CORRELATION BETWEEN PARAMETERS AND VARIABLES IN An Inventory Model of DETERIORATING ITEMS INVOLVING FUZZY WITH SHORTAGES,Exponential DEMAND and inFinite Production Rate

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ABSTRACT *In the present paper, correlation coefficients are calculated between parameters and various variables in an inventory model of deteriorating items involving fuzzy with shortages and exponential demand. The production rate is infinite.It is observed that the time of one cycle has negative high correlation with holding cost, purchasing cost, shotage cost and hazardous rate; the inventory periodhas negative high correlation with the holding cost, purchasing cost and hazardous ratebut positivehigh correlation with the shotage cost; the average total cost per unit timehas positive high correlation with the holding cost, purchasing cost, shotage cost and hazardous rate.*

Keywords: *Correlation, infinite production rate, deteriorating items,inventory model, fuzzy with shortages,exponential demand.*

1. Introduction

Inventory is defined as the stock of items to satisfy the future demands. Harris[1]developed the mathematical model to decide the number of products at ones. He also gave the concept of economic order quantity (EOQ). After himmany mathematical models have been developed for controlling the inventory. In several exciting models, it is assumed that the products have infinite shelf time. But actually deterioration plays a vital role in inventory. Deterioration is defined as decay, spoilage, loss of utility of products etc. The process of deterioration is observed in volatile liquids, beverages, medicines, blood components, sweets, fruits and vegetables. There are many other products in the real world which deteriorate with a significant rate. So it should not be neglected in the decision process of production lot size.

In recent years, mathematical ideas have been used in different areas in real life problems, particularly for controlling inventory. When the items of the commodity are kept in stock as an inventory for fulfilling the future demand, there may be deterioration of items in the inventory system.

At the end of the storage period, deterioration is studied by Whitin[2]for the fashion goods. Ghare and Schrader[3] analyzed the problem of decaying inventories exponentially and developed an EOQ model with constant demand. Covert and Philip[4] extended Ghare and Schrader's model by considering a two parameter Weibull's distribution for variable rate of deterioration. Shah and Jaiswal[5] developed an order-level inventory model for a system with constant rate of deterioration. Aggarwal[6] modified Shah and Jaiswal's model in calculating the average inventory holding cost. Yang and Wee [7] developed an integrated multi-lot-size production inventory model for deteriorating item. Sharma et al.[8] developed a deterministic production inventory model fordeteriorating products with exponentially declining demand and shortages. Baten and Kamil[9]studied the inventory management systems with two-parameter exponential distributed hazardous items in which production and demand rates are constant.Manna and Chiang [10] presented economic production quantity models for deteriorating items with ramp type demand. Mishra andRaju [11] developed a deterministic inventory model for deteriorating items with on-hand inventory dependent, variable type demand rate.Batenand Kamil [12] gaveoptimal fuzzy control with application to discounted cost production inventory planning problem. Sharma *et al*. [13] developed an inventory model for hazardous items of two-parameter exponential distribution with finite production rate. Sharma and Muhammad [14] analysed correlation between parameters and variables of EOQ models without shortages for hazardous items.Mishraand Singh [15] gave computational approach to an inventory model with ramp-type demand and linear deterioration. Mishra and Singh [16] studiedpartial backlogging EOQ model for queued customers with power demand and quadratic deterioration. Sharma and Muhammad [17] presented an EOQ model for hazardous items with uniform rate of demand. Dash*et al*. [18] gavean inventory model for deteriorating items with exponential declining demand. Sharma and Muhammad [19] developed anEOQ model with instantaneous production for hazardous items of two-parameter exponential distribution. Kumar*et al.*

[20]gavefuzzyinventory modelfordeterioratingitemswithtime dependent demand and partial backlogging.Sharmilaand Uthayakumar [21] presentedinventorymodelfor deteriorating items involvingfuzzywithshortagesand exponentialdemand.Sharmaand Muhammad [22] analyzed sensitivity of inventory model fordeteriorating items with on-hand inventory dependent demand rateand infinite production rate without shortages. Sharma et al. [23] correlateedhazardous rate and variables in an inventory model of two-parameter exponential distribution with finite production.

