Model equations of a thermoelastic medium with microtemperatures and microconcentrations

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ABSTRACT: *Present article is presented to obtain two-dimensional model of the equations of a thermoelastic medium with microtemperatures and microconcentrations in the context of linear theory of continuum mechanics. The medium is assumed to be homogeneous and isotropic. It can be 9erified that there exist various kinds of coupled longitudinal waves in addition to transverse, microtemperature and microconcentration waves in such type of medium.*

Introduction

Assuming that the microelements of a thermoelastic material have different temperatures, the concept of microtemperatures has been derived. Microtemperatures depend homogeneously on microcoordinates of the microelements, which are based on the microstructure of the continuum. The study of materials with microtemperatures is of great importance due to increasing scope and interest in the current research area. Grot [1] formulated a theory of thermodynamics of elastic bodies with microstructure whose microelements possess microtemperatures. The Clausius-Duhem inequality is modified to include microtemperatures and the first-order moment of the energy equations are added to the usual balance laws of a continuum with microstructure. A theory of heat conducting micromorphic continuum has been developed by Riha [2] to analyze the heat conduction in materials with inner structure. Iesan and Quintanilla [3] proposed a linear theory of thermoelastic material with microtemperatures, in which the particles of the material are subjected to classical displacement, temperature field and may possess microtemperatures. Iesan [4] put forward the theory of micromorphic elastic solids with microtemperatures.

The theory of thermoelastic bodies with microstructures and microtemperatures was presented by Iesan [5]. In this theory, the microelements of the material possess microtemperatures and can undergo microrotation, microstretch and translation. Scalia and Svanadze [6] presented the description of solutions of the theory of thermoelasticity with microtemperatures. Steeb et al. [7] examined time harmonic waves in thermoelastic material with microtemperatures. In this article, it has been found that there can exist three sets of coupled dilatational waves and a shear wave in an infinite thermoelastic half-space with microtemperatures. Kumar et al. [8] studied wave propagation at an interface of elastic and microstretchthermoelastic solid with microtemperatures. In this paper, the amplitude ratios of various reflected waves are found to be functions of angle of incidence and frequency of incident wave. The propagation of Rayleigh waves in a strongly elliptic thermoelastic material with microtemperatures was studied by Passarella and Tibullo [9].A strain gradient theory of thermoelastic bodies with microtemperatures which permits the transmission of heat as thermal waves at finite speed was proposed by Iesan [10]. Recently, Ailawalia et al. [11] discussed a two-dimensional problem in a thermoelastic solid with microtemperatures subjected to an internal heat source.

Diffusion is a spontaneous migration of the particles from an area of higher concentration to that of lower concentration, until equilibrium is reached. Diffusion is important in many life processes. The study of this phenomenon is of great concern due to its many geophysical and industrial applications. Thermodiffusionin an elastic solid is due to coupling of the fields of strain, temperature and mass diffusion in addition to heat and mass exchange with the environment.

Singh [12, 13] studied the reflection of P and SV waves from free surface of an elastic solid with generalized thermodiffusion. In the discussion of these articles, the reflection coefficients are found to depend upon the angle of incidence of P and SV waves, thermodiffusion constants and other material parameters. The influence of magnetic field on wave propagation in generalized thermoelastic solid with diffusion was measured by Abo-Dahab and Singh [14]. Kumar et al. [15] presented the study of propagation of plane waves in micropolarthermodiffusion elastic half-space in the context of generalized theories of thermoelasticity. A theory of thermoelastic diffusion with microtemperatures and microconcentrations was employed by Aouadi et al. [16]. In this paper, microelement temperature and mass diffusion are supposed to be linear functions of microcoordinates and the Clausius-Duhem inequality is also modified to include microtemperatures and microconcentrations.

Formulation of the model equations

According to Aouadi et al. [16], the balance equation of mass diffusion is given by

$$\dot{C} = \eta_{i,i}, (1)$$

where$C$ is the concentration of the diffusive material and η_i is the flux vector of mass diffusion and it is given by

$$\eta_i = hP_{,i} + h_1 C_i, (2)$$

where$h$ is the thermodiffusion constant, P is the chemical potential, h_1 is the constitutive coefficient and C_i are the microconcentrations vector.

The expression for chemical potential is given by

$$P = -\beta_2 e_{ij} - a\theta + bC, (3)$$

whereθ denotes temperature of the medium, $\beta_2 = (3\lambda + 2\mu)\alpha_t$, λ, μ are Lame's constants and α_t is the coefficient of linear thermal expansion.

From (1), (2) and (3), one can get

$$\dot{C} = h\left(-\beta_2 e_{ij} - a\theta + bC\right)_{,ii} + h_1 C_{i,i}\ , \quad (4)$$

whichis known as "**mass diffusion equation**".

