SOME LABELING PARAMETERS OF $D(vK_{1,n})$

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ABSTRACT: *Graph labeling was first introduced in the late 1960's. A graph labeling is the assignment of labels, traditionally represented by integers to the vertices/edges or both of a graph. It is a recent developing area in graph theory. In this paper, we established a general formula to label the vertices and edges of the duplication of $K_{1,n}$; n>1 by different labeling parameters. Now we compare the radio number with the minimum span of arithmetic, odd mean, even mean, graceful graphs. Also we have proved $S_o(G) = S_e(G)$ where $S_o(G)$ and $S_e(G)$ are minimum span (relative to the vertices) of odd mean and even mean graphs.*

Key Words: *Graph, Labeling, Radio number, Graph duplication.*

Introduction:

In 1967, A. Rosa published a pioneering paper on graph labeling problems (5). A labeling of a graph $G = (V, E)$ is a mapping f from the vertex set V into the set of non-negative integers that induces for each edge $uv \in E$, a label depending on the vertex labels f(u) and f(v). In 1973, E. Sampath Kumar introduced the concept of Graph duplication and proved some results (6).

Definition: 1.1

A **graph** G is an ordered pair (V(G),E(G)) consisting of a non-empty set V(G) of vertices and a set E(G), disjoint from V(G), of edges, together with an incidence function ψ_G that associate with each edge of G is an unordered pair of vertices of G.

Definition: 1.2

A **bipartite graph** is a graph whose vertex set can be partitioned into two subsets V_1 and V_2, such that each edge has one end in V_1 and one end in V_2.

A bipartite graph is said to be **complete** if every vertex of V_1 is joined to every vertex of V_2. A complete bipartite graph with $|V_1| = m$ and $|V_2| = n$ is denoted by $K_{m,n}$. $\mathbf{K_{1,n}}$ is called a **star** graph for n ≥ 1.

Definiton: 1.3

A **labeling** or valuation of a graph G is an assignment f of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y).

Definiton: 1.4

The **distance** between any two vertices u and v denoted by d(u,v) is the length of a shortest u-v path, also called a u-v geodesic.

Definiton: 1.5

Let G be a graph and v be a vertex of G. The **eccentricity** of the vertex v is the maximum distance from v to any other vertex of G.

That is $e(v) = max\{d(v, w): w \in V(G)\}$.

Definition: 1.6

The **diameter** of G is the maximum eccentricity among the vertices of G. Which is denoted by $diam(G) = max\{e(v): v \in V(G)\}$.

Definition: 1.7

The **span** of f is defined as max{$|f(u) - f(v)|: u, v \in V(G)$ }

Definition: 1.8

A **radio labeling** f of G is an assignment of nonnegative integers to the vertices of G satisfying, $|f(u) - f(v)| \geq diam(G) + 1 - d_G(u, v) \; \forall \, u, v \in V(G)$. The **radio number** denoted by $\boldsymbol{rn(G)}$ is the minimum span of a radio labeling for G.

Definition: 1.9

Let G be a graph with q edges and let a and d be positive integers. A labeling f of G is said to be (a,d) - **arithmetic** if the vertex labels are distinct non-negative integers and the edge labels induced by $f(x) + f(y)$ for each xy are a, a+d, a+2d, …, a+(q-1)d. A graph is called arithmetic if it is (a, d) - arithmetic for some a and d.

Definition: 1.10

A function f is called an **odd mean labeling** of a graph G with p vertices and q edges. If f is an injection from the vertices of G to the set $\{1,3,5, \dots ,2q - 1\}$ such that each edge uv is assigned the label $\frac{f(u)+f(v)}{2}$, then the resulting edge labels are distinct. A graph which admits an odd mean labeling is said to be odd mean grah.

Definition: 1.11

A function f is called an **even mean labeling** of a graph G with p vertices and q edges. If f is an injection from the vertices of G to the set $\{2,4,6, \dots ,2q\}$ such that each edge uv is assigned the label $\frac{f(u)+f(v)}{2}$, then the resulting edge labels are distinct. A graph which admits an even mean labeling is said to be even mean graph.

