TEMPORAL GENERATED N-PICTURE FUZZY SOFT DIMENSIONS VIA ALGBERAIC STRUCTURES

Dr.S.V.Manemaran[1] & Dr.R.Nagarajan[2]
[1]Professor, Department of Science and Humanities, Sri Ranganathar Institute of Engineering and Technology, Athipalayam, Coimbatore– 641 110. Tamilnadu, India.
[2]Professor, Department of Science and Humanities, J J College of Engineering and Technology, Tiruchirappalli– 620009.Tamilnadu, India.

Received: September 01, 2018 Accepted: October 21, 2018

ABSTRACT *In this paper, we apply the concept of Picture fuzzy soft set to group theory. The notion of temporal N-Picture fuzzy soft group [TPFSG] is introduced and their basic properties are presented. Union, intersection and difference operations of temporal N-Picture fuzzy soft groups are defined. Further we have defined temporal generated N-Picture fuzzy soft group[TGPFSG] and studied some related properties with supporting proofs.*

Keywords: *Fuzzy set, Picture fuzzy set, N-Picture fuzzy set, Temporal, Soft set, Generated set. Union, complement and intersection, N-Picture fuzzy soft family.*

1. INTRODUCTION

Due to unassociated sorts of unpredictable occurring in different areas of life like Economics, Engineering, Medical Sciences, Management Sciences, Psychology, Sociology, Decision making and Fuzzy set as noted and often effective mathematical instruments have been offered to make, be moving in and grip those unpredictable. Temporal intuitionistic fuzzy set (TIFS) was defined by Atanassov in 1991. In his definition, membership and non-membership degrees of an element change with both of the element and time moment. This is one of the most important extensions of IFS. Since the establishment of fuzzy set, several extensions have been made such as Atanassov's [4] work on intuitionistic fuzzy set (IFSs) was quite remarkable as he extended the concept of FSs by assigning non-membership degree say "N(x)" along with membership degree say "P(x)" with condition that $0 \leq P(x)+N(x) \leq 1$. Since, in 1965 Zadeh[20] established fuzzy set theory, it's become an essential tool to grip inaccurate and vagueness material in different areas of prevailing civilization. Such inaccuracies are associated with the membership function that belongs to [0,1]. Through membership function, we obtain information which makes possible for us to reach the conclusion. The fuzzy set theory, becomes a strong area of making observations in different areas like Medical Science, Social Sciences, Engineering, Management Sciences, Artificial intelligence, Robotics, Computer networks, Decision making and so on. Form last few decades, the IFS has been explored by many researchers and successfully applied to many practical fields like Medical diagnosis, Clustering analysis, Decision making pattern recognition [3, 4, 5, 6]. Strengthening the concept IFS suggest Pythagorean fuzzy sets which somehow enlarge the space of positive membership and negative membership by introducing some new condition that $0 \leq P^2(x) + N^2(x) \leq 1$. The structure of Cuong [9] PFS is considerably more close to human nature than that of earlier concepts and is one of the richest research area now. For undefined terms and notions related to these areas one may refer to [7, 8, 9, 10]. Molodtsov [14] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Molodtsov applied this theory to several directions [14, 15, 16], and then formulated the notions of soft number, soft derivative, soft integral, etc. in [17]. The soft set theory has been applied to many different fields with great success. Maji et al. [13] worked on theoretical study of soft sets in detail, and [12] presented an application of soft set in the decision making problem using the reduction of rough sets [18].Our objective is to introduce the concept of temporal generated N-Picture fuzzy soft group [TGPFSSG] and its properties. The remaining part of this paper is organized as follows. Section 2 contains basic definitions and notations that are used in the remaining parts of the paper. We investigated main results of temporalN-Picture fuzzy soft group and some operations in Section 3. In section 4, we present. Characteristictemporal N-Picture fuzzy soft group and its properties. Temporal generatedN-Picture fuzzy soft group and its characterization are studied in Section-5.Finally, conclusion is made in section 6.

2. PRELIMINARIES

2.1 Definition:[20]Let U be a non-empty set. Then by a fuzzy set on U is meant a function A : U→[0,1].A is called the membership function, A(x) is called the membership grade of x in A. We also write A= {(x, A(x)): x∈U}.