We introduce the first moment of mass diffusion by

$$\rho\dot{\Omega} = \eta_{ji,j} + \eta_i - \sigma_i, (5)$$

whereσ_i is micromass diffusion flux average and η_{ji} is the first mass diffusion flux moment. Here, σ_i and η_{ji} can be expressed as:

$$\sigma_i = (h - h_3)P_{,i} + (h_1 - h_2)C_i, \qquad (6)$$

$$\eta_{ji} = -h_4 C_{k,k}\delta_{ij} - h_5 C_{i,j} - h_6 C_{j,i}, \quad (7)$$

where$h_2, \dots, h_6$ are constitutive coefficients and δ_{ij} is Kronecker delta.

Now, we use the relation

$$\rho\Omega = -m_1 C_i - k_1 T_i, \qquad (8)$$

wherem_1 is microthermal conductivity, T_i are microtemperature components and k_1 is constant.

The combination of (2) and (5)-(8) provides us the field equation for a microconcentrative media.

Also by the previous procedure and using the constitutive relations given by Iesan [5], we can get the field equations of a thermoelastic medium with microtemperatures. Thus, in case of an isotropic and homogeneous solid, yhe constitutive equations are given by

$$\sigma_{ij} = \lambda e_{kk} + 2\mu e_{ij} - \beta_1\theta\delta_{ij} - \beta_2 C\delta_{ij},$$

$$q_i = k\theta_{,i} + k_1 T_i,$$

$$\varsigma_i = (k - k_3)\theta_{,i} + (k_1 - k_2)T_i,$$

$$q_{ji} = -k_4 T_{k,k}\delta_{ij} - k_5 T_{i,j} - k_6 T_{j,i}.$$

where$k_2, \dots, k_6$ are constitutive coefficients, q_{ij} is the first heat flux moment tensor, σ_{ij} are the components of stress, e_{ij} are the components of strain and q_i is the heat flux vector.

Conclusion

We have derived the linear equations of a thermoelastic medium with microtemperatures and microconcentrations. These are the simplest equations of elastic solids that takes into account the new variables. By describing thermal-diffusion interactions at the macroscopic and microscopic levels, the derived equations in this article are closer to the realistic constitutive structure of solids.

References

1. Grot R.A., 1969. Thermodynamics of a continuum with microstructure. Int. J. Eng. Sci. 7, 801-814.
2. Riha P., 1976. On the microcontinuum model of heat conduction in materials with inner structure. Int. J. Eng. Sci. 14, 529-535.
3. Iesan D., Quintanilla R., 2000. On a theory of thermoelasticity with microtemperatures. J. Therm. Stress. 23, 199-215.
4. Iesan D., 2001. On a theory of micromorphic elastic solids with microtemperatures. J. Therm. Stress. 24, 737-752.

5. Iesan D., 2007. Thermoelasticity of bodies with microstructures and microtemperatures. Int. J. Solids Struct. 44, 8648-8662.
6. Scalia A., Svanadze M., 2006. On the representation of solutions of the theory of thermoelasticity with microtemperatures. J. Therm. Stress. 29, 849-863.
7. Steeb H., Singh J., Tomar S.K., 2013. Time harmonic waves in thermoelastic material with microtemperatures. Mech. Res. Commun. 48, 8-18.
8. Kumar R., Kaur M., Rajvanshi S.C., 2013. Wave propagation at an interface of elastic and microstrechthermoelastic solids with microtemperatures. J. Solid Mech. 5, 226-244.
9. Passarella F., Tibullo V., 2016. On the propagation of Rayleigh waves in a strongly elliptic thermoelastic material with microtemperatures. J. Therm. Stress. 39, 1111-1118.
10. Iesan D., 2018. On a theory of thermoelasticity without energy dissipation for solids with microtemperatures. J. Appl. Math. Mech. 98, 1-16.
11. Ailawalia P., Budhiraja S., Singh J., 2018. Two-dimensional deformation in a thermoelastic solid with microtemperatures subjected to an internal heat source. Int. J. Appl. Mech. Eng. 23, 5-21.
12. SinghB., 2005. Reflection of P and SV waves from the free surface of an elastic solid with generalized thermodiffusion. J. Earth. Syst. Sci. 114, 159- 168.
13. Singh B., 2006. Reflection of SV waves from the free surface of an elastic solid with generalized thermoelastic diffusion. J. Sound Vib. 291, 764-778.
14. Abo-Dahab S. M., Singh B., 2009. Influences of magnetic field on wave propagation in generalized thermoelastic solid with diffusion. Arch. Mech. 61, 121-136.
15. Kumar R., Kaushal S., Marin M., 2018. Propagation of waves in micropolar thermodiffusion elastic half-space. Afr. Mat. 29, 451-462.
16. Aouadi M., Ciarletta M., Tibullo V., 2017. A thermoelastic diffusion theory with microtemperatures and microconcentrations. J. Therm. Stress. 40, 486- 501.