BISERIAL QUEUING MODEL WITH PROBABILISTIC BATCH ARRIVAL UNDER GEOMETRICAL DISTRIBUTION FOR TWO PARAMETERS

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ABSTRACT *Biserial batch arrival queuing models have tremendous applications in operations and service mechanism for evaluating the system performance. This paper deals with the steady state behaviour of bulk biserial queuing model with varying batch size under geometrical distribution. The performance measure of the model has been analysed for two parameters. Numerical illustration is provided to enhance the understanding of the model.*

Keywords: *Batch arrival, probabilistic, geometrical distribution.*

1. INTRODUCTION:

A bulk queue model provides an effectual mechanism for evaluating the characteristics measures of the queuing system. These models are extensively used in many realistic situations such as banking system, transportation, supermarkets, computer networks and production etc. The objective to develop a queuing system is to establish a balance between service to customers and economic consideration. Larger the service facility quicker it will disperse queue and therefore, more often the system will stand idle and hence it costs money to increase the service facility. Queue models are developed so that vacation period of the server can be utilized.

Generally bulk queue models studied by prominent researchers focused on single server like Bailey[1], Suzuki[2], Chaudhary and Templeton[4], Madan, K.C.[6] studied single server with different vacation policies. Maggu[3] studied the queues with two server in biseries. Biserial queues with batch arrival were studied by Hafiz Noor Mohammad et.al. [5]. Singh T.P. and Gupta Deepak [7] analysed a queue network model comprised of biserial and parallel channel linked with a common server. The queue network model having batch arrival with threshold effect was studied by Singh T.P., Gupta Deepak and Mittal Meenu [8].

Recently Mittal Meenu and Gupta Renu [9] studied the biserial bulk queue network model linked with common server. The present paper is an extension of the study carried out by Singh T.P, Gupta Deepak and Mittal Meenu [8] and Mittal Meenu and Gupta Renu [9] with an addendum of biserial queuing model with probabilistic batch arrival under geometrical distribution. Performance measure of the model has been analysed for two parameters.

2. DESCRIPTION OF THE MODEL:

The queue network model in the problem consists of two biserial service channels S_1, S_2 with queues q_1 and q_2 respectively. The customers demanding service arrive in batches of varying size say x with the arrival rate $\lambda_{1,x}$ and $\lambda_{2,x}$ and the associated probabilities $c_{1,x}$ and $c_{2,x}$ at S_1 and S_2 respectively. The number of customers in any arrival at either sever is a random variable x such that

$c_{1,x} = \frac{\lambda_{1x}}{\lambda_{1,T}}$ and $c_{2,x} = \frac{\lambda_{2x}}{\lambda_{2,T}}$

Where $\lambda_{1,T} = \sum_{k=1}^{n_1} \lambda_{1,k}$

and $\lambda_{2,T} = \sum_{m=1}^{n_2} \lambda_{2,m}$

are composites of all arrival at server S_1 and S_2 respectively. Customers coming at rate $\lambda_{1,x}$ after completion of service at S_1 will either join S_2 with the probabilities p_{12} or will depart from the system with probability p_{11} such that $p_{11} + p_{12} = 1$ and those coming at the rate $\lambda_{2,x}$ after completion of service at S_2 will either join S_1 with probability p_{22} or will depart from the system with probability p_{21} such that $p_{21} + p_{22} = 1$ respectively.

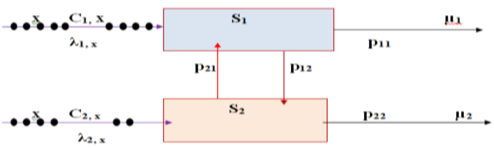

Figure: 1.1 Biserial Bulk Queue Network Model

2.1 PRACTICAL SITUATIONS:

The queue model under consideration is applicable in various fields like banking sector, administrative setup, game club, hospitals, shopping mall and many other similar situations.

2.2 MATHEMATICAL ANALYSIS:

Let $P_{n_1.n_2}$ (t) denotes the probability that there are n_1, n_2 numbers of customers in the system S_1 and S_2 at any time t where $n_1, n_2 \geq 0$.

