Posuielle flow of a non Newtonian fluid between two porous Plates in a Magnetic Field

Atendra Singh Yadav* **and Dr. Rajeev Kumar Khare**
Department of Mathematics and Statistics, Sam Higginbottom University of Agriculture, Technology and Sciences Allahabad U.P., India. 211007

Received: June 23 , 2018 Accepted: August 04, 2018

ABSTRACT *In this paper, the motion of steady laminar two dimensional flows of fluid which is viscous, electrically conducting, incompressible fluid flowing between two infinite parallel plates placed at distance of h, such that lower one is porous and both plates are so situated that they feel the influence of a transverse Magnetic field. A constant pressure gradient is applicable to maintain the flow. The x-axis is assumed to be existing in the middle of the channel parallel to the direction of the flow. After forming the governing equations, analytical expression for the velocity has been obtained using Non dimensional parameter. A graphical study for velocity profile by varying the Magnetic field and its effects are observed and discussed graphically. The results are useful in the metallic flow in channels in industries.*

Keywords: *Magneto hydrodynamics (MHD) Flow, Magnetic Field, Navier-Stokes Equations, Porous Plate, Non Newtonian Factor, Analytic solutions*

Introduction

The flow of liquid metals, metal iron and ionize gasses has an important placed in industries and this is studies in magneto hydrodynamics where the motion of Electrically conducting fluid occurs in the present of magnetic field and this induces an Electric current. Which affect its velocity by putting in extra force. The equations that describe magneto hydro dynamics flows in the form of continuity equation, Navier-Stokes equations and Maxwell's equations of electromagnetism. The concerned differential equations are formed and solved after that the graphical representation between the magnetic field and velocity has been consist of critical analysis of magnetic field's velocity profile.

Mathematical Formulation of problems

Assuming that a viscous incompressible and electrically conducting fluid flowing between two infinite parallel plates such that they are rest and a distance of *h* this flow is considered to be a plane posuielle flow having constant pressure gradient the magneto hydrodynamics equations are now applicable along with equations of fluid motion. Let us an electrically conducting fluid moving with velocity **V**. we apply a magnetic field, the field strength of which is represented by the vector **B.** here the combination of two field velocity and magnetic fields, and electric field , vector is denoted by **E** is induced at right angles to both **V** and **B.** this electric field is given by

$$E = V \times B \qquad \text{.....(1)}$$

The equations of motion are the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{.....(2)}$$

and the Navier-Stokes equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho} f_{1x} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad \text{.....(3)}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho} f_{1y} - \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad \text{.....(4)}$$

Where ρ is the fluid density and f_{1x}, f_{1y} ,u,v are the fluid components of the body force per unit mass of the fluid and the velocity in *x* and *y* directions, μ is the fluid viscosity and p is the pressure acting on the fluid.

The flow is practically horizontal, so we choose the axis of the channel formed by the two plates. Assume that flow in *x*- axis direction. If v=0, and u=u(y). Also V=0 then continuity equations collapses to

$$\frac{\partial u}{\partial x} = 0$$

Hence equations (4) and (5) become:

$$0 = \frac{1}{\rho} f_{1x} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \qquad \text{.....(5)}$$

$$0 = \frac{1}{\rho} f_{1y} - \frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \text{.....(6)}$$

Let the pressure gradient

$$-\frac{\partial p}{\partial x} = s$$

Now since p=*p(x)*, equations (6) Collapses and

$$\frac{\partial p}{\partial x} = \frac{dp}{dx} = -s \qquad \text{.....(7)}$$

Combining (5) and (7) we find

$$0 = \frac{1}{\rho} f_{1x} + \frac{s}{\rho} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \qquad \text{.....(8)}$$

The *x*- axis component of the Lorentz force in Equation (8) can be expressed as follows

$$\frac{1}{\rho} f_{1x} = \frac{\sigma}{\rho}\left[\left(u\, i \times j\, Bo\right) \times jBo\right] = -\frac{Bo^2\sigma}{\rho} ui \qquad \text{.....(9)}$$

Where Bo is the magnetic field strength component assumed to be applied to a direction perpendicular to fluid motion. Equation (7) is achieved when we use expressions of force and induced current. In equation (9) is unit rectangular vectors **i** and **j.** Hence Equation (8) becomes:

$$\frac{d^2u}{dy^2} - \frac{\sigma}{\mu} Bo^2 u + \frac{s}{\mu} = 0 \qquad \text{.....(10)}$$

$$\frac{d^2u}{dy^2} - \frac{\sigma}{\mu}\frac{Bo^2 u}{1+c^2} + \frac{s}{\mu} = 0 \qquad \text{.....(11)}$$

Non- dimensionalization of the Governing Equation (11)

Let

$$x^* = \frac{x}{l}, \quad y^* = \frac{y}{l}, \quad u^* = \frac{ul}{v}, \quad p^* = \frac{pl^2}{\rho v}$$

Where *l* is the characteristic length of plates and *v* is the kinematic viscosity. Using these quantities in Equation (11) and dropping the stars then we get

$$\frac{d^2u}{dy^2} - \frac{M^2 u}{1+c^2} + \frac{l^3 s}{\mu v} = 0 \qquad \text{.....(12)}$$

Since Q= l^2s

Where M=M^* and

$$M^* = lBo\sqrt{\frac{\sigma}{\mu}} = Ha$$

Where *Ha* is Hartmann number, *l* and *s* are constants and letting Q= l^2s is a constant, Equations (12) becomes

$$\frac{d^2u}{dy^2}-\frac{M^2}{1+c^2}u=-\frac{Ql}{\mu v} \qquad \text{.....(13)}$$