2.2 Example: Consider U = { a, b, c ,d } and A : U→[0,1] defined by A(a)=0, A(b)=0.7, A(c)=0.4, A(d)=1.

2.3 Definition:[16]Let U be the initial universe set and E be the set of parameters. Let P(U) denote the power set of U. Consider a non-empty set A, A⊂ E.A pair (F,A) is called a soft set over U, where F : A→P(U).

2.4 Example: Suppose that U is the set of houses under consideration, say U = {h_1, h_2, … . ., h_5}.
Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_8}, where e_1, e_2, . . ., e_8 standfor the attributes "expensive", "beautiful", "wooden", "cheap", "modern", and "in bad""repair",respectively.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on.
For example, the soft set (F, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:A={e_1, e_2, e_3, e_4, e_5};
F(e_1) = {h_2, h_3, h_5}, F(e_2) = {h_2, h_4}, F(e_3) = {h_1}, F(e_4) = U, F(e_5) = {h_3, h_5}.

2.5 Definition:[9]A Picture fuzzy soft set 'A' on the universe of discourse X is defined as A = {<x,$T_A(x), I_A(x), F_A(x)$>, x∈X } where T: X→[-1,0], I: X→[-1,0], F: X→[-1,0] and$-1 \le T_A(x) + I_A(x) + F(x) \le 0$.

2.6 Example: Assume that the universe of discourse U={x_1,x_2,x_3},where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is N-Picture fuzzy soft set (PFSS) of U, such that, A = {< x_1, -0.3, -0.5I, -0.4 >,< x_2,-0.4I, -0.2, -0.6 >,< x_3, -0.7, -0.3, -0.5I >}, where the degreeof goodness of capability is -0.3, degree of indeterminacy of capability is -0.5I and degree of falsity of capability is -0.4 etc.

2.7 Definition:[19]A N-Picture fuzzy soft set 'A' on the universe of discourse X is defined as A= <x,$T_A(x), I_A(x), F_A(x)$>, x∈X where$T_A(x)$: X→$[0^-, 1^+]$, $I_A(x)$: X→$[0^-, 1^+]$, $F_A(x)$: X→$[0^-, 1^+]$ and $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$,$T_A(x)$, $I_A(x)$and $F_A(x)$ are respectively truth membership, indeterminacy membership and falsity membership.

From philosophical point of view, the N-Picture fuzzy soft set takes from real standard and non-standard values of $[-1^-, 0]$. So instead of $[-1^-, 0]$we need to take the interval [-1,0] for technical applications, because [-1,0] will be difficult to apply in all the real applications such as scientific and engineering problems.

A N-Picture fuzzy soft set 'A' is contained in another N-Picture fuzzy soft set B, i.e. A⊆B then $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$,$F_A(x) \ge F_B(x)$ for all x∈X.

2.8 Definition:[19]Let U be the initial universal set and E be a set of parameters. Let P(U) denote the set of all N-Picture fuzzy soft set of U. Consider a non-empty set A, A⊂E. The collection (F,A) is termed to bethe N-Picture fuzzy soft set (PFSS) over U, where F: A→P(U).

2.9Example: Let U be the set of cars under consideration and E is the set of parameters (or qualities).Each parameter is a Picture fuzzy soft set word or sentence involving N-Picture fuzzy soft set words. Consider E = {Bright, Cheap, Costly, very costly, Colorful, Cotton, Polystyrene, long sleeve,expensive}. In this case, to define a N-Picture fuzzy soft set means to point out in the universe U given by, U = {b_1, b_2, b_3, b_4, b_5} and the set of parameters A = {e_1, e_2, e_3, e_4},where each e_i is a specific criterion for cars:

e_1 stands for 'Bright',e_2 stands for 'Cheap',e_3 stands for 'costly',e_4 stands for 'Colorful'.

