposed article:

S_v: Vendor's set up cost per set up.
S_b: Buyer's ordering cost per order.
C_v: Production cost per unit.
C_b: Buyer's purchase cost per unit.
C_c: The unit retail price to customers where $C_c > C_b > C_v$.
I_v: Vendor's inventory holding cost rate per unit per annum, excluding interest charges.
I_b: Buyer's inventory holding cost rate per unit per annum, excluding interest charges.
I_{v0}:Vendor's opportunity cost/$/unit time.
I_{b0}: Buyer's opportunity cost/$/unit time.
I_{be}: Buyer's interest earned/$/unit time.
ϱ: Capacity utilisation which is ratio of demand to the production rate, $\varrho < 1$ and known constant.
M: Allowable credit period for the buyer offered by the vendor.
Q: Buyer's order quantity.
T: cycle time (decision variable).
n : Number of shipments from vendor to the buyer.
θ: constant rate of deterioration.
TVP: Vendor's total profit per unit time.
TBP: Buyer's total profit per unit time.
π : TVP + TBP Joint total profit per unit time.

3. Assumptions

In addition, the following assumptions are made in derivation of the model:

- The supply chain under consideration comprise of single vendor and single buyer for a single product.
- Shortages are not allowed.
- The demand rate considered is time dependent, increasing demand rate. The constant part of quadratic demand pattern changes with each cycle.
- Replenishment rate is instantaneous for buyer
- The units in inventory are subject to deteriorate at a constant rate of θ, $0 < \theta < 1$. The deteriorated units can neither be repaired nor replaced during the cycle time.
- Finite production rate.
- Vendor produces the nQ items and then fulfils the buyer's demand, so at the beginning of production item, there is small possibility of deterioration in general. Moreover vendor is a big merchant who can have capacity to prevent deterioration. So in this model, deterioration cost is considered for buyer only at the rate θ is assumed to be constant.
- Vendor offers the buyer a permissible delay period M. During this permissible delay period, the buyer sells the items and uses the sales revenue to earn interest at a rate of I_{be} /unit/annum. At the end of this time period buyer settles the payments due against the purchase made and incurs opportunity cost at a rate of I_{b0} /unit/annum for unsold items in stock.

4. Mathematical Formulation

Let *I(t)* be the inventory level at any time *t*, ($0 \le t \le T$). Depletion due to deterioration and demand will occur simultaneously. The differential equation describing the instantaneous state of *I(t)* over (0,*T*) is given by:

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt + ct^2) \qquad 0 \le t \le T \qquad (1)$$

Solution to the equation (1) (using the boundary condition *I(t)* = 0 at *t* = *T*) is given by

$$I(t) = \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(e^{\theta(T-t)} - 1\right) + \left(\frac{b}{\theta} - \frac{2c}{\theta}\right)\left(Te^{\theta(T-t)} - t\right) + \frac{c}{\theta}\left(T^2 e^{\theta(T-t)} - t^2\right) \quad 0 \le t \le T \qquad (2)$$

Also at t = 0 I(t) = Q

$$Q = \left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)(e^{\theta T} - 1) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T} \qquad 0 \le t \le T \qquad (3)$$

4.1 Vendor's total profit per unit time

1. Sales revenue: the total sales revenue per unit time is $(C_b - C_V)\frac{Q}{T}$.

$$= \frac{(C_b - C_V)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)(e^{\theta T} - 1) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T}\right. \qquad (4)$$

2. Set-up cost : nQ units are manufactured in one production run by the vendor. Therefore the setup cost per unit time is $\frac{S_v}{nT}$
3. Holding cost : using method given by Joglekar (1988) vendor's average inventory per unit time is $\frac{C_v(I_v + I_{v0})}{T}[(n-1)(1-\varrho) + \varrho]\left\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3} - \frac{2c}{\theta^3}\right)(1 + \theta T - e^{\theta T}) + \left(-\frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(T - Te^{\theta T} + \frac{\theta T^2}{2}\right) - \frac{c}{\theta^2}\left(T^2 - T^2 e^{\theta T} + \theta\frac{T^3}{3}\right)\right\}$ (5)
4. Opportunity cost : opportunity cost per unit time because of offering permissible delay period is $\frac{C_b I_{v0} MQ}{T}$

$$= \frac{C_b I_{v0} M}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)(e^{\theta T} - 1) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T}\right\} \qquad (6)$$

Hence the total profit per unit time for vendor is = Sales revenue – Set up cost – Holding cost – Opportunity cost.

