On Strongly Nano Regular Generalized and Strongly Nano Generalized Regular Closed Sets in Nano Topological Spaces

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ABSTRACT: *The purpose of this paper is to define and study a newclass of sets called Strongly nano regular generalized and Strongly nano generalized regular closed sets in nano topological spaces. Basic properties of Stronglynano regular generalized closed sets and Strongly nano generalized regular closed sets are analysed. Notions like nano regular generalized closure and nano generalized regular closure and their relation with already existing well known sets are also investigated.*

Key Words: : *Nano closed set, Nano regular closed, Strongly Nano Regular generalized closed set, Strongly Nano generalized regular closed set.*

1. INTRODUCTION

In 1970, Levine introduced the concept of generalized closed sets as a generalization of closed sets in Topological space. Later on N.Palaniappan studied the concept of regular generalized closed set in a topological space. In 2011, SharmisthaBhattacharya have introduced the notion of generalized regular closed sets in topological space. The notion of nano topology was introduced by LellisThivagar which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. In this paper, we have introduced a new class of closed sets on nano topological spaces called strongly nano regular generalized closed sets and strongly nano generalized regular closed sets and the relation of these new sets with the help of existing sets.

2. PRELIMINARIES

Definition 2.1: Let U be a non-empty finite set of objectscalled the universe and R be an equivalence relation of U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with in another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = U \{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = U \{R(x): R(x) \cap X \neq \phi\}$

(iii)The boundary region of X with respect to R is theset of all objects, which can be classified neither X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2: If (U, R) is an approximation space and $X, Y \subseteq U$, then

- $L_R(X) \subseteq X \subseteq U_R(X)$
  $L_R(\phi) = U_R(\phi) =$ & $L_R(U) = U_R(U) = U$

[1] $U_R(XUY) = U_R(X)\ U\ U_R(Y)$

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vi) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(vii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

(viii) $U_R U_R\ (X) = L_R\ U_R(X) = U_R(X)$

(ix) $L_R\, L_R(X) = U_R\, L_R(X) = L_R(X)$

Definition 2.3: Let U be the universe, R be an equivalencerelation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.3, $\tau_R(X)$ satisfies the following axioms:
(i) U and ϕ belongs to $\tau_R(X)$.
(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U,\tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. The elements of $(\tau_R(X))^C$ are called as nano closed sets.
Remark 2.4: If $\tau_R(X)$ is the Nano topology on U with respectto X, then the set $B=\{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5: If $(U, \tau_R(X))$ is a Nano topological space withrespect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of the set A is defined as the union of all Nano open subsets contained in A and it is denoted by NInt(A). That is NInt(A) is the largest Nano open subset of A. The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and it is denoted by NCl(A). That is NCl(A) is the smallest Nano closed set containing A.
Definition 2.6: A Nano topological space $(U, \tau_R(X))$ is said to be extremely disconnected, if the Nano closure of each Nano open set is Nano open.
Definition 2.7: Let $(U,_R(X))$ be a Nano topological spaceand $A \subseteq U$. Then A is said to be

I. Nano Semi open if $A \subseteq NCl(NInt (A))$
II. Nano Pre-open if $A \subseteq NInt(NCl (A))$
III. Nano α-open if $A \subseteq NInt(NCl(NInt (A)))$
IV. Nano Regular open if $A = NInt(NCl (A))$
V. (v) Nano Regular closed if $NCl(NInt (A)) = A$

NSO(U,X), NPO(U,X), NRO(U,X) and $\tau_R^{\alpha}(X)$ respectively, denote the families of all Nano semi-open, Nano Pre-open, Nano Regular open, Nano Regular closed and Nano α-open subsets of U.
Definition 2.8: A subset A of a Nano topological space$(U, \tau_R(X))$ is called Nano Generalized closed set if $NCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $\tau_R(X)$.
Result 2.9: Every Nano closed set is nano generalizedclosed set.
Definition 2.10: A subset A of a Nano topological space$(U, \tau_R(X))$ is called :
(i) Nano semi generalized closed set if $NSCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $\tau_R(X)$.
(ii) Nano generalized semi closed set if $NSCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $\tau_R(X)$.
(iii)Nano generalized α-closed set if $N\alpha Cl(A) \subseteq V$ whenever $A \subseteq V$and V is Nano open in $\tau_R(X)$.
(iv) Nano α generalized closed set if $N\alpha Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano α open in $\tau_R(X)$.

3. STRONGLY NANO REGULAR GENERALIZED CLOSED SETS AND STRONGLY NANO GENRALIZED REGULAR CLOSED SETS

In this section, we introduce Stronglynano regular generalized closed set and investigate some of their properties.
Definition 3.1: If $(U,\tau_R(X))$ is a nano topological space withrespect to X where $X \subseteq U$ and if $A \subseteq U$ then
(i) The nano regular closure of A is defined as the intersection of all nano regular closed sets containing A and it is denoted by NrCl(A). NrCl(A) is the smallest nano regular closed set containing A.
(ii) The nanoregular interior of A is defined as the union of all nano regular open subsets of A contained in A and it is denoted by NrInt(A). NrInt(A) is the largest nano regular open subset of A.