Definition: 1.12

Let G be a (p,q) graph. Let V(G) and E(G) be the vertex set and edge set of G. A function f is called **graceful labeling** on G if $f: V(G) \to \{0,1,2, \dots , q\}$ is injective and the induced function $f^*: E(G) \to \{1,2, \dots , q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is injective. The graph which admits graceful labeling is called graceful graph.

Definition: 1.13

Let G = (V,E) be a (p,q) - graph. The graph G is said to be (a,r) - **geometric** if it's vertices can be assigned distinct positive integers so that the values of the edges obtained as the product of the numbers assigned to their end vertices, can be arranged as a geometric progression $a, ar, ar^2, \dots , ar^{q-1}$.

Definition: 1.14

A **magic** graph is a graph whose edges are labeled by positive integers, so that the sum over the edges incident with any vertex is the same, independent of the choice of vertex; or it is a graph that has such a labeling.

Definition: 1.15

A graph with q edges is called **antimagic** if its edges can be labeled with $1,2, \dots , q$ so that the sum of the labels of the edges incident to each vertex are distinct.

Definition: 1.16

Duplication of a vertex v of a graph G produces a new graph G′ by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G′. For a graph G, the graph obtained by duplication of all the vertices of G is denoted by $\boldsymbol{D(vG)}$.

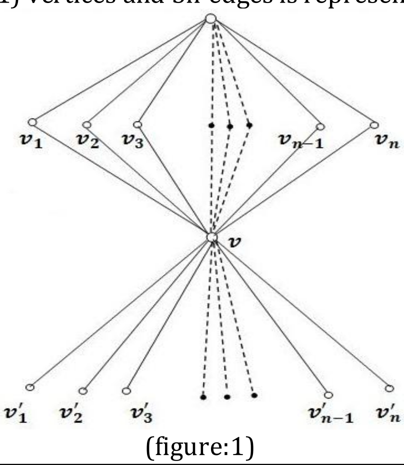
Theorem: 2.1

Let $G = D(vG^*)$ where $G^* = K_{1,n};\ n > 1$ then $rn(G) = 4n + 1$.

Proof:

Given $G = D(vG^*)$ is the duplication of all the vertices of $K_{1,n};\ n > 1$.

Therefore, G consist of 2(n+1) vertices and 3n edges is represent in (figure:1) as below



(figure:1)

The vertex set of G is denoted by
$V(G) = \{v, v', v_i, v_i' /\ 1 \leq i \leq n\}$ where
deg (v) = n, deg (v_i) = 1;

$v, v_i \epsilon\, G^*$and v', v_i' are the duplication
of v, v_i; $1 \leq i \leq n$ respectively and diam(G) = 3
Define $f: V(G) \rightarrow N \cup \{0\}$.
The label to the vertices of G as below.

$$\left.\begin{array}{l} f(v_1') = t \ ('t' \text{ be any non negative integer}) \\ f(v_i') = 2(i-1)+t;\ 2 \leq i \leq n \\ f(v_i) = 2(n+i-1)+t;\ 1 \leq i \leq n \\ f(v') = t+1,\ f(v) = 4n+t+1 \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

In equation (1), $f(v_1') = t$ is minimum and $f(v) = 4n + t + 1$ is maximum for any i.
Therefore, $rn(G) = |f(v) - f(v_1')|$
$\Rightarrow \quad rn(G) = 4n + 1$
Hence the proof. □

Theorem: 2.2

Let $G = D(vG^*)$ where $G^* = K_{1,n}; n > 1$ then G is an arithmetic graph.