Suppose that,
F(Bright)={<b_1, -0.5, -0.6I, -0.3>,<b_2,-0.4, -0.7, -0.2I>,<b_3,-0.6I, -0.2, -0.3>,<b_4, -0.7I, -0.3, -0.2>,<b_5,-0.8, -0.2, -0.3I>}.
F(Cheap)={<b_1, -0.6I, -0.3, -0.5>,<b_2,- 0.7, -0.4I, -0.3>,<b_3, -0.8I, -0.1, -0.2>,<b_4, -0.7, -0.1, -0.3I>,<b_5,- 0.8I, -0.3, -0.4}.
F(Costly)={<b_1, -0.7I, -0.4,- 0.3>,<b_2, -0.6, -0.1I, -0.2>,<b_3, -0.7, -0.2, -0.5I>,< b_4, -0.5I, -0.2, -0.6>,<b_5,-0.7, -0.3I,-0.2>}.
F(Colorful)={<b_1, -0.8, -0.1I, -0.4>,<b_2, -0.4, -0.2I, -0.6>,<b_3, -0.3I , -0.6, -0.4>,<b_4, -0.4,-0.8,-0.5I>,<b_5, -0.3, -0.5I, -0.7>}.

The N-Picture fuzzy soft set (PFSS)(F, E) is a parameterized family of all Picture fuzzy soft sets of U and describes a collection of approximation ofan object. The mapping F here is 'blouses (.)', where dot(.) is to be filled up by a parameter $e_i \in E$. Therefore, $F(e_1)$ means 'blouses (Bright)' whose functional-value is the N-Picture fuzzy soft set {< b_1, -0.5, -0.6I, -0.3 >,< b_2, -0.4, -0.7, -0.2I>, < b_3, -0.6I, -0.2, -0.3 >,< b_4, -0.7I, -0.3, -0.2>,< b_5, -0.8, -0.2, -0.3I>}.

Thus we can view the N-Picture fuzzy soft set (PFSS) (F,A) as a collection of approximation as below:

(F, A) = { Bright blouses= {< b_1, -0.5, -0.6I, -0.3 >,< b_2, -0.4, -0.7, -0.2I >, < b_3, -0.6I, -0.2, -0.3>,<b_4,-0.7I,-0.3,-0.2 >,< b_5,-0.8,-0.2,-0.3I >}, Cheap blouses= {< b_1,-0.6I,-0.3,-0.5 >,< b_2,-0.7,-0.4I,-0.3>,<b_3,-0.8I,-0.1,-0.2>, <b_4, -0.7,-0.1,-0.3I>,<b_5, -0.8I,-0.3,-0.4>}, costly blouses= {< b_1,-0.7I,-0.4,-0.3>,<b_2,-0.6, -0.1I, -0.2 >,< b_3, -0.7, -0.2, -0.5I >,< b_4, -0.5I,-0.2,-0.6 >,< b_5, -0.7, -0.3I, -0.2 >}, Colorful blouses= {<b_1, -0.8, -0.1I,-0.4 >,< b_2, -0.4, -0.2I,-0.6>,< b_3, -0.3I, -0.6,-0.4 >, < b_4, -0.4,- 0.8, -0.5I>,< b_5, -0.3, -0.5I, -0.7 >}}.In order to store anN-Picture fuzzy soft set in a computer, we could represent it in the form of a table as shown below (corresponding to the N-Picture fuzzy soft set in the above example). In this table, the entries are c_{ij} corresponding to the blouse b_i and the parameter e_j, where c_{ij} = (true-membership value of b_i,indeterminacy-membership value of b_i, falsity membership value of b_i) in $F(e_j)$. The Picture fuzzy soft set (F, A) described as above

$$\begin{array}{c} b1 \\ b2 \\ b3 \\ b4 \\ b5 \end{array} \begin{pmatrix} (-0.5, -0.6I, -0.3) & (-0.6I, -0.3, -0.5) & (-0.7I, -0.4, -0.3) & (-0.8, -0.1I, -0.4) \\ (-0.4, -0.7, -0.2I) & (-0.7, -0.4I, -0.3) & (-0.6, -0.1I, -0.2) & (-0.4, -0.2I, -0.6) \\ (-0.6I, -0.2, -0.3) & (-0.8I, -0.1, -0.2) & (-0.7, -0.2, -0.5I) & (-0.3I, -0.6, -0.4) \\ (-0.7I, -0.3, -0.2) & (-0.7, -0.1, -0.3I) & (-0.5I, -0.2, -0.6) & (-0.4, -0.8, -0.5I) \\ (-0.8, -0.2, -0.3I) & (-0.8I, -0.3, -0.4) & (-0.7, -0.3I, -0.2) & (-0.3, -0.5I, -0.7) \end{pmatrix}$$

2.10 Definition:[4] A temporal fuzzy set is an object of the form $A(T)=\{h(x,t), \mu_A(x,t):(x,t)\in E\times T\}$ where (a) $A \subset E$ is a fixed set

(b) $0 \le \mu_A(x, t) \le 1$ for every $(x, t) \in E \times T$.