The differential difference equations for queuing system in transient form are defined as follows:

For $n_1 \geq 1$, $n_2 \geq 1$

$$P'_{n_1.n_2}(t) = -(\lambda_{1,T} + \lambda_{2,T} + \mu_1 + \mu_2) P_{n_1.n_2}(t) + \mu_1 p_{11} P_{n_1+1.n_2}(t) + \mu_1 p_{12} P_{n_1+1.n_2-1}(t) + \lambda_{1,T} \sum_{k=1}^{n_1} P_{n_1-k.n_2}(t)\, c_{1,k} + \mu_2 p_{22} P_{n_1.n_2+1}(t) + \mu_2 p_{21} P_{n_1-1.n_2+1}(t) + \lambda_{2,T} \sum_{m=1}^{n_2} P_{n_1.n_2-m}(t)\, c_{2,m} \quad (1)$$

For $n_1 \geq 1$, $n_2 = 0$

$$P'_{n_1.0}(t) = -(\lambda_{1,T} + \lambda_{2,T} + \mu_1) P_{n_1.0}(t) + \mu_1 p_{11} P_{n_1+1.0}(t) + \lambda_{1,T} \sum_{k=1}^{n_1} P_{n_1-k.0}(t)\, c_{1,k} + \mu_2 p_{22} P_{n_1.1}(t) + \mu_2 p_{21} P_{n_1-1.1}(t) \quad (2)$$

For $n_1 = 0$, $n_2 \geq 1$

$$P'_{0.n_2}(t) = -(\lambda_{1,T} + \lambda_{2,T} + \mu_2) P_{0.n_2}(t) + \mu_1 p_{11} P_{1.n_2}(t) + \mu_1 p_{12} P_{1.n_2-1}(t) + \mu_2 p_{22} P_{0.n_2+1}(t) + \lambda_{2,T} \sum_{m=1}^{n_2} P_{0.n_2-m}(t)\, c_{2,m} \quad (3)$$

For $n_1 = 0$, $n_2 = 0$

$$P'_{0.0}(t) = -(\lambda_{1,T} + \lambda_{2,T}) P_{0.0}(t) + \mu_1 p_{11} P_{1.0}(t) + \mu_2 p_{22} P_{0.1}(t) \quad (4)$$

The corresponding Steady- State equations for $t \longrightarrow \infty$, where the behaviour of system becomes independent of time are as follows:

For $n_1 \geq 1$, $n_2 \geq 1$

$$0 = -(\lambda_{1,T} + \lambda_{2,T} + \mu_1 + \mu_2) P_{n_1.n_2} + \mu_1 p_{11} P_{n_1+1.n_2} + \mu_1 p_{12} P_{n_1+1.n_2-1} + \lambda_{1,T} \sum_{k=1}^{n_1} P_{n_1-k.n_2}\, c_{1,k} + \mu_2 p_{22} P_{n_1.n_2+1} + \mu_2 p_{21} P_{n_1-1.n_2+1} + \lambda_{2,T} \sum_{m=1}^{n_2} P_{n_1.n_2-m}\, c_{2,m} \quad (5)$$

For $n_1 \geq 1$, $n_2 = 0$

$$0 = -(\lambda_{1,T} + \lambda_{2,T} + \mu_1) P_{n_1.0} + \mu_1 p_{11} P_{n_1+1.0} + \lambda_{1,T} \sum_{k=1}^{n_1} P_{n_1-k.0}\, c_{1,k} + \mu_2 p_{22} P_{n_1.1} + \mu_2 p_{21} P_{n_1-1.0} \quad (6)$$

For $n_1 = 0$, $n_2 \geq 1$

$$0 = -(\lambda_{1,T} + \lambda_{2,T} + \mu_2) P_{0.n_2} + \mu_1 p_{11} P_{1.n_2} + \mu_1 p_{12} P_{1.n_2-1} + \mu_2 p_{22} P_{0.n_2+1} + \lambda_{2,T} \sum_{m=1}^{n_2} P_{0.n_2-m}\, c_{2,m} \quad (7)$$

For $n_1 = 0$, $n_2 = 0$

$$0 = -(\lambda_{1,T} + \lambda_{2,T}) P_{0.0} + \mu_1 p_{11} P_{1.0} + \mu_2 p_{22} P_{0.1} \quad (8)$$

2.3 SOLUTION PROCESS:

In order to solve the above system of equations we apply Generating Function Technique. For this we define the generating function as:

$$F(Y, Z) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1.n_2}\, Y^{n_1} Z^{n_2}, \quad Y \leq 1, Z \leq 1 \quad (9)$$

$$\left.\begin{aligned} \mathbf{C_1(Y)} &= \sum_{k=1}^{\infty} \boldsymbol{c_{1,k} Y^k} \\ \mathbf{C_2(Z)} &= \sum_{k=1}^{\infty} \boldsymbol{c_{2,m} Z^m} \end{aligned}\right\} \quad (10)$$