Proof:

Given $G = D(vG^*)$ where $G^* = K_{1,n};\ n > 1$.
Therefore, $|V(G)| = 2(n+1)$, $|E(G)| = 3n$ and the graph is represented in (figure:1).
$V(G) = \{v, v', v_i, v_i' /1 \leq i \leq n\}$ where deg (v) = n, deg (v_i) = 1; $v, v_i \epsilon\, G^*$and
v', v_i' are the duplication of v, v_i; $1 \leq i \leq n$ respectively.
Define $f: V(G) \rightarrow \mathbb{N}$ where $\mathbb{N}$ is a set of non negative integers and the vertex labels are as below.

$$\begin{array}{l} f(v) = \text{any non negative integer} \\ \left.\begin{array}{l} f(v') = f(v)+1,\ f(v_1) = f(v')+1 \\ f(v_i') = f(v')+2i-1;\ 1 \leq i \leq n \\ f(v_i) = f(v')+2n+i;\ 1 \leq i \leq n \end{array}\right\} \quad \ldots\ldots\ldots\ldots (2) \end{array}$$

The edge labels induced by $f(uv) = f(u) + f(v)$ are as follows.

$$\left.\begin{array}{l} f(vv_i') = f(v)+f(v')+2i-1;\ 1 \leq i \leq n \\ f(vv_i') = f(v)+f(v')+2n+i;\ 1 \leq i \leq n \\ f(v'v_i) = 2(f(v')+i)-1;\ 1 \leq i \leq n \end{array}\right\} \quad \ldots\ldots\ldots\ldots (3)$$

Using a constant distance 'd' where d be any positive integer, we can label to the vertices of G as

$$\left.\begin{array}{lcl} f(v) & = & t \ (\text{'t' be any non negative integer}) \\ f(v') & = & t+d \\ f(v_i') & = & d(2i-1)+t+1;\ 1 \leq i \leq n \\ f(v_i') & = & d(2n+i)+t+1;\ 1 \leq i \leq n \end{array}\right\} \quad \ldots\ldots\ldots\ldots (4)$$

In similar, the edges of G also can be labeled as

$$\left.\begin{array}{lcl} f(vv_i') & = & d(2i-1)+2t+1;\ 1 \leq i \leq n \\ f(vv_i') & = & d(2n+i)+2t+1;\ 1 \leq i \leq n \\ f(v'v_i) & = & 2(id+t)+1;\ 1 \leq i \leq n \end{array}\right\} \quad \ldots\ldots\ldots\ldots (5)$$

The above equation (2), (3), (4), (5) gives the edge values of G are of the form
$f(E(G)) = \{a, a+d, \ldots, a+(q-1)d\}$.
Therefore, f is an arithmetic labeling.
Hence, G is an (a,d) – arithmetic graph. □

Corollary: 2.3

Let $G = D(vG^*)$ where $G^* = K_{1,n};\ n > 1$, $\mathcal{S}_a$ be the minimum span (relative to the vertices) of arithmetic graph with d = 1 then $\mathcal{S}_a = rn(G) - n$.

Proof:

Given, $G = D(vG^*)$where $G^* = K_{1,n};\ n > 1$ which is represented in (figure:1) and let $\mathcal{S}_a$ be the minimum span (relative to the vertices) of arithmetic graph.
To prove $\mathcal{S}_a = rn(G) - n$

By theorem : 1, $rn(G) = 4n + 1$
By equation (4), $f(v)$ = t is minimum and $f(v_i^{'}) = 3n + t + 1$ is maximum for any i (since d = 1).

Therefore, $\mathcal{S}_a = \max|fv_i^{'}) - f(v)|$

⇨ $\mathcal{S}_a = 3n + 1$

⇨ $\mathcal{S}_a + n = 3n + 1 + n$

⇨ $\mathcal{S}_a + n = 4n + 1$

⇨ $\mathcal{S}_a + n = rn(G)$

⇨ $\mathcal{S}_a = rn(G) - n$

Hence the proof. □

Theorem: 2.4

Let $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ then G is an odd mean graph.

Proof:

Given $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ is represented in (figure:1) and $V(G) = \{v, v^{'}, v_i, v_i^{'} \,/\, 1 \le i \le n\}$ where deg (v) = n, deg (v_i) = 1; $v, v_i \epsilon\, G^*$ and $v^{'}, v_i^{'}$ are the duplication of v, v_i; $1 \le i \le n$ respectively.