(c) $\mu A(x, t)$ are the degrees of membership of the element $x \in E$ at the time-moment $t \in T$.

2.11 Example: Suppose that is a temporal fuzzy set (Table 1) defined on = $\{x_1,x_2,x_3\}$ with respect to the time moment set = $\{t_1,t_2\}$:

Table 1. Temporal fuzzy set A

t/x	t_1	t_2
x_1	0.2	0.5
x_2	1.0	0.7
x_3	0.5	0.4

2.12 Definition: Let X be a non-empty set and A = <$x, T_A(x,t), I_A(x,t), F_A(x,t)$>, B = <$x, T_B(x,t), I_B(x,t), F_B(x,t)$> are temporal N-picture fuzzy soft sets. Then union, intersection and difference sets defined as

$T_{A\cup B}(x,t)= \max\{T_A(x,t),T_B(x,t)\}$,$I_{A\cup B}(x,t) = \max\{I_A(x,t), I_B(x,t)\}$,$F_{A\cup B}(x,t) = \min\{F_A(x,t), F_B(x,t)\}$,
$T_{A\cap B}(x,t) = \min\{T_A(x,t),T_B(x,t)\}$,$I_{A\cap B}(x,t) = \min\{I_A(x,t), I_B(x,t)\}$,$F_{A\cap B}(x,t) = \max\{F_A(x,t), F_B(x,t)\}$ for all $x \in X$, *and*

$A/B = T_{A/B}(x,t) = \min\{T_A(x,t),= I_{A/B}(x,t) = \min\{I_A(x,t), 1 - I_B(x,t)\}= F_{A/B}(x,t) = \max\{F_A(x,t), T_B(x,t)\}$

2.13Definition: A pair (F,A) is called temporal N-Picture fuzzy soft subgroup if the following conditions are satisfied:

(TNPFSG1):$T_A(x*y,t) \ge \min\ \{T_A(x,t),T_A(y,t)\}$, $F_A(x*y,t) \le \max\ \{F_A(x,t),F_A(y,t)\}$, $I_A(x*y,t) \le \max\ \{I_A(x,t),I_A(y,t)\}$ for all $x, y \in$ X.

(TNPFSG2): $T_A(x^{-1},t) \ge T_A(x,t)$, $F_A(x^{-1},t) \le F_A(x,t)$, $I_A(x^{-1},t) \le I_A(x,t)$ for all $x \in$ X.

2.14Example:(1) Assume that the universe of discourse X = {*x, y, z*}. A = {<x, -0.1, -0.3I, -0.5>,<y,-0.2,-0.5,-0.6I>,< z,-0.3I,-0.4,-0.5>} where the degrees of goodness of capability is -0.1, degree of Indeterminacy of capability is -0.3I and degree of falsity capability is -0.5.

(2) Consider a universe X = {FOX, RAT, LION}. A TPFSS 'A' of X could beA={< FOX,(-0.3I,-0.2,-0.1)>, < RAT,(-0.3,-0.4I,-0.6)>,< LION,(-0.1,-0.3,-0.4I)>}

2.15Definition:A temporal N-Picture fuzzy soft set is said to be zero TPFSS if $T_A(x,t)$= -1,$I_A(x,t)$= -1,$F_A(x,t)$= 0for all $x \in$ X. It is denoted by -1_N.A temporal N-Picture fuzzy soft set is said to be unit TPFS if $T_A(x,t)$= 0, $I_A(x,t)$= -1,$F_A(x,t)$= -1for all $x \in$ X. It is denoted by 0_N.

2.16Proposition: Zero TPFSS and unit TPFSS of a group X are trivial TPFSSG of X.
It is obvious.

2.17 Definition: The α-cut of the TPFSS A is a crisp subset A_α of the set X is given by A_α= { x,x ∈ X / $T_A(x,t) \geq \alpha$ }.