Solving above differential equation then

$$U=Ae^{\frac{MY}{\sqrt{1+C^2}}}+Be^{-\frac{MY}{\sqrt{1+C^2}}}+\frac{Ql}{\mu\upsilon}\frac{\left(1+c^2\right)}{M^2} \qquad \text{.....(15)}$$

Whose solution under the boundary Condition $u=0,\ \ y=\pm 1$ is given by

$$A=-\frac{Ql}{\mu\upsilon}\frac{\left(1+c^2\right)}{M^2}\frac{1}{2Cosh\frac{M}{\sqrt{\left(1+c^2\right)}}}$$

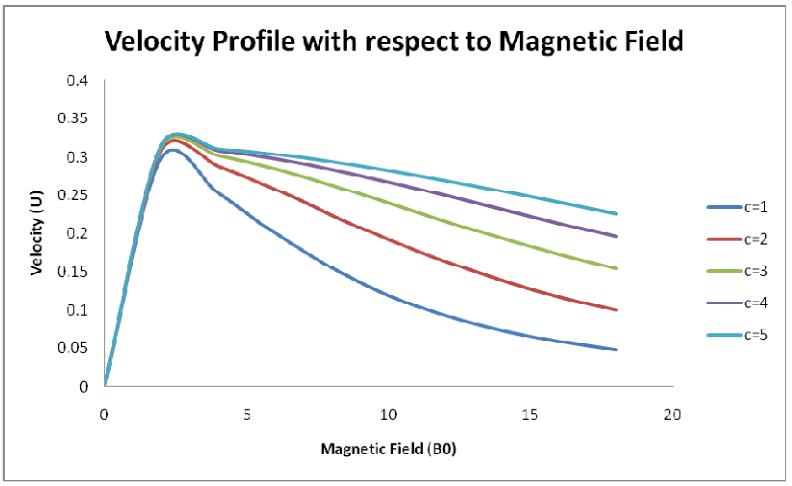
$$B=-\frac{Ql}{\mu\upsilon}\frac{\left(1+c^2\right)}{M^2}\frac{1}{2Cosh\frac{M}{\sqrt{\left(1+c^2\right)}}}$$

A and *B* value put in Equation (15) then

$$U=\frac{Ql}{\mu\upsilon}\frac{\left(1+c^2\right)}{M^2}\left(1-\frac{Cosh\frac{MY}{\sqrt{\left(1+c^2\right)}}}{Cosh\frac{M}{\sqrt{\left(1+c^2\right)}}}\right)$$

Result and Discussion

Magnetic Field(Bo)	Velocity of Fluid(U)				
	c=1	c=2	c=3	c=4	c=5
0	0	0	0	0	0
2	0.298203	0.309386	0.313298	0.314937	0.315754
4	0.252367	0.287784	0.301842	0.308028	0.31118
6	0.200474	0.257666	0.284467	0.297149	0.303837
8	0.155171	0.224557	0.263187	0.283123	0.29411
10	0.119791	0.1925	0.240005	0.266884	0.282463
12	0.09333	0.163675	0.21657	0.249348	0.269396
14	0.073747	0.138852	0.194045	0.23132	0.255398
16	0.059187	0.117982	0.173132	0.213439	0.240914
18	0.048239	0.100648	0.154168	0.196174	0.226321



The above graph plotted shows that the velocity profile of the fluid with respect to magnetic field for different values of non-Newtonian factor. As magnetic field increasing velocities also increases from its zero values and going a maximum values around M=3, its starts decreasing and after this the curve is split for all values of non-Newtonian factor. Also as the value of non Newtonian factor is increasing and velocities also increases. Since this term is represented with hyperbolic function of cosine therefore a combined effects is seen. Hence the curves show resonating characters. Near for the values of M=3, and this can be utilized increases for controlling the metallic flow through the channels placed in magnetic field. Results are also supported by the derived relation.

References

1. Alfred W. Manyonge, Jacob K. Bitok, Dionysius W. Kiema, 2013; On the Steady MHD Poiseuille Flow between Two Infinite Parallel Porous Plates in an Inclined Magnetic Field, American Journal of Computational and Applied Mathematics 2013, 3(4): 220-224.
2. Alfven H. 1942: 'Existence of Electromagnetic waves': Nature, vol.150, issue 3805, pp. 405- 406.
3. She cliff, J.A., 1956: 'Entry of conducting and Non-conducting Fluids in pipes': Journal of Mathematical proc. of the Cambridge Philosophical Soc. Vol.52, pp 573-583.
4. Davision, P.A.: An Introduction to Magneto hydrodynamics, Ed.1, Cambridge University Press, UK, 2001.
5. Ganesh, S. and Krishnambal, S., 2007: Unsteady MHD stokes Flow of Viscous Fluid between two Parallel Porous Plate': Journal of Applied Sciences Vol.7, pp.374-379
6. Raisinghniya M.D.,2014: "Fluid dynamics with complete Hydrodynamics and boundary Layer Theory". S. Chand & Company PVT. LTD.
7. Al-hadharmi, A.K., Ellot, L., Ingham, M.D. and Wen, X., 2003: 'Flow through horizontal channels of porous materials': International journal of Energy Research, Vol.27, pp.875-889.
8. Singh, C.B., 1993: 'Magneto hydrodynamics Steady Flow of liquid Between two Parallel Plates': Proc. of First conference of Kenya Mathematical society, pp. 24-26.
9. Ram, P.C., Singh, C.B. and Singh, U., 1984: 'Unsteady Magneto hydrodynamics fluid Flow Through a channel': Journal of Scientific Research, Vol. XXVIII, No. 2.
10. Drake, D.G., 1965:'On the Flow in a channel due to a Periodic Pressure Gradient:' Ouart. J. of Mech. And appl. Maths., Vol.18, No.1.