Define $f: V(G) \to \{1,3,5, \dots, 2q - 1\}$ as below

$$\left.\begin{array}{l} \text{Let } f(v) = 3 \\ f(v^{'}) = f(v) + 2,\ f(v_1) = f(v^{'}) - f(v) - 1 \\ f(v_i) = 6i - 1;\ 2 \le i \le n \\ f(v_1^{'}) = f(v) + f(v^{'}) - f(v_1) \\ f(v_i^{'}) = 3(2i - 1);\ 2 \le i \le n \end{array}\right\} \qquad \dots\dots\dots\dots (6)$$

Then, label to the edges of G are

$$\left.\begin{array}{l} f(vv_1) = \frac{f(v)+f(v_1)}{2},\ f(vv_1^{'}) = \frac{f(v)+f(v_1^{'})}{2} \\ f(v^{'}v_1) = \frac{f(v^{'})+f(v_1)}{2} \\ f(vv_i^{'}) = 3i + 1;\ 2 \le i \le n, \\ f(vv_i) = 3i;\ 2 \le i \le n \\ f(v^{'}v_i) = 3i + 2;\ 2 \le i \le n \end{array}\right\} \qquad \dots\dots\dots\dots (7)$$

The labeling of vertices and edges are satisfies the odd mean labeling.

Hence G is an odd mean graph. □

Corollary: 2.5

Let $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$, $\mathcal{S}_o$ be the minimum span (relative to the vertices) of odd mean graph then $\mathcal{S}_o = rn(G) + (2n - 3)$.

Proof:

Given $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ is represented in (figure:1) and let $\mathcal{S}_o$ be the minimum span (relative to the vertices) of odd mean graph.

To prove $\mathcal{S}_o = rn(G) + 2n - 3$
By theorem:1, $rn(G) = 4n + 1$
Equation (6) ⇨ $f(v_1) = 1$ is minimum and $f(v_i) = 6n - 1$ is maximum for all i.

Therefore, $\mathcal{S}_o = \max|f(v_i) - f(v_1)|$

⇨ $\mathcal{S}_o = 6n - 2$

⇨ $\mathcal{S}_o - (2n - 3) = 6n - 2 - (2n - 3)$

⇨ $\mathcal{S}_o - (2n - 3) = 4n + 1$

⇨ $\mathcal{S}_o - (2n - 3) = rn(G)$

⇨ $\mathcal{S}_o = rn(G) + (2n - 3)$

Hence the proof. □

Theorem: 2.6

Let $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ then G is an even mean graph.

Proof:

Given $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ is represented in (figure:1) and $V(G) = \{v, v^{'}, v_i, v_i^{'} \,/\, 1 \le i \le n\}$ where deg (v) = n, deg (v_i) = 1; $v, v_i \epsilon\, G^*$ and $v^{'}, v_i^{'}$ are the duplication of v, v_i; $1 \le i \le n$ respectively.

Define $f: V(G) \to \{2,4,6, \dots, 2q\}$ and the label to the vertices of G as below.

Let $f(v) = 4$
$f(v_1) = f(v) - 2$
$f(v_i) = 6i;\ 2 \le i \le n$
$f(v') = f(v) + 2\ ,\ f(v_1') = 2f(v)$
$f(v_i') = 2(3i-1); 2 \le i \le n$ ………… (8)

E(G) can be labeled as
$f(vv_1) = f(v) - 1$
$f(vv_i) = 3i + 2;\ 2 \le i \le n$
$f(vv_1') = \frac{f(v)+f(v_1')}{2}$
$f(vv_i') = 3i + 1;\ 2 \le i \le n$
$f(v'v_1) = \frac{f(v')+f(v_1)}{2}$
$f(v'v_i) = 3(i+1);\ 2 \le i \le n$ ………… (9)

Clearly equations (8) and (9) satisfies the even mean labeling.
Hence G is an even mean graph. □

Corollary: 2.7

Let $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$, $\mathcal{S}_e$ be the minimum span (relative to the vertices) of even mean graph then $\mathcal{S}_e = rn(G) + (2n-3)$.