Where F(Y, Z) is generating function for steady state probability $P_{n_1.n_2}$

$C_1(Y)$ & $C_2(Z)$ are generating functions for batch size $c_{1,x}$ and $c_{2,x}$. Also for convenience we define partial generating function as:

$$\left.\begin{aligned} F_{n_2}(y) &= \sum_{n_1=0}^{\infty} P_{n_1.n_2}\, Y^{n_1} \\ F(Y,Z) &= \sum_{n_2=0}^{\infty} F_{n_2}(y)\, Z^{n_2} \end{aligned}\right\} \quad (11)$$

Multiplying equation (5) by Y^{n_1} and taking summation over n_1 from 0 to ∞ and using equations (7) and (10), we get

$$0 = -(\lambda_{1,T} + \lambda_{2,T} + \mu_1 + \mu_2) F_{n_2}(y) + \mu_1 P_{0.n_2} + \frac{\mu_1 p_{11}}{Y}[F_{n_2}(y) - P_{0.n_2}] + \frac{\mu_1 p_{12}}{Y}[F_{n_2-1}(Y) - P_{0.n_2-1}] + \lambda_{1,T} \sum_{k=1}^{\infty} c_{1,k} Y^k \sum_{n_1=k}^{\infty} P_{n_1-k.n_2} Y^{n_1-k} + \mu_2 p_{22} F_{n_2+1}(Y) + \mu_2 p_{21} Y F_{n_2+1}(Y) + \lambda_{2,T} \sum_{m=1}^{n_2} F_{n_2-m}(Y) c_{2,m} \quad (12)$$

Further simplifying equation (12), we get:

$$(\lambda_{1,T} + \lambda_{2,T} + \mu_1 + \mu_2) F_{n_2}(Y) - \mu_1 P_{0.n_2} = \frac{\mu_1 p_{11}}{Y}[F_{n_2}(Y) - P_{0.n_2}] + \frac{\mu_1 p_{12}}{Y}[F_{n_2-1}(Y) - P_{0.n_2-1}] + \lambda_{1,T} C_1(Y) F_{n_2}(Y) + \mu_2 p_{22} F_{n_2+1}(Y) + \mu_2 p_{21} Y F_{n_2+1}(Y) + \lambda_{2,T} \sum_{m=1}^{n_2} F_{n_2-m}(Y) c_{2,m} \quad (13)$$

Now multiply equation (6) by Y^{n_1} and using equation (8) & (11), we obtain

$$(\lambda_{1,T} + \lambda_{2,T} + \mu_1) F_0(Y) - \mu_1 P_{0.0} = \frac{\mu_1 p_{11}}{Y}[F_0(Y) - P_{0.0}] + \lambda_{1,T} Y C_1(Y) F_0(Y) + \mu_2 p_{21} Y F_0(Y) + \mu_2 p_{22} F_1(Y) \quad (14)$$

Now multiply equation (13.1) by Z^{n_2} & taking summation over n_2 from 0 to ∞ and using equation (11) & (14), we get:

$$(\lambda_{1,T} + \lambda_{2,T} + \mu_1 + \mu_2) F(Y,Z) - \mu_2 F_0(Y) - \mu_1 G_0(Z) = \frac{\mu_1 p_{11}}{Y}[F(Y,Z) - F_0(Z)] + \frac{\mu_1 p_{12}}{Y} Z[F(Y,Z) - F_0(Z)] + \lambda_{1,T} C_1(Y) F(Y,Z) + \frac{\mu_2 p_{22}}{Z}[F(Y,Z) - F_0(Y)] + \frac{\mu_2 p_{21}}{Z} Y [F(Y,Z) - F_0(Y)] + \lambda_{2,T} C_2(Z) F(Y,Z) \quad (15)$$

Simplifying equation (15), we get

$$F(Y,Z)[\lambda_{1,T}(1 - C_1(Y)) + \lambda_{2,T}(1 - C_2(Z)) + \mu_1(1 - \frac{p_{11}}{Y} - \frac{p_{12}}{Y}Z) + \mu_2(1 - \frac{p_{22}}{Z} - \frac{p_{21}}{Z}Y)] = F_0(Y)\mu_2(1 - \frac{p_{22}}{Z} - \frac{p_{21}}{Z}Y)] + F_0(Z)\mu_1(1 - \frac{p_{11}}{Y} - \frac{p_{12}}{Y}Z)] \quad (15.1)$$

