Effect of Central Point Load on FGM Plate Subject to Various Boundary Conditions

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ABSTRACT: *Functionally Graded Material (FGM) is a composite in which material properties vary gradually. The properties of the FGM depends upon material volume fraction. The volume fraction is varied by Power law (P-FGM) and Sigmoid law (S-FGM). FGMs are used in various geometry forms e.g. shells, plates, beams etc. FSDT and TSDT are found reliable for the FGM plates. FSDT is suitable for thin plates as the effect of transverse shear stress is less. FEM has been popular among researchers. In the present work FSDT has been used for the analysis of FGM plates. An FGM plate is subjected to point load and various boundary conditions. The results for P-FGM and S-FGM plate under central point load has been determinedin terms of non dimensional parameters e.g. deflection, stress and strain.*

Key Words: *FEM, FGM, FSDT, Deflection, stress.*

1. Introduction

Heterogeneous composite materials are very popular in engineering applications. The constituents of heterogeneous composite materials can be two or more different material phases and compositions. One of the recently developed heterogeneous materials is Functionally Graded Material (FGM). FGM is a typical composite material in which the material properties vary gradually in one or more directions in space. The properties of the FGM depends upon the distribution of the material volume fraction. The volume fraction is varied by certain laws among which Power law (P-FGM) and Sigmoid laws (S-FGM) are two of the most used laws. Cheng and Batra (2000),Qian et.al. (2004), used power law for estimation of volume fraction in FGM while Shyang (2006), Bhandari and Purohit (2014) used Sigmoid law for the same since sigmoid law gives smoother variation in volume fraction. FGMs are used in various geometry forms e.g. shells, plates, beams etc. Since plate exhibits 2-D behavior they provide an intermediate level prototype for formulating and testing. A number of theories have used by various researchers to analyze the performance of the FGM plates. First Order Shear Deformation Theory (FSDT) and Third Order Shear Deformation Theory (TSDT) are found reliable for the FGM plates in which the volume fractions vary continuously with position in a predefined manner. FSDT has been used by Dai et. al. (2005) and Alshorbagyet. al. (2013)and concluded that FSDT is suitable for thin plates as the effect of transverse shear stress is relatively less. while TSDT has been used by Reddy (1998) and Suresh et. al. (2005).Further, it is also observed that the researchers have employed various boundary conditions, which include simply supported (S) at all edges, by Reddy (1998), Sharma et.al. (2016) and clamped (C) at all edges by Cheng and Batra (2000), Bhandari and Purohit (2015). Although experimental, analytical and numerical work has been carried out on FGM, finite element method has been very popular among the researchers. Craig et. al. (2002) used FEM for design and analysis of functionally graded materials. Ki-Hoon (2006) introduced a method for FEA-based design of heterogeneous objects like FGM. Yasser and Naotake (2008) used the homogenization method (HM) based on the finite element method (FEM) Moitaet. al. (2018) implemented finite element model based on a non-conforming triangular flat plate.Various combinations of ceramic and metal have been used and analyzed by the researchers e.g. J. N. Reddy (2000) and Dai et. al. (2005) used Aluminum-Zirconia, Ki-Hoon (2006) used Aluminum-Alumina, Cheng and Batra (2000) used Monel-Zirconiaetc. In the literature the combination of Aluminum (metal) and Zirconia (Ceramic) has widely been used as functionally graded material (FGM).
In the present work Aluminum-Zirconia (Al-ZrO_2) is used as FGM. Al-ZrO_2. The FSDT has been used for the analysis of FGM plates. An FGM plate is subjected to point load and various boundary conditions i.e. combination of simply supported (S), clamped (C) and free (F) boundary condition. The FE analysis has been done and results for P-FGM and S-FGM plate under central point load has been determined. The results are presented in terms of non dimensional parameters Bhandari and Purohit (2014) e.g. deflection, stress and strain.