2.18Proposition: Let A be TPFSSG of a group X.Then for $\alpha \in$ [-1,0], α-cut A_α is a crisp subgroup of X.

Proof: For all $x, y \in A_\alpha$. We have $T_A(x,t) \geq \alpha$,$T_A(y,t) \geq \alpha$.
Now $T_A(x* y^{-1},t) \geq$ min {$T_A(x,t)$,$T_A(y,t)$} =α. Therefore $x*y^{-1} \in A_\alpha$. Hence proved.

3. MAIN RESULTS

3.1Theorem: Let A be a TPFSSG and S be aTPFSS of A. Then S is a PFS subgroup of A (written as S<A) iff $T_S(x* y^{-1},t) \geq$ min {$T_S(x,t)$,$T_S(y,t)$},$I_S(x* y^{-1},t) \leq$ max {$I_S(x,t)$,$I_S(y,t)$}, $F_S(x* y^{-1},t) \leq$max {$F_S(x,t)$,$F_S(y,t)$} for all $x, y \in$ S.

Proof: Let S be aN-Picture fuzzy soft subgroup of A.
Form S is a TPFSSG, (TPFSSG1) and (TPFSSG2) are satisfied. Hence we obtain that

$$T_S(x* y^{-1},t) \geq \min \{T_S(x,t),T_S(y^{-1},t)\} = \min \{T_S(x,t),T_S(y,t)\}.$$

Conversely, let the inequality $T_S(x* y^{-1},t) \geq$ min {$T_S(x,t)$,$T_S(y,t)$} be satisfied.
Choosing y=x, we get that

$$T_S(x* x^{-1},t) = T_S(e,t) \geq \min \{T_S(x,t),T_S(x^{-1},t)\} = T_S(x,t).$$

Hence for x=e

$$T_S(e* y^{-1},t)=T_S(y^{-1},t) \geq \min \{T_S(e,t),T_S(y,t)\} = T_S(y,t).$$

Consequently,

$$T_S(x*(y^{-1})^{-1},t) \geq \min \{T_S(x,t),T_S(y^{-1},t)\} = \min \{T_S(x,t),T_S(y,t)\}.$$

Also

$$I_S(x* y^{-1},t) \leq \max \{I_S(x,t),I_S(y^{-1},t)\} = \max \{I_S(x,t),I_S(y,t)\}.$$

Conversely, let $I_S(x* y^{-1},t) \leq$ max {$I_S(x,t)$,$I_S(y^{-1},t)$} inequality be satisfied.
Choosing y=x, we get that

$$I_S(x*x^{-1},t) = I_S(e,t) \leq \max \{I_S(x,t),I_S(x^{-1},t)\} = I_S(x,t).$$

Hence for x=e

$$I_S(e* y^{-1},t) = I_S(y^{-1},t) \leq \max \{I_S(e,t),I_S(y,t)\} = I_S(y,t).$$

Consequently,

$$I_S(x* y^{-1},t) \leq \max \{I_S(x,t),I_S(y^{-1},t)\}= \max \{I_S(x,t),I_S(y,t)\}.$$

Similarly, we can prove $F_S(x* y^{-1},t) \leq$ max {$F_S(x,t)$,$F_S(y,t)$},for all $x, y \in$ S.

3.2Theorem: Let A and B be temporal N-Picture fuzzy soft group in X, then so is $A \cup B$.

Proof: Since A and B be temporal N-Picture fuzzy soft groups in X. Then clearly TPFSSG1 and TPFSSG2are satisfied.
Now, Let x,y ε X

(TPFSGS1):
$$\begin{aligned} T_{A\cup B}(x*y,t) &= \max \{T_A(x*y,t),T_B(x*y,t)\} \\ &\geq \max\{\min\{T_A(x,t),T_A(y,t)\},\min\{T_B(x,t),T_B(y,t)\}\} \\ &\geq \min\{\max\{T_A(x,t),T_A(y,t)\}, \max \{T_B(x,t),T_B(y,t)\}\} \\ &\geq \min\{\max\{T_A(x,t),T_B(x,t)\}, \max \{T_A(y,t),T_B(y,t)\}\} \\ &\geq \min\{T_{A\cup B}(x,t), T_{A\cup B}(y,t)\} \end{aligned}$$