Definition 3.2: A subset A of a Nano topological space $(U, \tau_R(X))$ is called Strongly nano regular generalized closed set if $NrCl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano regular generalized open.
Example 3.3: Let U={a,b,c,d} withU/R= {{a}, {c}, {b,d}} and X={a,b} Then $\tau_R(X)$={U,ϕ, {a}, {a,b,d}, {b,d}} which are open sets. The nano regular closed sets = {ϕ ,U,{b,c,d},{a,c}}
The nano regular open sets = {U, ϕ, {a}, {b,d}, {a,b,d}}
The nano generalized regular closed set ={U, ϕ ,{c},{b,c},{c,d},{a,c}, {a,b,c}, {b,c,d}, {a,c,d}}
The strongly nano regular – generalized closed sets = {U, ϕ ,{c},{b,c},{c,d},{a,c}, {a,b,c}, {b,c,d}, {a,c,d}}

The strongly nano regular – generalized open sets ={ ϕ , U, {a}, {b}, {d}, {a,b}, {a,d},{b,d}, {a,b,d}}
The strongly nano generalized regular open sets ={U, ϕ ,{a},{b},{d},{a,b},{a,d},{b,d},{a,b,d}}
The strongly nano generalized regular closed sets ={ ϕ ,U,{c},{b,c},{c,d}, {a,c},{a,b,c},{b,c,d}, {a,c,d}}

Theorem 3.4:Every nano regular closed set is strongly nano regulargeneralized closed.

Proof:Let A be a nano regular closed set in X such that A⊆V,V is Nano regular open. That is NCl(NInt (A)) = A. Since A is nano regular open, NInt(A) = A. Since A is nano regular closed. Therefore NCl(A) = A⊆V. NCl(A)⊆ V. Since A ⊆V then NCl(A) ⊆V whenever V is nano regular open. Hence A is strongly nano regular generalized closed.

The converse of the above theorem is not true which can be seen from the following example.

Example 3.5:Let U={a,b,c,d} with U/R = {{a}, {c}, {b,d}}. Then the nano topology$\tau_R(X)$ = {U,ϕ , {a}, {a,b,d}, {b,d}}. Here {a,c,d} is strongly nano regular generalized closed sets but it is not nano regular closed.

Theorem 3.6:Every nano regular closed set is strongly nano generalizedregular closed set.

Proof:Let A be a nano regular closed set in X such that A⊆V,V is Nano open. That is NCl(NInt (A)) = A. Since A is nano open, NInt(A) = A. Every Nano regular closed set is nanoclosed(by above example). Therefore NCl(A) = A ⊆V. NCl(A)⊆V. Since A⊆V then NCl(A)⊆V whenever V is nano open. Hence every nano regular closed set is strongly nano generalized regular closed.

The converse of the above theorem is not true which can be seen from the following example.

Example 3.7:Let U={a,b,c,d}with U/R = {{a}, {c}, {b,d}}. Then the nano topology
$\tau_R(X)$ = {U,ϕ , {a}, {a,b,d}, {b,d}}. Here {a,b,c} is strongly nano generalized regular closed sets but it is not nano regular closed.

Theorem 3.8:The union of two stronglynano regular generalizedclosed sets in (U, $\tau_R(X)$) is also a strongly nano regular generalized closed set in (U, $\tau_R(X)$).

Proof: *Let A and B be two* strongly*nano*regular generalized closed setsin (U, $\tau_R(X)$) and V be any nano regular open set in U such that A⊆V and B⊆V. Then we have AUB⊆V. As A and B are strongly nano regular generalized closed sets in (U, $\tau_R(X)$).

Therefore NrCl(A)⊆V; NrCl(B)⊆V.Now NrCl(AUB)=(NrCl(A))U(NrCl(B))⊆V.Thus we have NrCl(AUB))⊆V whenever AUB⊆V, V is nano regular open set in U. AUB is a strongly nano regular generalized closed set in U.

Theorem 3.9:The intersection of any two subsets of strongly nanoregular generalized closed sets in (U, $\tau_R(X)$) is strongly nano regular generalized closed set in (U, $\tau_R(X)$).

Proof:Let A and B are any two strongly nano regular generalized closedsets. A⊆ V; V is annano regular open and B⊆ V; V is nano regular open. Then NrCl(A)⊆ V; NrCl(B) ⊆V. Therefore NrCl(A∩B) ⊆V; V is nano regular open in X. Since A and B are strongly nano regular generalized closed sets. Hence A∩B is a strongly nano regular generalized closed set.

Theorem 3.10: If A is both Nano regular open and strongly nano regulargeneralized closed set in X, then A is nano regular closed set.

Proof:Since A is nano regular open and strongly nano regulargeneralized closed in X, NrCl(A) ⊆V But A⊆NrCl(A). Therefore A = NrCl(A). Since A is nano closed NInt(A)=A. NrCl(A)) = A. Hence A is nano regular closed.

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