2. Methodology

1. **Material:**FGM is made up of ceramic and metal with controlled variation of volume fraction of the material. The metal for the FGM is taken as Aluminum and the ceramic is Zirconia (Al-ZrO_2). The physical properties e.g. modulus of elasticity, Poisson's ratio, density etc. of Aluminum and Zirconia are used as input to calculate the variation in the volume fraction.
2. **Gradation:**The material gradation is governed by Power law and Sigmoid law. The volume fraction is calculated using these two laws and accordingly material properties are evaluated along the thickness of the plate. The analysis is performed various values of the volume fraction exponent (n) in P-FGM and S-FGM. The values of volume fraction exponent (n) are: n=0 (ceramic), 0.1, 0.2, 0.5, 1, 2, 5, 10, 100, n=infinity (metal)
3. **Load and dimensions:** A square FGM plate (1mx1m) is considered here i.e. aspect ratio is taken unity. The mechanical analysis is performed by applying central point load (10^6N) for various boundary conditions.
4. **Boundary conditions:** The boundary conditions are combination of simply supported (S), clamped (C) and free (F) boundary conditions.
5. **Numerical method:**Finite element method is used to analyse and calculate various non-dimensional parameters e.g. Non-dimensional deflection, Non-dimensional Tensile stress, Non-dimensional Shear stress, strain and shear strain.

3. Results

3.1 Variation of boundary conditions with point load

In this section the results of analysis performed on a square size (a/b =1) P-FGM and S-FGM plate with various boundary conditions subject to constant point load are discussed. The results are reported in terms of non-dimensional parameters i.e. a. non-dimensional deflection ($\overline{u_z}$), b. non-dimensional tensile stress ($\overline{\sigma_x}$), c. non-dimensional shear stress ($\overline{\sigma_{xy}}$), d. strain (e_x) and e. shear strain (e_{xy}).

a. Non-Dimensional Deflection ($\overline{u_z}$)

Fig.1 and Fig.2 show the comparative bar charts of non-dimensional deflection ($\overline{u_z}$) for various boundary conditions of a square plate under point load for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM have been presented.

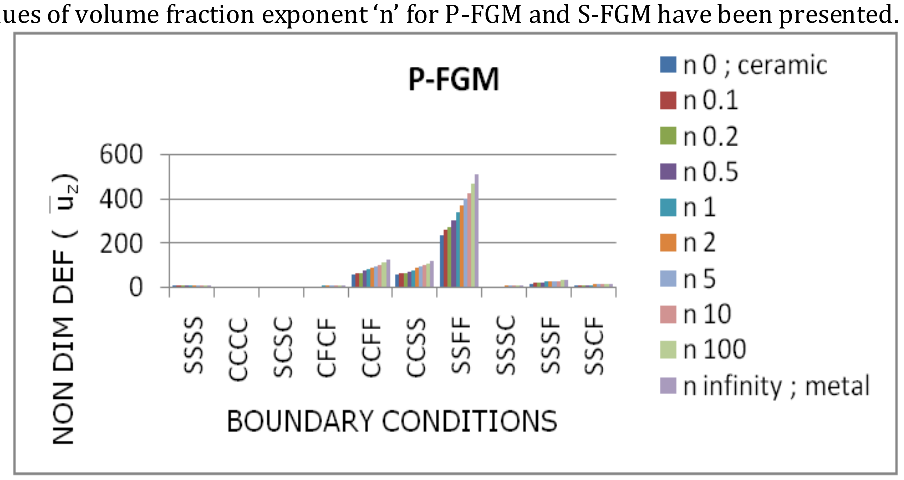

Fig. 1: Effect of boundary conditions on non-dimensional deflection ($\overline{u_z}$) of a square plate under point load for P-FGM

Comparing Fig.1 and Fig.2 one can conclude that:

(i) The maximum non-dimensional deflection for pure metal and pure ceramic plate under SSFF boundary condition is approximately 510 and 237 respectively. The non-dimensional deflection increases with increasing value of volume fraction exponent 'n'.

(ii) It is also found that the maximum deflection occurs for simply supported-free (SSFF) boundary conditions and minimum for all sides clamped (CCCC) boundary condition among all the cases considered.