$$\begin{aligned} I_{A\cup B}(x*y,t) &= \max \{I_A(x*y,t),I_B(x*y,t)\} \\ &\leq \max\{\max\{I_A(x,t),I_A(y,t)\},\max\{I_B(x,t),I_B(y,t)\}\} \\ &\leq \max\{\max\{I_A(x,t),I_B(x,t)\},\max\{I_A(y,t),I_B(y,t)\}\} \\ &\leq \max\{I_{A\cup B}(x,t), I_{A\cup B}(y,t)\} \end{aligned}$$

$$\begin{aligned} F_{A\cup B}(x*y,t) &= \min \{F_A(x*y,t),F_B(x*y,t)\} \\ &\leq \min\{\max\{F_A(x,t),F_A(y,t)\}, \max \{F_B(x,t),F_B(y,t)\}\} \\ &\leq \max\{\min\{F_A(x,t),F_A(y,t)\}, \min \{F_B(x,t),F_B(y,t)\}\} \\ &\leq \max\{\min\{F_A(x,t),F_B(x,t)\}, \min \{F_A(y,t),F_B(y,t)\}\} \\ &\leq \max\{F_{A\cup B}(x,t), F_{A\cup B}(y,t)\} \end{aligned}$$

(TPFSSG2): $T_{A\cup B}(x^{-1},t)$ = max {$T_A(x^{-1},t)$,$T_B(x^{-1},t)$}≥max{$T_A(x,t)$,$T_B(x,t)$}≥ $T_{A\cup B}(x,t)$.
$I_{A\cup B}(x^{-1},t)$ = max {$I_A(x^{-1},t)$,$I_B(x^{-1},t)$}≤max {$I_A(x^{-1},t)$,$I_B(x^{-1},t)$} = $I_A(x^{-1},t)$

$F_{A\cup B}(x^{-1},t)$ = max $\{F_A(x^{-1},t),F_B(x^{-1},t)\}\leq$ max $\{F_A(x^{-1},t),F_B(x^{-1},t)\}$ = $F_A(x^{-1},t)$

3.3Theorem: If A and B be temporal N-Picture fuzzy soft group in X, then so is $A\cap B$.

Proof: The proof is similar to that of Theorem3.2.

3.4 Theorem: If A and B be temporal N-Picture fuzzysoft group in X, then A/B also N-Picture fuzzysoft group in X.

Proof: Let x,y ε X

Now $T_{A/B}(x*y)$ = min $\{T_A(x*y),F_B(x*y)\}$
$\geq$min {min$\{T_A(x,t),T_A(y,t)\}$,1-$F_B^c(x*y)\}$
= min {min$\{T_A(x,t),T_A(y,t)\}$,1-max$\{F_B^c(x,t),F_B^c(y,t)\}\}$
= min {min$\{T_A(x,t),T_A(y,t)\}$,min$\{F_B(x,t),F_B(y,t)\}$
= min {min$\{T_A(x,t),F_B(x,t)\}$,min$\{T_A(y,t),F_B(y,t)\}$
= min $\{T_{A/B}(x,t),T_{A/B}(y,t)\}$

Also $I_{A/B}(x*y)$ = min $\{I_A(x*y),1-I_B(x*y)\}$
$\leq$min {max$\{I_A(x,t),I_A(y,t)\}$,max$\{1-I_B(y,t),1-I_B(x,t)\}$
$\leq$max {min$\{I_A(x,t),1-I_B(x,t)\}$, min$\{I_A(y,t),1-I_B(y,t)\}$
$\leq$ max $\{I_{A/B}(x,t),I_{A/B}(y,t)\}$

Also $F_{A/B}(x*y)$ = max $\{F_A(x*y),T_B(x*y)\}$
$\leq$max {max$\{F_A(x,t),F_A(y,t)\}$,1- $T_B^c(x*y,t)\}$
$\leq$max {max$\{F_A(x,t),F_A(y,t)\}$,max$\{T_B(x,t),T_B(y,t)\}\}$
$\leq$max {max$\{F_A(x,t),T_B(x,t)\}$,max$\{F_A(y,t),T_B(y,t)\}\}$
$\leq$max $\{F_{A/B}(x,t),F_{A/B}(y,t)\}$

$\therefore$ $A/B$is also temporal N-Picture fuzzysoft group in X.