On further simplification we have,

$$F(Y,Z) = \frac{\mu_2 F_0(Y) Y[Z - p_{22} - p_{21} Y] + \mu_1 F_0(Z) Z[Y - p_{11} - p_{12} Z]}{\lambda_{1,T}[1 - C_1(Y)]YZ + \lambda_{2,T}[1 - C_2(Z)]YZ + \mu_1 Z[Y - p_{11} - p_{12} Z] + \mu_2 Y[Z - p_{22} - p_{21} Y]} \quad (16)$$

For convenience, we define

$F_0(Z) \equiv F_1$, $F_0(Y) \equiv F_2$

The above equation (16) can be rewritten as:

$$F(Y,Z) = \frac{\mu_2 F_2 Y[Z - p_{22} - p_{21} Y] + \mu_1 F_1 Z[Y - p_{11} - p_{12} Z]}{\lambda_{1,T}[1 - C_1(Y)]YZ + \lambda_{2,T}[1 - C_2(Z)]YZ + \mu_1 Z[Y - p_{11} - p_{12} Z] + \mu_2 Y[Z - p_{22} - p_{21} Y]} \quad (17)$$

For Y = Z = 1 & F (1, 1) = 1the above equation (17) reduces to inderminate form $(\frac{0}{0})$.

Letting Z =1& limit as Y → 1in equation (17)

$$1 = \lim_{Y \to 1} \frac{\mu_2 F_2 Y[1 - p_{22} - p_{21} Y] + \mu_1 F_1[Y - p_{11} - p_{12}]}{\lambda_{1,T}[1 - C_1(Y)]Y + \mu_1[Y - p_{11} - p_{12}] + \mu_2 Y[1 - p_{22} - p_{21} Y]}$$

$$1 = \lim_{Y \to 1} \frac{\mu_2 F_2 p_{21}[Y - Y^2] + \mu_1 F_1[Y - 1]}{\lambda_{1,T}[1 - C_1(Y)]Y + \mu_1[Y - 1] + \mu_2 p_{21}[Y - Y^2]} \quad (\text{using } p_{21} + p_{22} = 1) \quad (17.1)$$

Applying L'Hospital rule for limits in (17.1)

$$1 = \lim_{Y \to 1} \frac{\mu_2 F_2 p_{21}[1 - 2Y] + \mu_1 F_1}{\lambda_{1,T}[1 - C_1(Y)] + \lambda_{1,T} Y(-\frac{d}{dY} C_1(Y)) + \mu_1 + \mu_2 p_{21}[1 - 2Y]}$$

$$1 = \frac{-\mu_2 F_2 p_{21} + \mu_1 F_1}{\lambda_{1,T}\left[1 - C_1(Y) - \frac{d}{dY} C_1(Y)\right] + \mu_1 - \mu_2 p_{21}}$$

$$1 = \frac{\mu_2 F_2 p_{21} - \mu_1 F_1}{\lambda_{1,T} E_1(x) - \mu_1 + \mu_2 p_{21}} \quad (17.2)$$

where $E_1(x) = C_1(Y) + Y \frac{d}{dY} C_1(Y) - 1$, is expected batch size for server S_1

Similarly letting Y=1 & taking limit as Z → 1 in (17), we have

$$1 = \lim_{z \to 1} \frac{\mu_2 F_2[Z - 1] + \mu_1 F_1 p_{12}[Z - Z^2]}{\lambda_{2,T} Z[1 - C_2(Z)] + \mu_1 p_{12}[Z - Z^2] + \mu_2[Z - 1]} \quad (\text{using } p_{11} + p_{12} = 1) \quad (17.3)$$

Applying L'Hospital rule for limits in (17.3)

$$1 = \frac{\mu_1 F_1 p_{12} - \mu_2 F_2}{\lambda_{2,T} E_2(x) + \mu_1 p_{12} - \mu_2} \quad (17.4)$$

$where\ E_2(x) = C_2(Z) + Z\frac{d}{dZ}C_2(Z) - 1$, is expected batch size for server S_2.
Solving (17.2) & (17.4) for F_1 and F_2, we get :

$$\lambda_{1,T}E_1(x) - \mu_1 + \mu_2 p_{21} = -\mu_1 F_1 + \mu_2 F_2 p_{21} \quad (17.5)$$

$$\lambda_{2,T}E_2(x) + \mu_1 p_{12} - \mu_2 = \mu_1 F_1 p_{12} - \mu_2 F_2 \quad (17.6)$$

Multiplying (17.6) by p_{21} and then adding, we get

$$F_1 = 1 - \frac{\lambda_{1,T}E_1(x) + \lambda_{2,T}E_2(x)p_{21}}{\mu_1(1-p_{12}p_{21})} = 1 - \rho_1 \quad (17.7)$$