TVP = $\frac{(C_b - C_V)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)(e^{\theta T} - 1) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T} - \frac{S_v}{nT} - \frac{C_v(I_v + I_{v0})}{T}[(n-1)(1-\varrho) + \varrho]\left\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3} - \right.\right.$ *2cθ31+θT-eθT+(-bθ2+2cθ2)(T- TeθT+θT22) – cθ2(T2-T2eθT+θT33)}* - $\frac{C_b I_{v0} M}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)(e^{\theta T} - 1) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T}\right\}$ (7)

4.2 Buyer's total profit per unit time

1. Sales revenue: The total sales revenue per unit time is $\frac{(C_c - C_b)Q}{T}$

$$= \frac{(C_c - C_b)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)(e^{\theta T} - 1) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T}\right\} \qquad (8)$$

2. Ordering cost : Ordering cost per unit time is $\frac{S_b}{T}$
3. Holding cost: The buyer's holding cost (excluding interest charges) per unit time is $\frac{C_b I_b}{T}\left\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3} - \frac{2c}{\theta^3}\right)(1 + \theta T - e^{\theta T}) + \left(-\frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(T - Te^{\theta T} + \frac{\theta T^2}{2}\right) - \frac{c}{\theta^2}\left(T^2 - T^2 e^{\theta T} + \theta\frac{T^3}{3}\right)\right\}$ (9)
4. Deteriorating cost : Deteriorating cost per unit time is $\frac{C_b}{T}\left[Q - \int_0^T (a + bt + ct^2)dt\right]$

$$= \frac{C_b}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)(e^{\theta T} - 1) + \left(\frac{bT}{\theta} + \frac{cT^2}{\theta^2} - \frac{2cT}{\theta}\right)e^{\theta T} - aT - \frac{bT^2}{2} - \frac{cT^3}{3}\right\} \qquad (10)$$

Based on the length of the credit period offered by the vendor, two cases arise namely $M < T$ and $M \ge T$.

Case 1] When $M < T$

In this case buyer starts getting the sales revenue and earns interest on average sales revenue for the time period till M, at M accounts are settled, if the stock still remains, finances are to be arranged to make payments to the vendor.

5. Interest earned per unit time during the credit period [0, M] is $\frac{I_{be} C_c}{T}\int_0^M (a + bt + ct^2)tdt$

$$= \frac{I_{be} C_c}{T}\left[\frac{aM^2}{2} + \frac{bM^3}{3} + \frac{cM^4}{4}\right] \qquad (11)$$

6. Interest payable per unit time during time span [M, T] is $\frac{C_b I_{b0}}{T}\int_M^T I(t)dt$

$$= \frac{C_b I_{b0}}{T}\left\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3} - \frac{2c}{\theta^3}\right)\left(1 + \theta(T - M) - e^{\theta(T-M)}\right) + \left(-\frac{b}{\theta^2} + \frac{2c}{\theta^2}\right)\left(T - Te^{\theta(T-M)} + \theta\left(\frac{T^2}{2} - \frac{M^2}{2}\right)\right) - \frac{c}{\theta^2}\left(T^2 - T^2 e^{\theta(T-M)} + \theta\left(\frac{T^3}{3} - \frac{M^3}{3}\right)\right)\right\} \qquad (12)$$

Therefore profit of the buyer in this case can be expressed as :-

TBP1= Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned – Interest paid.