4. CHARACTERISTIC TEMPORAL N-PICTURE FUZZY SOFT GROUP

Our work in this section is to define characteristic temporal N-Picture fuzzy soft group (TCPFSG) and to study their properties. For this, first of all we define the notationsT_A^θ,I_A^θ,F_A^θ which will be useful in our next discussion.

4.1 Definition: Let A be a temporal N-Picture fuzzy soft set of a group G. Let θ :G→G be a map. Define the maps T_A^θ:G→[-1,0],I_A^θ:G→[-1,0],F_A^θ:G→[-1,0] given by, respectively $T_A^\theta(x,t)$=$T_A(\theta(x,t))$,$I_A^\theta(x,t)$=$I_A(\theta(x,t))$,$F_A^\theta(x,t)$=$F_A(\theta(x,t))$ for all x ∈X.

4.2 Definition: A TPFSG 'A' of a group G is called characteristic temporal N-Picture fuzzy soft group (TCPFSG) of G if T_A^θ= T_A, I_A^θ= I_A and F_A^θ= $F_A$for every automorphism θ of G.

We now prove the following properties.

4.1 Proposition: If A is TPFSSG of a group G and θ is a homomorphism of G, then the N-Picture fuzzy soft set A^θ of G given by A^θ= {<x,T_A^θ,I_A^θ,F_A^θ >/ x ∈G } also TPFSSG of G.

Proof: Let x,y ∈G. Then

(TPFSSG1): $T_A^\theta(x*y,t)$=$T_A(\theta(x*y),t)$ =$T_A(\theta(x,t)\theta(y,t))$
$\geq$min $\{T_A(\theta(x,t)),T_A(\theta(y,t))\}$
= min $\{T_A^\theta(x,t),T_A^\theta(y,t)\}$.

$I_A^\theta(x*y,t)$=$I_A(\theta(x*y),t)$ = $I_A(\theta(x,t)\theta(y,t))$
$\leq$ max $\{I_A(\theta(x,t)),I_A(\theta(y,t))\}$
= max $\{I_A^\theta(x,t),I_A^\theta(y,t)\}$.

$F_A^\theta(x*y,t)$=$F_A(\theta(x*y),t)$ = $F_A(\theta(x,t)\theta(y,t))$
$\leq$ max $\{F_A(\theta(x,t)),F_A(\theta(y,t))\}$
= max $\{F_A^\theta(x,t),F_A^\theta(y,t)\}$.

And (TPFSSG2): $T_A^\theta(x^{-1},t)$ = $T_A(\theta(x^{-1}),t)\geq T_A(\theta(x,t))$= $T_A(\theta(x,t))$.

Similarly, we can prove $I_A^\theta(x^{-1},t)$ = $I_A(\theta(x,t))$ and $F_A^\theta(x^{-1},t)$= $F_A(\theta(x,t))$.

Therefore,A^θ is TPFSSG of G.

5. GENERATED TEMPORALN-PICTURE FUZZY SOFT SUBGROUP

The main underlying idea of this work on temporal generated N-Picture fuzzy soft group is based on generated group andN-Picture fuzzy soft group with temporal set.Before defining the temporal generated N-Picture fuzzy soft group, we will need the following well known definition.

If 'a' is an element of a group A, then the set S=$\{a^n / n \in Z\}$ is a subgroup of A.
The group S is generated if and only if there exists an element a in S such that S=$\{a^n / n \in Z\}$, and it willbe denoted by <a >, the element a is called the generator of the group S.
Now we shall define a new class of temporalN-Picture fuzzy soft group. Let A=<a > be a cyclic group. If $\tilde{A}$ = {<a^n, $(T_A(a^n,t)),(I_A(a^n,t)),(F_A(a^n,t))$>/ $n \in Z$} is N-Picture fuzzy soft group, then $\tilde{A}$ is called a temporal generated N-Picture fuzzy soft group [TGFSSG] generated by $(a, T_A(a,t), I_A(a,t), F_A(a,t))$ and will be denoted by <$a, T_A(a,t), I_A(a,t), F_A(a,t)$>.