Multiplying (17.5) by p_{12} and then adding, we get

$$F_2 = 1 - \frac{\lambda_{1,T}E_1(x)p_{12} + \lambda_{2,T}E_2(x)}{\mu_2(1-p_{12}p_{21})} = 1 - \rho_2$$

Where
$$\rho_1 = \frac{\lambda_{1,T}E_1(x) + \lambda_{2,T}E_2(x)p_{21}}{\mu_1(1-p_{12}p_{21})} \quad (17.8)$$

$$\rho_2 = \frac{\lambda_{1,T}E_1(x)p_{12} + \lambda_{2,T}E_2(x)}{\mu_2(1-p_{12}p_{21})} \quad (17.9)$$

and $E_1(x) = C_1(Y) + Y\frac{d}{dY}C_1(Y) - 1$

$$E_2(x) = C_2(Z) + Z\frac{d}{dZ}C_2(Z) - 1 \quad (17.10)$$

Assume that batch sizes are geometrically distributed

$$\left.\begin{aligned} c_{1,x} &= (1-a_1)a_1^{x-1} \\ c_{2,x} &= (1-a_2)a_2^{x-1} \end{aligned}\right\} \quad 0 < a_1, a_2 < 1, \text{ are parameters} \quad (17.11)$$

Substituting these values in equation (10), we have

$$C_1(Y) = \sum_{k=1}^{\infty}(1-a_1)\, a_1^{k-1}Y^k$$

$$C_1(Y) = (1-a_1)[Y + a_1 Y + a_1^2 Y^2 + \ldots\ldots\ldots\ldots\infty]$$

$$C_1(Y) = \frac{(1-a_1)Y}{1-a_1 Y} \quad \text{and, similarly}$$

$$C_2(Z) = \frac{(1-a_2)Z}{1-a_2 Z} \quad (17.12)$$

Using (17.12) in (17.10), we get

$$E_1(x) = \frac{1}{1-a_1} \text{ and } E_2(x) = \frac{1}{1-a_2} \quad (17.13)$$

Substituting these values from (17.12) & (17.13) in equation (17), we get:

$$F(Y,Z) = \frac{\mu_2(1-\rho_2)Y[Z-p_{22}-p_{21}Y] + \mu_1(1-\rho_1)Z[Y-p_{11}-p_{12}Z]}{\lambda_{1,T}\left[\frac{(1-Y)}{1-a_1 Y}\right]YZ + \lambda_{2,T}\left[\frac{(1-Z))}{1-a_2 Z}\right]YZ + \mu_1 Z[Y-p_{11}-p_{12}Z] + \mu_2 Y[Z-p_{22}-p_{21}Y]}$$

$$= \frac{\dfrac{\{\mu_2(1-\rho_2)Y[Z-p_{22}-p_{21}Y] + \mu_1(1-\rho_1)Z[Y-p_{11}-p_{12}Z]\}}{\{(1-a_1Y)(1-a_2Z)\}}}{\lambda_{1,T}[1-Y](1-a_2Z)YZ + \lambda_{2,T}[1-Z](1-a_1Y)YZ + \mu_1 Z[Y-p_{11}-p_{12}Z](1-a_1Y)(1-a_2Z) + \mu_2 Y[Z-p_{22}-p_{21}Y](1-a_1Y)(1-a_2Z)} \quad (17.14)$$

Taking Z = 1, $n_2 = 0$ and hence there will be no departure from S_2 as a result of which $\mu_2 = 0$.
Under these assumptions, from (17.14)

$$F(Y,1) = \frac{\mu_1(1-\rho_1)(1-a_2)[Y-1](1-a_1Y)}{\lambda_{1,T}(1-Y)(1-a_2)Y + \mu_1[Y-1]((1-a_1Y)(1-a_2)}$$

$$F(Y,1) = \frac{\mu_1(1-\rho_1)(1-a_1Y)}{-\lambda_{1,T}Y + \mu_1(1-a_1Y)} \quad (17.15)$$

Again solving (17.8) & (17.9) for $\lambda_{1,T}$ & $\lambda_{2,T}$ we have

$$\lambda_{1,T} = \frac{\rho_1\mu_1 - \rho_2\mu_2 p_{21}}{E_1(x)} \text{ and } \lambda_{2,T} = \frac{\rho_1\mu_1 p_{12} - \rho_2\mu_2}{E_2(x)}$$