Proof:

Given $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ is represented in (figure:1) and let $\mathcal{S}_e$ be the minimum span (relative to the vertices) of even mean graph.
To prove $\mathcal{S}_e = rn(G) + (2n-3)$
By theorem:1, $rn(G) = 4n + 1$
From equation (8), $f(v_1) = 2$ is minimum and $f(v_i) = 6n$ is maximum for any i.
Therefore, $\mathcal{S}_e = \max|f(v_i) - f(v_1)|$
⇨ $\mathcal{S}_e = 6n - 2$
⇨ $\mathcal{S}_e - (2n-3) = 6n - 2 - (2n-3)$
⇨ $\mathcal{S}_e - (2n-3) = 4n + 1$
⇨ $\mathcal{S}_e - (2n-3) = rn(G)$
⇨ $\mathcal{S}_e = rn(G) + (2n-3)$
Hence proved. □

Result: 2.8

Minimum span (relative to the vertices) of odd mean graph is same as the minimum span (relative to the vertices) of even mean graph.

Proof:

The result is trivial from corollary 2.5 and corollary 2.7.

Theorem: 2.9

Let $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ then G is a graceful graph.

Proof:

Given, $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ which is represented in (figure:1) and $V(G) = \{v, v', v_i, v_i' \,/\, 1 \le i \le n\}$ where deg (v) = n, deg (v_i) = 1; $v, v_i \epsilon G^*$ and v', v_i' are the duplication of v, v_i; $1 \le i \le n$ respectively.
Define $f: V(G) \to \{0,1,2,\dots,q\}$
Let $f(v) = 0$ and $f(v') = 1$
$f(v_i) = 2i;\ 1 \le i \le n$
$f(v_i') = 2n + i;\ 1 \le i \le n$ ………… (10)

The edges of G are labeled by
$f^*: E(G) \to \{1,2,\dots,q\}$ are as follows.
$f(vv_i') = 2i;\ 1 \le i \le n$
$f(vv_i) = 2n + i;\ 1 \le i \le n$
$f(v'v_i) = 2i - 1;\ 1 \le i \le n$ ………… (11)

Clearly the vertices and edges of G are graceful.
Hence G is a graceful graph. □

Corollary: 2.10

Let $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$, $\mathcal{S}_g$ be the minimum span (relative to the vertices) of graceful graph then $\mathcal{S}_g = rn(G) - (n+1)$.

Proof:

Given, $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ is represented in (figure:1) and let $\mathcal{S}_g$ be the minimum span (relative to the vertices) of graceful graph.

To prove $\mathcal{S}_g = rn(G) - (n+1)$

By theorem:1, $rn(G) = 4n + 1$

From equation (10) of theorem: 2.11, $f(v) = 0$ is minimum and $f(v_i^{'}) = 3n$ is maximum for any i.

Therefore, $\mathcal{S}_g = 3n$

⇨ $\mathcal{S}_g + (n+1) = 3n + (n+1)$

⇨ $\mathcal{S}_g + (n+1) = 4n + 1$

⇨ $\mathcal{S}_g + (n+1) = rn(G)$

⇨ $\mathcal{S}_g = rn(G) - (n+1)$

Hence proved. □

Theorem: 2.11

Let $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ then G is an (a,r) - geometric graph.

Proof:

From (figure:1) represented by the given graph $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$. $V(G) = \{v, v^{'}, v_i, v_i^{'}/1 \le i \le n\}$ where deg (v) = n, deg (v_i) = 1; $v, v_i \epsilon\, G^*$ and $v^{'}$, $v_i^{'}$ are the duplication of v, v_i; $1 \le i \le n$ respectively.