Some commodities were observed to shrink with time by a proportion which can be approximated by a negative exponential function of time. The probability density function of a two-parameter exponential distribution is given by

$f(t;\mu,\eta) = \frac{1}{\eta}e^{-\frac{(t-\mu)}{\eta}}$, $t \geq \mu, \eta > 1$,

where μis the location parameter andη is the scale parameter.

The unreliabilityfunction is given by

$$F(t;\mu,\eta) = 1 - e^{-\frac{(t-\mu)}{\eta}}.$$

The failure or hazard rate function of on-hand inventory is given by

$H(t;\mu,\eta) = \frac{f(t;\mu,\eta)}{1-F(t;\mu,\eta)} = \frac{1}{\eta}$, $t \geq \mu, \eta > 1$.

So the hazardous rate followed by the two-parameter exponential distribution is constant.

In this paper, our objective is to correlate all the parameters and variables by the Karl-Pearson's coefficient of correlation. The results are summarized in tables.

The Karl-Pearson's coefficient of correlation between two variables x and y is given by

$$r = r_{xy} = \frac{covariance(x,y)}{\sqrt{\text{variance}(x)}\sqrt{\text{variance}(y)}}, -1 \leq r_{xy} \leq 1.$$

2. Assumptions and Notations

1. d is the rate of demand, which is known.
2. I(t) is the on hand inventory at any timet.
3. $H(t;\mu,\eta) = \frac{1}{\eta}$ is constant hazardous rate per unit time. It follows two-parameterexponential distribution(0< H <1).
4. C_p is the production cost of one item.
5. C_h is the inventory holding cost coefficient per unit time.
6. C_s is the shortage cost per unit time.
7. C_o is the operating cost per order.
8. $C_d$is the deterioration cost per unit time.
9. z is the order level to which the inventory raised at the beginning of cycle
10. s is the shortage at the end of time T
11. C is the average total cost per unit time.
12. T is the time of one cycle.
13. t_1 is the time of production.
14. I_1 is the maximum inventory level
15. Lead time is zero.
16. The inventory system deals with only one item.
17. d, p,C_p, C_h,C_s, C_o,C_d, z, s,C,I, t_1, I_1> 0;t ≥ 0.

3. Mathematical Model

The behavior of the inventory system is describing by the differential equations

$\frac{dI(t)}{dt} = -d - \frac{1}{\eta} I(t)$ for $0 \leq t \leq t_1$and $\frac{dI(t)}{dt} = -d$ for $t1 \leq t \leq T$ ... (1)

with boundary conditions $I(0) = z$, $I(t_1) = 0$ and $I(T) = -s$. ... (2)

The solution of above system is obtained as

$I(t) = \eta d\left(e^{\frac{t_1-t}{\eta}} - 1\right)$ for $0 \leq t \leq t_1$ and $I(t) = -d(t-t_1)$ for $t_1 \leq t \leq T$ ... (3)

with $z = \eta d\left(e^{\frac{t_1}{\eta}} - 1\right)$and $s = d(T - t_1)$ ... (4)

So the lot size per production run is $q = z + s = \eta d\left(e^{\frac{t_1}{\eta}} - 1\right) + d(T - t_1)$... (5)

The situation of inventory is illustrated in figure 1.