Also at above boundary conditions, since $\mu_2 = 0$

$$\therefore \lambda_{1,T} = \frac{\rho_1\mu_1}{E_1(x)} = \rho_1\mu_1(1-a_1)$$

Using this value of $\lambda_{1,T}$ in (17.15), we get

$$F(Y,1) = \frac{\mu_1(1-\rho_1)(1-a_1Y)}{-\rho_1\mu_1(1-a_1)Y + \mu_1(1-a_1Y)}$$

$$F(Y,1) = \frac{(1-\rho_1)(1-a_1Y)}{1-\{a_1+\rho_1(1-a_1)\}Y}$$

$$F(Y,1) = (1-\rho_1)\left[\frac{1}{1-\{a_1+\rho_1(1-a_1)\}Y} - \frac{a_2Y}{1-\{a_1+\rho_1(1-a_1)\}Y}\right] \quad (17.16)$$

Terms inside the bracket form the sum of infinite G.P. By using the sum of geometric series, the equation (17.16) can be written as

F (Y, 1) = $(1-\rho_1)$

$$\left[\sum_{n_{1=0}}^{\infty}(\{a_1+\rho_1(1-a_1)\}Y)^{n_1}-a_1 Y\sum_{n_{1=0}}^{\infty}(\{a_1+\rho_1(1-a_1)\}Y)^{n_1}\right]$$

F (Y, 1) = $(1-\ \rho_1)$
$(\sum_{n_{1=0}}^{\infty}(\{a_1+\rho_1(1-a_1)\}Y)^{n_1}-a_1\sum_{n_{1=0}}^{\infty}\{a_1+\rho_1(1-a_1)\}^{n_1}Y^{n_1+1}$ (17.17)

The coefficient of Y^{n_1} in F (Y, 1), is

$P_{n_{1.0}}=\rho_1(1-\ \rho_1)(1-a_1)\{a_1+\rho_1(1-a_1)\}^{n_1-1}$ (18)

Similarly taking Y = 1, n_1 = 0 & μ_1= 0 and letting $\lambda_{2,T}=\ \rho_2\mu_2(1-a_2)$ in (17.14)

F (1, Z) = $[1-\ \rho_2]$
$\left[\sum_{n_{2=0}}^{\infty}(\{a_2+\rho_2(1-a_2)\}z)^{n_2}-a_2\sum_{n_{2=0}}^{\infty}\{a_2+\rho_2(1-a_2)\}^{n_2}z^{n_2+1}\right]$ (19)

The coefficient of Z^{n_2} in F(1, Z) is

$P_{0.n_2}=\rho_2(1\text{-}\ \rho_2)(1-a_2)\{a_2+\rho_2(1-a_2)\}^{n_2-1}$ (20)

$P_{n_1.n_2}$= Steady- State probability of $n_1$and n_2 units in the system
= Coefficient of $Y^{n_1}.Z^{n_2}$
$=\rho_1(1-\ \rho_1)(1-a_1)\{a_1+\rho_1(1-a_1)\}^{n_1-1}.\ \rho_2(1\text{-}\ \rho_2)\ (1-a_2)$
$\{a_2+\rho_2(1-a_2)\}^{n_2-1}$

$P_{n_1.n_2}\ =\ \rho_1\rho_2(1-\ \rho_1)(1\text{-}\ \rho_2)\ (1-a_1)(1-a_2)\{a_1+\rho_1(1-a_1)\}^{n_1-1}$
$\{a_2+\rho_2(1-a_2)\}^{n_2-1}$
(21)

3. MODEL ANALYSIS FOR TWO PARAMETERS:

To calculate the performance measure we analyse the model for two parameters.
From equation (10), we have

$C_1(Y)=\ \sum_{k=1}^{2}C_{1,k}\,Y^k$

$C_1(Y)=C_{1,1}Y+C_{1,2}Y^2$

$$\left.\begin{aligned}C_1(Y)&=\frac{\lambda_{1,1}Y+\lambda_{1,2}Y^2}{\lambda_{1,T}}\\ C_2(Z)&=\frac{\lambda_{2,1}Z+\lambda_{2,2}Z^2}{\lambda_{2,T}}\end{aligned}\right\}\quad(22)$$

Substituting these values from (22) in (17), we get

$$F(Y,Z)=\frac{\mu_2(1-\rho_2)Y[Z-p_{22}-p_{21}Y]+\mu_1(1-\rho_1)Z[Y-p_{11}-p_{12}Z]}{\lambda_{1,T}\left[1-\frac{\lambda_{1,1}Y+\lambda_{1,2}Y^2}{\lambda_{1,T}}\right]YZ+\lambda_{2,T}\left[1-\frac{\lambda_{2,1}Z+\lambda_{2,2}Z^2}{\lambda_{2,T}}\right]YZ+\mu_1 Z[Y-p_{11}-p_{12}Z]+\mu_2 Y[Z-p_{22}-p_{21}Y]}\quad(23)$$

