Simplicity of groups with Sylow's theorems

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Received: February 01, 2019 Accepted: March 23, 2019

ABSTRACT: *The aim of this paper is to check whether the group is simple or not. We will give some example.*

Key Words: *GROUP, SET, PRIME, SIMPLE GROUP, SYLOW SUBGROUP.*

Introduction:
The Sylow theorems are three powerful theorems in modern algebra which help us to show that group of certain order is not simple. Sylow theorem is partial converse of lagrange's theorem. In this article we discuss the properties of some groups of certain order using Sylow theorem.

Defination: Let G be a group and p be a prime number.

1. A group of order p for some aϵN is called a pgroup.
2. If G is a group of order p m, where płm, then a

Subgroup of order p is called a sylow p subgroup of G.

SIMPLEGROUP:
Let G be group. Then, G is said to be simple group if G has no proper normal subgroup.

Sylow's 1st Theorem:
Let G be a group of order p^am,wherep is prime and płm, then Sylowp-subgroup of G exist.

Sylow's 2nd Theorem:
Any two sylow p subgroup of G are conjugate in G.

Sylow's 3rd Theorem:
The number of sylow p subgroups of G is of the form 1+kp where 1+kp divides m.

THEOREM2:
A Sylow p subgroup of a finite group G is normal subgroup of G if it is the only p subgroup of G.

THEOREM3:
Any group of prime order is always cyclic.

Ex.1 Let G be a group of order 132.
$O(G)=132=2^2.3.11$

Bysylow's 1st theorem ,
it has 2 Sylow subgroup, 3 Sylow subgroup,11 Sylow subgroup

By sylow's 3rd
Possibility for 2 Sylow subgroup=1,3,11,33
Possibility for 3 Sylow subgroup =1,4,22,44
Possibilityfor 11 Sylow subgroup=1,12

By theorem 2,
if any of Sylow subgroup is unique if it is normal.
If all Sylow subgroup of G are not unique ,then Minimum number of 2 sylow subgroup=3 Minimum number of 3 sylow subgroup=4
Minimum number of 11 sylow subgroup=12

Order of 11 sylow subgroup is=11

By theorem 3, 11 sylow subgroup is cyclic.So it has 10 elements of order 11.Thus, group G has 120 elements of order 11.
Now Order of 3 sylow subgroup is=3

By theorem3, 3 sylow subgroup is cyclic.
So it has 2 elements of order 3.
Thus, group G has 8 elements of order 3.
So this group has 120 elements of order 11 ,8 elements of order 3,one
element of order 1.Thus we have 129 elements . So we left with 3 elements .From these 3 elements we can construct only one 2 sylow subgroup. So it has to be unique. Thus G is not simple.
Hence group of order 132 is not simple.

EX.2 Let G be a group of order 105.
O(G)=7.5.3

BYsylow's 1st theorem 1, it has 3 sylow subgroup, 7 sylow subgroup, 5 sylow subgroup

By sylow's 3rd theorem
Possibility for 7 Sylow subgroup=1,15
Possibility for 5 Sylow subgroup=1,21

By theorem 2, if any of sylow subgroup is unique iff it is normal. If all of Sylow subgroup of G are not unique then number of 7 sylow subgroup=15
Then G has 15 subgroups $H_1, H_2, \ldots H_{15}$ such that order of each H_i is 7. Hence each H_i contains 6 elements of order 7. Then G has 15×6=90 elements of order 7.
Now if number of 5 sylow subgroup=21
Then G has 21 subgroups $K_1, K_2, \ldots K_{21}$ such that order of each K_i is 5. Hence each K_i contains 4 elements of order 5. Then G has 21×4=84 elements of order 5.
So if number of 7 sylow subgroup=15 and number of 5 sylow subgroup=21 then G must contain 90 elements of order 7 and 84 elements of order 5 i.e. G contain 90+84=174 elements which is a contradication. Hence G is not simple group

Hence group of order 108 is not simple.

CONCLUSION:

1. A group of order 132 is not simple.
2. A group of order 108 is not simple.

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