Define $f: V(G) \to \mathcal{M}$ where $\mathcal{M}$ is a set of positive integers and the vertex labeling of G is

$$\left.\begin{aligned} f(v) &= u;\ u > 0 \\ f(v^{'}) &= ut;\ u\ and\ t > 0\ and\ u\ \neq t \\ f(v_1) &= r;\ r > 0\ and\ r\ \neq u\ and\ t \\ f(v_i) &= rt^{2(i-1)};\ 1 \le i \le n \\ f(v_i^{'}) &= rt^{(2n+i-1)};\ 1 \le i \le n \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (12)$$

The edges of G are labeled as

$$\left.\begin{aligned} f(vv_i) &= urt^{2(i-1)};\ 1 \le i \le n \\ f(vv_i^{'}) &= urt^{(2n+i-1)};\ 1 \le i \le n \\ f(v^{'}v_i) &= urt^{(2i-1)};\ 1 \le i \le n \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (13)$$

The edges are labeled in the form of $a, ar, ar^2, \ldots, ar^{q-1}$

Hence, G is an (a,r) – geometric graph. □

Theorem: 2.12

Let $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ then G is an antimagic graph.

Proof:

Given, $G = D(vG^*)$ where $G^* = K_{1,n}$; $n > 1$ is represented in (figure:1).

$|V(G)| = 2(n+1), |\ E(G)| = 3n$.

Define $f: E(G) \to \{1,2, \ldots, q\}$ and the edge labels are as follows:

Case(i)

If $\boldsymbol{n = 2}$ then

$f(vv_i) = i;\ 1 \le i \le n$

$f(vv_i^{'}) = i + 2;\ 1 \le i \le n$

$f(v^{'}v_i) = i + 4;\ 1 \le i \le n$

Case(ii)

If $\boldsymbol{n \equiv 0 (mod\ 4)}$ then

$f(vv_i^{'}) = i;\ 1 \le i \le n$

$$f(vv_i) = \begin{cases} n+1 & if \quad i = 1 \\ n + \frac{n}{4} + 1 & if \quad i = 2 \end{cases}$$

$n+\frac{n}{4}+i \quad ; \quad 3 \le i \le n$

$$f(v'v_i) = \begin{cases} n+1+i & ; \quad 1 \le i \le \left(\frac{n}{4}-1\right) \\ \frac{5n}{4}+2 & if \quad i = \frac{n}{4} \\ 2n+i & ; \quad \left(\frac{n}{4}+1\right) \le i \le n \end{cases}$$

Case(iii)

If $\boldsymbol{n \equiv 1 (mod\, 4)}$ then

$f(vv_i') = i; \ 1 \le i \le n$

$$f(vv_i) = \begin{cases} n+\left\lfloor\frac{n}{4}\right\rfloor & if \quad i=1 \\ n+\left\lfloor\frac{n}{4}\right\rfloor+i & ; \quad 2 \le i \le n \end{cases}$$

$$f(v'v_i) = \begin{cases} n+i & ; \quad 1 \le i \le \left(\left\lfloor\frac{n}{4}\right\rfloor-1\right) \\ \left(\frac{5(n-1)}{4}\right)+2 & if \quad i = \left\lfloor\frac{n}{4}\right\rfloor \\ 2n+i & ; \quad \left(\left\lfloor\frac{n}{4}\right\rfloor+1\right) \le i \le n \end{cases}$$

Case(iv)

If $\boldsymbol{n \equiv 2 (mod\, 4)}$ then

$$f(vv_i') = \begin{cases} i & ; \quad 1 \le i < n \\ \left\lfloor\frac{n}{4}\right\rfloor+n & if \quad i = n \end{cases}$$

$$f(vv_i) = \left\lfloor\frac{n}{4}\right\rfloor+n+i \quad ; \quad 1 \le i \le n$$

$$f(v'v_i) = \begin{cases} n+i-1 & ; \quad 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor \\ 2n+i & ; \quad \left(\left\lfloor\frac{n}{4}\right\rfloor+1\right) \le i \le n \end{cases}$$