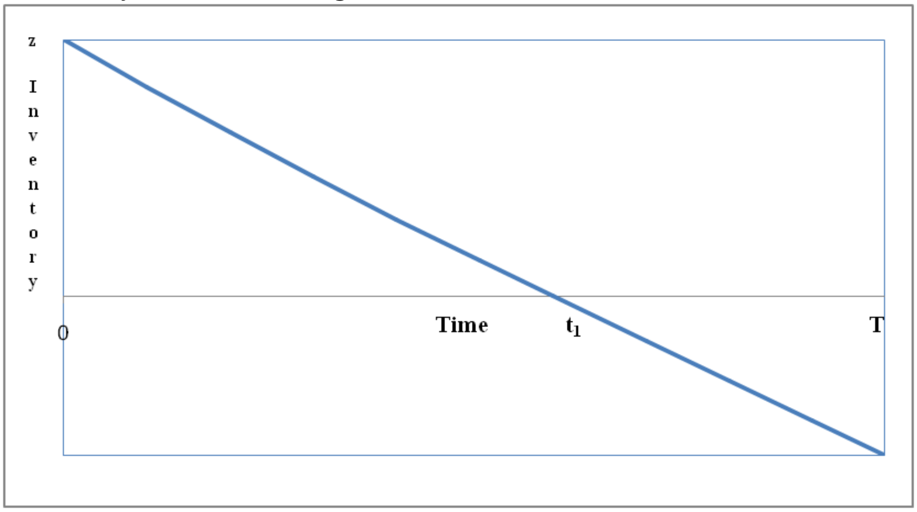

Figure 1

The total cost for one cycle includes the inventory holding cost, the shortage cost, the production cost and the ordering cost per cycle. So the total cost for one cycle of time T is

$TC = HC + SC + PC + OC.$

Hence the average total cost per unit time is given by

$$C = \frac{TC}{T} = \frac{1}{T}\left[\eta d C_h\left\{\eta\left(e^{\frac{t_1}{\eta}} - 1\right) - t_1\right\} + \frac{1}{2}dC_s(T - t_1)^2 + C_p\left\{\eta d\left(e^{\frac{t_1}{\eta}} - 1\right) + d(T - t_1)\right\} + C_o\right]$$

Using $T = t_1 + t_2$, we have

$$C = \frac{1}{(t_1 + t_2)}\left[\eta d\left\{(\eta C_h + C_p)\left(e^{\frac{t_1}{\eta}} - 1\right) - C_h t_1\right\} + \frac{1}{2}dC_s(t_2)^2 + dC_p t_2 + C_o\right] = C(t_1, t_2)$$

For minimum value of C, Newton's method is applied and hence we have obtained the optimum values t_1^*, t_2^*.

4. Numerical Example

Taking the suitable values of parameters, the corresponding values of variables are obtained. Then the correlation coefficient r is calculated between parameters and variables, which is shown in the following tables:

C_h	T	r
3	1.325	-0.907461636
4	1.026	
5	0.9456	
6	0.8752	
7	0.8496	

C_p	T	r
16	0.9561	-0.814437791
18	0.9263	
20	0.9248	
22	0.9215	
24	0.92	

C_s	T	r
11	0.9621	-0.942648087
13	0.9153	
15	0.9018	
17	0.8915	

H	T	r
0.006	0.8931	-0.954423268
0.008	0.8903	
0.01	0.8896	
0.012	0.8815	

19	0.8768

0.014	0.8754

C_h	t_1	r
3	0.7652	-0.983456985
4	0.7321	
5	0.6548	
6	0.6376	
7	0.5931	

C_p	t_1	r
16	0.6376	-0.940672056
18	0.6341	
20	0.6301	
22	0.6102	
24	0.6096	

C_s	t_1	r
11	0.6456	0.995817783
13	0.6541	
15	0.6661	
17	0.6792	
19	0.6863	

H	t_1	r
0.006	0.6846	-0.988602836
0.008	0.6714	
0.01	0.6604	
0.012	0.6563	
0.014	0.6428	

C_h	C	r
3	348.23	0.984388557
4	368.14	
5	398.25	
6	412.56	
7	423.45	

C_p	C	r
16	423.45	0.996925781
18	425.32	
20	427.64	
22	429.71	
24	431.09	

C_s	C	r
11	410.15	0.983597105
13	423.12	
15	436.74	
17	442.91	
19	449.53	

H	C	r
0.006	450.55	0.999377429
0.008	456.12	
0.01	461.47	
0.012	468.09	
0.014	473.3	

5. Conclusion

In this paper a correlation analysis is presented betweenparameters and variables. It is observed from above tables that the time of one cycle has negative high correlation with holding cost, purchasing cost, shotage cost and hazardous rate; the inventory period has negative high correlation with the holding cost, purchasing cost and hazardous rate but positive high correlation with the shotage cost; the average total cost per unit time has positive high correlation with the holding cost, purchasing cost, shotage cost and hazardous rate. Hence we conclude that the average total cost increases highly with hazardous rate. Therefore it should be controlled for maximization of profit.

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