If we consider in (23)

$$F(Y,Z)=\frac{f(Y,Z)}{g(Y,Z)}\quad(24)$$

Where

$f(Y,Z)=\mu_2(1-\rho_2)Y[Z-p_{22}\ -p_{21}Y]+\mu_1(1-\rho_1)Z[Y-p_{11}-\ p_{12}Z]$ (24.1)

$g(Y,Z)$ = YZ $(\lambda_{1,T}-\lambda_{1,1}Y-\lambda_{1,2}Y^2+\ \lambda_{2,T}-\lambda_{2,1}Z-\lambda_{2,2}Z^2)$
$+\ \mu_1 Z[Y-p_{11}-\ p_{12}Z]+\ \mu_2 Y[Z-p_{22}\ -p_{21}Y]$ (24.2)

Calculating the partial derivatives at Y= Z =1

$\frac{\partial f}{\partial Y}_{(1,1)}=-\mu_2(1-\rho_2)\,p_{21}+\mu_1(1-\rho_1)$

$\frac{\partial^2 f}{\partial Y^2}_{(1,1)}=0$

$\frac{\partial g}{\partial Y}_{(1,1)}=-\lambda_{1,1}-2\lambda_{1,2}+\mu_1-\mu_2 p_{21}$

$\frac{\partial^2 g}{\partial Y^2}_{(1,1)}=-2\lambda_{1,1}-6\ \lambda_{1,2}-2\ \mu_2 p_{21}$

$\frac{\partial f}{\partial Z}_{(1,1)}=\mu_2(1-\rho_2)-\mu_1(1-\rho_1)\ p_{12}$

$\frac{\partial^2 f}{\partial Z^2}_{(1,1)}=\mu_1(1-\rho_1)p_{11}$

$\frac{\partial g}{\partial Z}_{(1,1)}=-\lambda_{2,1}-2\ \lambda_{2,2}-\mu_1 p_{12}+\mu_2$

$\frac{\partial^2 g}{\partial Z^2}_{(1,1)}=-2\,\lambda_{2,1}-6\ \lambda_{2,2}-2\mu_1 p_{12}$

3.1 MEAN QUEUE LENGTH:

L_{q_1}= Marginal queue length at first server S_1

$$= \frac{\frac{\partial g}{\partial Y}_{(1,1)} \frac{\partial^2 f}{\partial Y^2}_{(1,1)} - \frac{\partial f}{\partial Y}_{(1,1)} \frac{\partial^2 g}{\partial Y^2}_{(1,1)}}{2\left[(\frac{\partial g}{\partial Y})_{(1,1)}\right]^2}$$

$$= \frac{[-\mu_2(1-\rho_2)p_{21}+\mu_1(1-\rho_1)][\lambda_{1,1}+3\lambda_{1,2}+\mu_2 p_{21}]}{[\lambda_{1,1}+2\lambda_{1,2}-\mu_1+\mu_2 p_{21}]^2} \quad (25)$$

Since for server S_1, $\mu_2 = 0$ from (25) we get

$$L_{q_1} = \frac{\mu_1(1-\rho_1)(\lambda_{1,1}+3\lambda_{1,2})}{[\lambda_{1,1}+2\lambda_{1,2}-\mu_1]^2} \quad (25.1)$$

Dividing numerator and denominator by $\mu_1^2(1-\rho_1)$, from equation (25.1), we get

$$L_{q_1} = \frac{(\lambda_{1,1}+3\lambda_{1,2})/\mu_1}{\left[\lambda_{1,1}+2\lambda_{1,2}-\mu_1/\mu_1\right]^2}(1-\rho_1) \quad (25.2)$$

Consider $\rho_{11} \equiv \frac{\lambda_{1,1}}{\mu_1}$ and $\rho_{12} \equiv \frac{\lambda_{1,2}}{\mu_1}$ (25.3)