Case(v)

If $\boldsymbol{n+1 \equiv 0 (mod\, 4)}$ then

$f(vv_i') = i; \ 1 \le i \le n$

$$f(vv_i) = \begin{cases} n+i & ; \quad 1 \le i \le 2 \\ \left(\frac{n+1}{4}\right)+n+i & ; \quad 3 \le i \le n \end{cases}$$

$$f(v'v_i) = \begin{cases} n+i+2 & ; \quad 1 \le i \le \left(\frac{n+1}{4}\right) \\ 2n+i & ; \quad \left(\left(\frac{n+1}{4}\right)+1\right) \le i \le n \end{cases}$$

$V(G) = \{v, v', v_i, v_i' / \ 1 \le i \le n\}$ Where deg (v) = n, deg (v_i) = 1; $v, v_i \epsilon G^*$ and v', v_i' are the duplication of v, v_i; $1 \le i \le n$ respectively.

The vertex labels are as follows:

Case (1)

If $\boldsymbol{n = 2}$ then

$f(v_i') = 2+i$, $f(v_i) = 2(i+2)$, $f(v) = n^3+2$, $f(v') = n^3+3$.

Case (2)

If $\boldsymbol{n \equiv 0 (mod\ 4)}$ then

$$f(v_i) = \begin{cases} n + \left(\frac{5n}{4}\right) + 3 & if \quad i = 1\ and\ n = 4 \\ 3n + \left(\frac{n}{4}\right) + i + 1 & if \quad i = 2\ and\ n = 4 \\ 3n + \left(\frac{n}{4}\right) + 2i & ; \quad 3 \le i \le n\ and\ n = 4 \\ n + \left(\frac{6n}{4}\right) + 3 & if \quad i = 2\ and\ n = 8 \\ 2(n+1) + i & if \quad i = 1\ and\ n > 4 \\ 2(n+1) + \left(\frac{n}{4}\right) + i & if \quad i = 2\ and\ n > 8 \\ 2(n+i) + \left(\frac{n}{4}\right) + 1 & ; \quad 3 \le i \le \left(\frac{n}{4} - 1\right) \\ n + \left(\frac{6n}{4}\right) + i + 2 & ; \quad i = \frac{n}{4}\ and\ n > 8 \\ 3n + \left(\frac{n}{4}\right) + 2i & ; \quad \left(\frac{n}{4} + 1\right) \le i \le n\ and\ n > 4 \end{cases}$$

$$f(v_i^{'}) = i\ ;\ 1 \le i \le n$$

$$f(v) = \left(\frac{3n(3n+1)}{4}\right) - 1$$

$$f(v^{'}) = \left(\frac{3n(3n+1)}{4}\right) + 1$$

Case (3)

If $\boldsymbol{n \equiv 1 (mod\ 4)}$ then

$$f(v_i) = \begin{cases} n + \left\lfloor\frac{n}{4}\right\rfloor + \frac{5(n-1)}{4} + 2 & if \quad i = 1\ and\ n = 5 \\ 2n + \left\lfloor\frac{n}{4}\right\rfloor + i & if \quad i = 1\ and\ n > 5 \\ n + \left\lfloor\frac{n}{4}\right\rfloor + \left(\frac{5(n-1)}{4}\right) + i + 2 & if \quad i = \left\lfloor\frac{n}{4}\right\rfloor\ and\ n > 5 \\ 2n + \left\lfloor\frac{n}{4}\right\rfloor + 2i & ; \quad 2 \le i \le \left(\left\lfloor\frac{n}{4}\right\rfloor - 1\right) \\ 3n + 2i + \left\lfloor\frac{n}{4}\right\rfloor & ; \quad \left(\left\lfloor\frac{n}{4}\right\rfloor + 1\right) \le i \le n \end{cases}$$

$$f(v_i^{'}) = i\ ;\ 1 \le i \le n$$

$$f(v) = \left(\frac{3n(3n+1)}{4}\right) - 1$$

$$f(v^{'}) = \left(\frac{3n(3n+1)}{4}\right) + 1$$

Case (4)