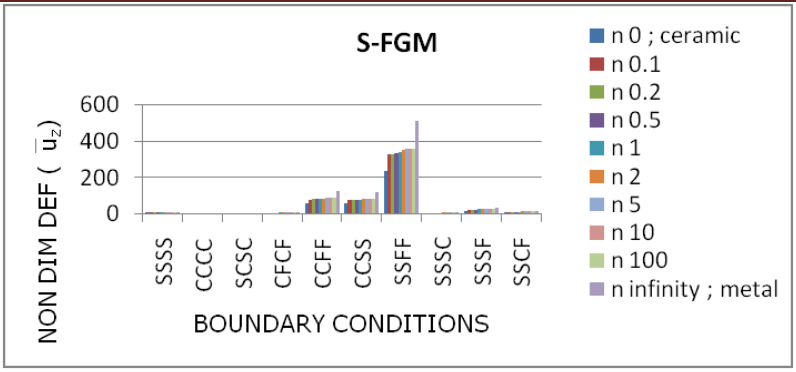

Fig. 2: Effect of boundary conditions on non-dimensional deflection ($\overline{u_z}$) of a square plate under point load for S-FGM

b. Non-Dimensional Tensile Stress ($\overline{\sigma_x}$)

The numerical results for variation of non-dimensional tensile stress ($\overline{\sigma_x}$) for various boundary conditions of a square plate under point load for P-FGM and S-FGM are shown in Fig.3 and Fig.4 respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM have been presented.

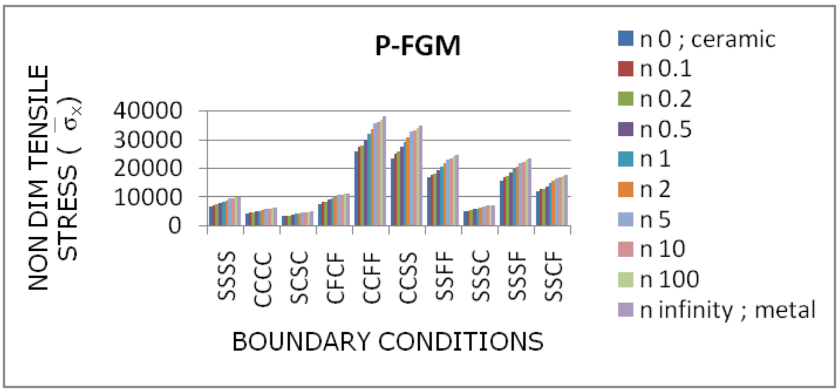

Fig.3: Effect of boundary conditions on non-dimensional tensile stress ($\overline{\sigma_x}$) of a square plate under point load for P-FGM

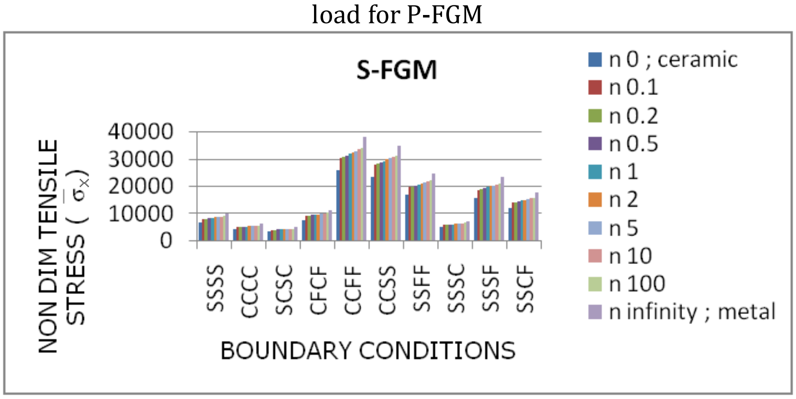

Fig.4: Effect of boundary conditions on non-dimensional tensile stress ($\overline{\sigma_x}$) of a square plate under point load for S-FGM