$$= \frac{(C_c-C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\} - \frac{S_b}{T}-\frac{C_bI_b}{T}\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}-\frac{2c}{\theta^3}\right)(1+\theta T-e^{\theta T})+\left(-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)-\frac{c}{\theta^2}\left(T^2-T^2e^{\theta T}+\theta\frac{T^3}{3}\right)\right\}-\frac{C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}-aT-\frac{bT^2}{2}-cT33+IbeCcTaM22+bM33+cM44-CbIb0T\{-a\theta 2+b\theta 3-2c\theta 31+\theta T-M-e\theta T-M+(-b\theta 2+2c\theta 2)(T-Te^{\theta(T-M)}+\theta(\frac{T^2}{2}-\frac{M^2}{2}))-\frac{c}{\theta^2}(T^2-T^2e^{\theta(T-M)}+\theta(\frac{T^3}{3}-\frac{M^3}{3}))\}$$

(13)

Case2] When M ≥ T.

The first 4 components of the profit function remain same. The sixth cost component does not exist for M ≥ T. The interest earned per unit time during time span [0, M] is

$$\frac{I_{be}C_c}{T}\left\{\int_0^T(a+bt+ct^2)tdt+Q(M-T)\right\}$$

$$=\frac{I_{be}C_c}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}\right]+\frac{I_{be}C_c}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\}(M-T) \qquad (14)$$

In this case profit for the buyer is given by

TBP2 = Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned.

$$TBP2 = \frac{(C_c-C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}\right\} - \frac{S_b}{T}-\frac{C_bI_b}{T}\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}-\frac{2c}{\theta^3}\right)(1+\theta T-e^{\theta T})+\left(-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)-\frac{c}{\theta^2}\left(T^2-T^2e^{\theta T}+\theta\frac{T^3}{3}\right)\right\}-\frac{C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}+\frac{2c}{\theta^2}\right)(e^{\theta T}-1)+\left(\frac{bT}{\theta}+\frac{cT^2}{\theta^2}-\frac{2cT}{\theta}\right)e^{\theta T}-aT-bT22-cT33+IbeCcTaT22+bT33+cT44+IbeCcTa\theta-b\theta 2+2c\theta 2e\theta T-1+bT\theta+cT2\theta 2-2cT\theta e\theta T(M-T) \qquad (15)$$

4.3 Joint total profit per unit time

In integrated system, the vendor and the buyer to take joint decision which maximizes the profit of the supply chain, the joint total profit per unit time for integrated system is

$$\pi = \pi 1 = TVP + TBP1 \qquad M < T$$

$$\pi 2 = TVP + TBP2 \qquad M \geq T$$

Considering $e^{\theta T} = 1+\theta T+\frac{\theta^2T^2}{2}$

$$TVP = (C_b - C_v - C_bI_{v0}M)(a+\frac{a\theta T}{2}+\frac{bT}{2}-cT+\frac{b\theta T^2}{2}+\frac{cT}{\theta^2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-c\theta T^2)-\frac{S_v}{nT}-C_v(I_v+I_{v0})[(n-1)(1-\varrho+\varrho bT22-cT2+aT2+2cT23\theta+cT3 \qquad (16)$$

$$TBP1 = (C_c-C_b)\{a+\frac{a\theta T}{2}+\frac{bT}{2}-cT+\frac{b\theta T^2}{2}+\frac{cT}{\theta^2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-c\theta T^2\}-\frac{S_b}{T}-C_bI_b\left\{\frac{bT^2}{2}-cT^2+\frac{aT}{2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-cT33\theta-Cb\{a\theta T2-cT+b\theta T22+cT\theta 2+cT2\theta+cT32-c\theta T2-cT23\}+IbeCcT\ aM22+bM33+cM44-\frac{C_bI_{b0}}{T}\{\frac{a(T-M)^2}{2}-\frac{b(T-M)^2}{2\theta}+\frac{c(T-M)^2}{\theta}+\frac{bT(T-M)}{\theta}+\frac{bT(T-M)^2}{2}-\frac{b}{\theta}\left(\frac{T^2}{2}-\frac{M^2}{2}\right)-\frac{2cT(T-M)}{\theta}+\frac{c(T^2-M^2)}{\theta}-cT(T-M)^2+\frac{cT^2(T-M)}{\theta}+\frac{cT^2(T-M)^2}{2}-\frac{c}{\theta}(\frac{T^3}{3}-\frac{M^3}{3})\}$$