If $\boldsymbol{n \equiv 2 (mod\ 4)}$ then

$$f(v_i) = \begin{cases} 2n + \left\lfloor\frac{n}{4}\right\rfloor + 2i - 1 & ; \quad 1 \le i \le \left\lfloor\frac{n}{4}\right\rfloor \\ 3n + 2i + \left\lfloor\frac{n}{4}\right\rfloor & ; \quad \left(\left\lfloor\frac{n}{4}\right\rfloor + 1\right) \le i \le n \end{cases}$$

$$f(v_i^{'}) = \begin{cases} i & ; \quad 1 \le i < n \\ \left\lfloor\frac{n}{4}\right\rfloor + n & if \quad i = n \end{cases}$$

$$f(v) = \left\lfloor\frac{3n(3n+1)}{4}\right\rfloor$$

$$f(v^{'}) = \left\lfloor\frac{3n(3n+1)}{4}\right\rfloor + 1$$

Case (5)

If $\boldsymbol{n+1 \equiv 0\ (mod\ 4)}$ then

$$f(v_i) = \begin{cases} 2(n+i+1) & if \quad i = 1\ and\ n = 3 \\ 3n+2i & if \quad i = 2\ and\ n = 3 \\ \left(\frac{n+1}{4}\right) + 3n + 2i & if \quad i = 3\ and\ n = 3 \\ 2(n+i+1) & ; \quad 1 \le i \le 2\ \ and\ \ n > 3 \\ \left(\frac{n+1}{4}\right) + 2\ (n+i+1) & ; \quad 3 \le i \le \frac{n+1}{4} \text{ and n} > 3 \\ \left(\frac{n+1}{4}\right) + 3\ n + 2i & ; \quad \left(\frac{n+1}{4} + 1\right) \le i \le n \text{ and n} > 3 \end{cases}$$

$$f(v_i^{'}) = i\ ;\ 1 \le i \le n$$

$$f(v) = \left\lfloor \frac{3n(3n+1)}{4} \right\rfloor$$

$$f(v^{'}) = \left\lfloor \frac{3n(3n+1)}{4} \right\rfloor + 1$$

The labeling of vertices and edges are satisfies the antimagic labeling.

Hence G is an antimagic graph. □

REFERENCE

1. Harary .F, Graph Theory, Narosa Publishing House (1969).
2. Hegde .S.M and Shankaran .P, Geometric Labeled Graphs, AKCE J. Graphs. Combin., 5, No. 1 (2008), pp. 83-97.
3. Joseph .A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, (2013).
4. Revathi .N, Vertex Odd Mean And Even Mean Labeling Of Some Graph, IOSR Journal Of Mathematics, Volume 11, Issue 2 Ver .IV (Mar-Apr.2015) PP 70-74
5. Rosa. A, On Certain Valuations of The Vertices of A Graph Theory of Graphs (Internet, Symposium, Rome, July 1966), Gordon and Dunod Paris (1967).
6. Sampathkumar.E, On Buplicate Graphs, Journal of The Indian Math.Soc, 37(1973) 285-293.
7. Stephen Joh . B and Jovit Vinish Melma. G., Arithmetic Labeling of $C_m \times P_n$ and $P_m \times P_n$, International Journal Of Mathematical Archive, 9(4), 2018.
8. Tao – Ming Wang, Guang – Hui Zhang, On Antimagic Labeling Of Odd Regular Graphs, International Workshop On Combinatorial Algorithms, 162-168, 2012.
9. Uma. R and Murugesan. N, Graceful Labeling Of Some Graphs And Their Subgraphs, Asian Journal Of Current Engineering and Maths 1:6 Nov –Dec (2012) 367-370.
10. Vaidya. S.K and Bantva. D.D Radio Number For Total Graph Of Path, Hindawi Publishing Corporation, ISRN Combinatorics, Volume 2013, Article ID 326038.