5.1Theorem: If 'A' is temporal generated N-Picture fuzzy soft group, then
$A^m = \{(a^n,\ (T_A(a^n,t))^m, (I_A(a^n,t)), (F_A(a^n,t))/n \in Z, m \in N\}$ is also a temporal generated N-Picture fuzzy soft group

5.1Definition: Let 'e' be the identity element of the group A. We shall define identity temporal N-Picture fuzzy soft group E by
E = { $e, T_A(e,t), I_A(e,t), F_A(e,t)$ /$T_A(e,t) = I_A(e,t) = F_A(e,t) = 1$ and e∈A}.

5.2 Theorem: The temporal N-Picture fuzzy soft group A^n is a temporal N-Picture fuzzy soft subgroupof A^m if m≤n.

Proof: Immediate from the above Theorem5.1.

5.3Theorem: If A^i and A^j are temporal generated N-Picture fuzzy soft group, then $A^i \cup A^j$ is also temporal generated N-Picture fuzzy soft group, $i, j \in N$.

Proof: It is enough to consider only membership function. Let m≤n.
In this case (TPFSSG1), since $A^i \supset A^j$,

$T_{A^i \cup A^j}(a^n * a^m, t)$ = max $\{T_{A^i}(a^n * a^m, t), T_{A^j}(a^{n*}a^m, t)\}$
$\geq$max $\{(T_A(a^n * a^m, t))^i, (T_A(a^n * a^m, t))^j\}$
$\geq$max { min$\{(T_A(a^n,t))^i, (T_A(a^m,t))^i\}$,min $\{(T_A(a^n,t))^j, (T_A(a^m,t))^j\}$}
$\geq$min {max$\{T_{A^i}(a^n,t), T_{A^i}(a^m,t)\}$,max$\{T_{A^j}(a^n,t), T_{A^j}(a^m,t)\}$}
$\geq$min {max$\{T_{A^i}(a^n,t), T_{A^j}(a^n,t),\}$,max$\{T_{A^i}(a^m,t), T_{A^j}(a^m,t)\}$}
$\geq$ min $\{T_{A^i \cup A^j}(a^n,t), T_{A^i \cup A^j}(a^m,t)\}$

Similarly we can prove $F_{A^i \cup A^j}$and $I_{A^i \cup A^j}$.

(TPFSSG2) $T_{A^i \cup A^j}(x^{-1},t)$ = max $\{T_{A^i}(x^{-1},t), T_{A^j}(x^{-1},t)\}$
$\geq$max $\{(T_A(x^{-1}))^i, (T_A(x^{-1}))^j\}$
$\geq$max $\{(T_A(x,t))^i, (T_A(x,t))^j\}$
$\geq$max $\{T_{A^i}(x,t), T_{A^j}(x,t)\}$
$\geq T_{A^i \cup A^j}(x,t)$.

Similarly $F_{A^i \cup A^j}(x^{-1},t)$ and $I_{A^i \cup A^j}(x^{-1},t)$ is proved.

5.4Theorem: If A^i and A^j are temporal generated N-Picture fuzzy soft groups, then $A^i \cap A^j$ is also temporal generated N-Picture fuzzy soft group, $i, j \in N$.

Proof: This theorem may be proved similarly as the Theorem5.3.

5.2Definition: Let A be a temporal generated N-Picture fuzzy soft group. Then the following set of the temporal generated N-Picture fuzzy soft group {A, A^2, A^3,..., A^m,....,E} is called temporal generated N-Picture fuzzy soft group family generated by A. It will be denoted by < A >.

5.5 Theorem: Let <A> = {A, A^2, A^3,..., A^m,....,E}. Then $\bigcup_{n=1}^{\infty} A^n$ = A and $\bigcap_{n=1}^{\infty} E$.

6. CONCLUSION AND FUTURE WORK

In this paper, the notion of temporal generated N-Picture fuzzy soft group [TGPFSSG] is introduced and their basic properties are presented. Union, intersection and difference operations of temporal N-Picture fuzzy soft group are defined.Further we have defined temporal generated N-Picture fuzzy soft group and studied some related properties with supporting proofs.The further research can be prepared in some other algebraic structures such as rough groups, factor groups and Hamilton groups.

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