Expected batch size may be defined as $E_1(x) = \frac{\lambda_{1,1}+2\lambda_{1,2}}{\lambda_{1,T}}$

and $\rho_1 = \rho_{11} + 2\rho_{12}$

Substituting these values in (24.2), we obtain:

$$L_{q_1} = \frac{\rho_{11}+3\rho_{12}}{1-\rho_{11}-2\rho_{12}} \quad (26)$$

L_{q_2}= Marginal queue length at first server S_2

$$= \frac{\frac{\partial g}{\partial Z}_{(1,1)} \frac{\partial^2 f}{\partial Z^2}_{(1,1)} - \frac{\partial f}{\partial Z}_{(1,1)} \frac{\partial^2 g}{\partial Z^2}_{(1,1)}}{2\left[(\frac{\partial g}{\partial Z})_{(1,1)}\right]^2} \quad (27)$$

Substituting values from (25) and also applying condition that for server S_2, $\mu_2 = 0$, in (27), we get

$$L_{q_2} = \frac{(\lambda_{2,1}+3\lambda_{2,2})/\mu_2}{\left[\lambda_{2,1}+2\lambda_{2,2}-\mu_2/\mu_2\right]^2}(1-\rho_2) \quad (28)$$

Consider $\rho_{21} \equiv \frac{\lambda_{2,1}}{\mu_2}$ and $\rho_{12} \equiv \frac{\lambda_{2,2}}{\mu_2}$ (28.1)

Expected batch size may be defined as $E_2(x) = \frac{\lambda_{2,1}+2\lambda_{2,2}}{\lambda_{2,T}}$

and $\rho_2 = \rho_{21} + 2\rho_{22}$

With these values we get

$$L_{q_2} = \frac{\rho_{21}+3\rho_{22}}{1-\rho_{21}-2\rho_{22}} \quad (29)$$

Mean queue length of the system is given by

$$L_q = L_{q_1} + L_{q_2}$$

$$L_q = \frac{\rho_{11}+3\rho_{12}}{1-\rho_{11}-2\rho_{12}} + \frac{\rho_{21}+3\rho_{22}}{1-\rho_{21}-2\rho_{22}} \quad (30)$$

$$\frac{\partial L_q}{\partial \rho_{11}} = \frac{1+\rho_{11}}{(1-\rho_{11}-2\rho_{12})^2} > 0$$

$$\frac{\partial L_q}{\partial \rho_{12}} = \frac{3-\rho_{11}}{(1-\rho_{11}-2\rho_{12})^2} > 0$$

$$\frac{\partial L_q}{\partial \rho_{21}} = \frac{1+\rho_{21}}{(1-\rho_{21}-2\rho_{22})^2} > 0$$

$$\frac{\partial L_q}{\partial \rho_{22}} = \frac{3-\rho_{21}}{(1-\rho_{21}-2\rho_{22})^2} > 0$$

Therefore by first derivative test for extreme values, the equation (30) gives the minimum queue length.

4.NUMERICAL ILLUSTRATION :

For numerical illustration consider the data

$\lambda_{1,1}$= 10, $\lambda_{1,2}$ =6, μ_1= 25

$\lambda_{2,1}$= 8, $\lambda_{2,2}$ = 5, μ_2= 34

$\lambda_{1,T} = \lambda_{1,1} + \lambda_{1,2} = 15$

$\lambda_{2,T} = \lambda_{2,1} + \lambda_{2,2} = 13$

Substituting these values in (25.3) & (28.1),

$\rho_{11} \equiv \frac{\lambda_{1,1}}{\mu_1} = 10/25$ and $\rho_{12} \equiv \frac{\lambda_{1,2}}{\mu_1} = 6/25$

$\rho_{21} \equiv \frac{\lambda_{2,1}}{\mu_2} = 8/34$ and $\rho_{12} \equiv \frac{\lambda_{2,2}}{\mu_2} = 5/34$

Results

With these values from (26) and (29), we get

$$L_q = \frac{\rho_{11} + 3\rho_{12}}{1-\rho_{11} - 2\rho_{12}} + \frac{\rho_{21} + 3\rho_{22}}{1-\rho_{21} - 2\rho_{22}}$$

$$L_q = \frac{10/25 + 3(6/25)}{1-10/25 - 2(6/25)} + \frac{8/34 + 3(5/34)}{1-8/34 - 10/34}$$

$$= \frac{28/25}{1-22/25} + \frac{23/34}{1-18/34}$$

$$= 28/3 + 23/16$$

$$= 517/48$$

$$= 10.77 \text{ units}$$

CONCLUSION :

In the existing model the concept of probabilistic batch arrival under geometrical distribution has been analysed. The steady – state solution of the model is obtained with the help of Generating function technique. This model is applicable in many realistic situations. Performance measures of the model has been analysed for two parameters in detail. Numerical illustration is provided to enlighten the model. More complex models can be developed by considering more parameters or by linking with other servers.

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