(17)

$$TBP2 = \left(C_c-C_b+I_{be}C_c(M-T)\right)\{a+\frac{a\theta T}{2}+\frac{bT}{2}-cT+\frac{b\theta T^2}{2}+\frac{cT}{\theta^2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-c\theta T^2\}-\frac{S_b}{T}-C_bI_b\left\{\frac{bT^2}{2}-cT^2+aT2+cT2\theta+cT32-cT33\theta-Cb\{a\theta T2-cT+b\theta T22+cT\theta 2+cT2\theta+cT32-c\theta T2-cT23\}+IbeCcT\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}\right]\right. \qquad (18)$$

$$\pi 1 = (C_b-C_v-C_bI_{v0}M)(a+\frac{a\theta T}{2}+\frac{bT}{2}-cT+\frac{b\theta T^2}{2}+\frac{cT}{\theta^2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-c\theta T^2)-\frac{S_v}{nT}-C_v(I_v+I_{v0})[(n-1)(1-\varrho)+\varrho bT22-cT2+aT2+2cT23\theta+cT3+Cc-Cb\{a+a\theta T2+bT2-cT+b\theta T22+cT\theta 2+cT2\theta+cT32-c\theta T2\}-\frac{S_b}{T}-C_bI_b\left\{\frac{bT^2}{2}-cT^2+\frac{aT}{2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-\frac{cT^3}{3\theta}\right\}-C_b\{\frac{a\theta T}{2}-cT+\frac{b\theta T^2}{2}+\frac{cT}{\theta^2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-c\theta T^2-\frac{cT^2}{3}\}+\frac{I_{be}C_c}{T}\left[\frac{aM^2}{2}+bM33+cM44-\frac{C_bI_{b0}}{T}\{\frac{a(T-M)^2}{2}-\frac{b(T-M)^2}{2\theta}+\frac{c(T-M)^2}{\theta}+\frac{bT(T-M)}{\theta}+\frac{bT(T-M)^2}{2}-\frac{b}{\theta}\left(\frac{T^2}{2}-\frac{M^2}{2}\right)-\frac{2cT(T-M)}{\theta}+\frac{c(T^2-M^2)}{\theta}-cT(T-M)^2+\frac{cT^2(T-M)}{\theta}+\frac{cT^2(T-M)^2}{2}-\frac{c}{\theta}(\frac{T^3}{3}-\frac{M^3}{3})\}$$

(19)

$$\pi 2 = (C_b-C_v-C_bI_{v0}M)(a+\frac{a\theta T}{2}+\frac{bT}{2}-cT+\frac{b\theta T^2}{2}+\frac{cT}{\theta^2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-c\theta T^2)-\frac{S_v}{nT}-C_v(I_v+I_{v0})[(n-1)(1-\varrho+\varrho bT22-cT2+aT2+2cT23\theta+cT3+Cc-Cb+IbeCcM-T\{a+a\theta T2+bT2-cT+\frac{b\theta T^2}{2}+\frac{cT}{\theta^2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-c\theta T^2\}-\frac{S_b}{T}-C_bI_b\left\{\frac{bT^2}{2}-cT^2+\frac{aT}{2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-\frac{cT^3}{3\theta}\right\}-C_b\{\frac{a\theta T}{2}-cT+\frac{b\theta T^2}{2}+\frac{cT}{\theta^2}+\frac{cT^2}{\theta}+\frac{cT^3}{2}-c\theta T^2-\frac{cT^2}{3}\}+\frac{I_{be}C_c}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}\right]$$

(20)

The optimum value of cycle time can be obtained by setting $\frac{d\pi}{dT} = 0$ for fixed n. The necessary condition for maximising total profit is $\frac{d^2\pi}{dT^2} < 0$.

5.1 Numerical examples

To illustrate the above developed model, an inventory system with the following data is considered a=100, b= 15 ,c= 5, ɵ=0.1, ϱ=0.7, C_v= \$5/unit, C_b= \$25/ unit, C_c = \$ 55 / unit, S_v =\$1500/setup, S_b= \$100/order, I_v= 1%/unit/annum, I_b=1%/unit/annum, I_{v0}= 2%/unit/annum, I_{b0}= 5%/unit/annum, I_{be} = 8%/unit/annum and M = 30days

Using computational procedure optimum cycle time T* for above data is 23 days for n = 5. The buyer's order quantity Q* are 26,54,100 units/order. Vendor's total profit TVP is \$3,20,780 and buyer's total profit TBP is \$ 26,10,400. The maximum total joint profit of the integrated system π is \$29,31,200.

5.2 Sensitivity analysis

Here we study the effects of changes in the system parameters a,b, c,ɵ and ϱ on the optimal length of order cycle T^*, the optimal order quantity per cycle Q^*, vendor's profit TVP, buyer's profit TBP, total profit of the collaborative vendor-buyer inventory system π. The results are shown in Table 1.

Table 1: Effect of change in various parameters of Example1

Parameter	Value	T	Q	Vendor	Buyer	Joint Profit
a	50	23	26,49,600	3,20,410	26,02,900	29,23,300
	100	23	26,54,100	3,20,780	26,10,400	29,31,200
	150	23	26,58,600	3,21,150	26,17,900	29,39,000
	200	23	26,63,100	3,21,530	26,25,300	29,46,900
b	10	23	26,47,100	3,20,210	25,98,600	29,18,800
	15	23	26,54,100	3,20,780	26,10,400	29,31,200
	20	23	26,61,100	3,21,350	26,22,200	29,43,500
	25	23	26,68,100	3,21,920	26,33,900	29,55,900
c	3	23	16,04,400	1,93,450	15,86,300	17,79,800
	5	23	26,54,100	3,20,780	26,10,400	29,31,200
	10	23	52,78,300	6,39,110	51,70,400	58,09,600
	15	23	79,02,500	9,57,440	77,30,500	86,88,000
ɵ	0.05	21	25,26,400	5,25,360	50,34,200	55,59,500
	0.1	23	26,54,100	3,20,780	26,10,400	29,31,200
	0.15	23	37,40,800	2,47,050	20,55,700	23,02,800
	0.2	23	66,72,500	2,15,280	18,15,000	20,30,200
ϱ	0.6	23	26,54,100	3,17,140	26,10,400	29,27,500
	0.7	23	26,54,100	3,20,780	26,10,400	29,31,200
	0.8	23	26,54,100	3,24,420	26,10,400	29,34,800
	0.9	23	26,54,100	3,28,060	26,10,400	29,38,400

Based on the results of Table 1, we can obtain the following managerial insights.

(1) Increase in the value of the parameters a, b and c will result in increase of Q*, vendor's profit, buyer's profit and joint total profit. T* remain same.
(2) Decrease in the value of the parameters a, b and c will result in decrease of Q*, vendor's profit, buyer's profit and joint total profit. T* remain same.
(3) Increase in the value of the parameters ɵ will result in increase of Q* but decrease of vendor's profit, buyer's profit and joint total profit.
(4) Decrease in the value of the parameters ɵ will result in increase of Q*, vendor's profit, buyer's profit and joint total profit but decrease of T*.
(5) Increase in the value of the parameter ϱ will result in increase of vendor's profit and joint total profit but Q*, buyer's profit and T* remain unchanged.

6 Conclusion

In this paper, we formulated collaborative vendor-buyer inventory model for deteriorating items under trade credit. The deterministic demand rate is assumed to be a quadratic function of time. The objective is to

determine the retailer's optimal ordering quantity and the optimal length of order cycle for maximizing the profit. Particularly, the model developed in the paper could be useful in the area of supply chain management. Numerical example is provided to demonstrate the applicability of the proposed models. The sensitivity analysis of the optimal solution with respect to the parameters is also included. The outcome shows that a higher value of deterioration rate causes lower value of joint total profit. This work can be extended for deteriorating items with a two parameter weibull distribution. The proposed model can also be extended for order quantity dependent trade credit and two level trade credit.

7 References

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