A close comparison of these graphs reveals the following information:

The non-dimensional tensile stress increases with increasing value volume fraction exponent 'n'. The maximum tensile stress (37777) occurs for clamped-free (CCFF) boundary conditions and the second highest value of tensile stress (34716) is obtained for clamped-simply supported (CCSS) boundary condition. The minimum (3459) is obtained for simply supported–clamped (SCSC) boundary condition amongst all the cases considered here.

c. Non-dimensional Shear Stress ($\overline{\sigma_{xy}}$)

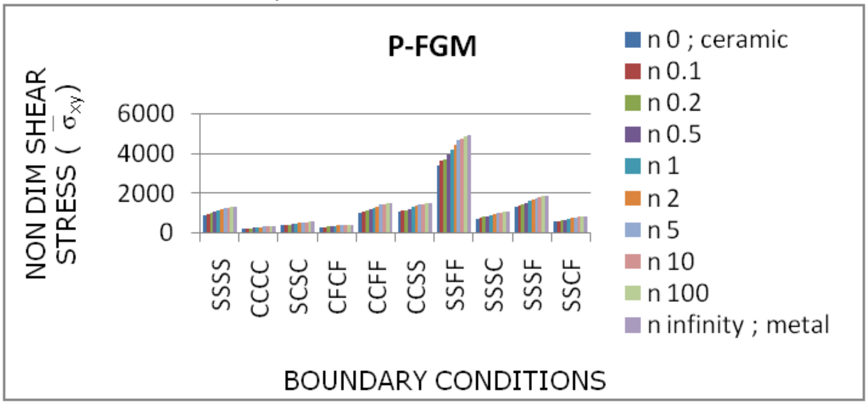

Fig.5: Effect of boundary conditions on non-dimensional shear stress ($\overline{\sigma_{xy}}$) of a square plate under point load for P-FGM

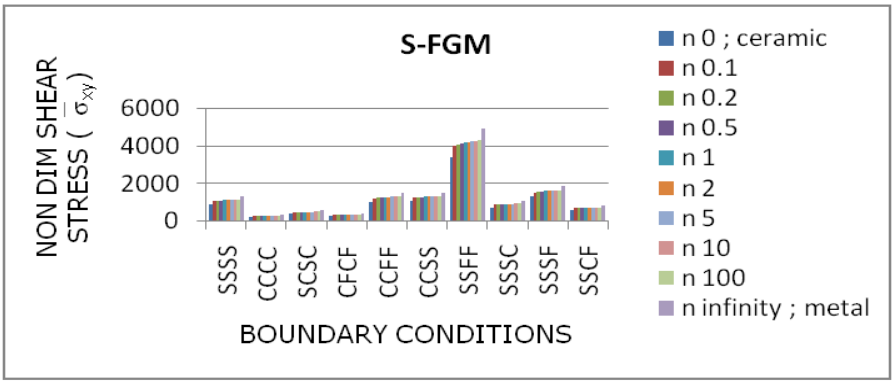

Fig.6: Effect of boundary conditions on non-dimensional shear stress ($\overline{\sigma_{xy}}$) of a square plate under point load for S-FGM

Fig. 5 and Fig. 6 show the variation of non-dimensional shear stress ($\overline{\sigma_{xy}}$) for various boundary conditions of a square plate under point load for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM have been presented.
The following observation are made by comparing the non-dimensional shear stress ($\overline{\sigma_{xy}}$) for various types of boundary conditions and various types of FGM:
The non-dimensional shear stress ($\overline{\sigma_{xy}}$) increases with increasing value of volume fraction exponent 'n'. In case of SSSS boundary condition, for n=0, the value of shear stress is approximately 921, which increases to 1332 for n=∝. The maximum non-dimensional shear stress ($\overline{\sigma_{xy}}$) for simply supported-free (SSFF) boundary conditions is of the order of 4922 while its minimum value is 245 for clamped (CCCC) boundary condition, among all the cases considered here.

d. Strain (e_x)

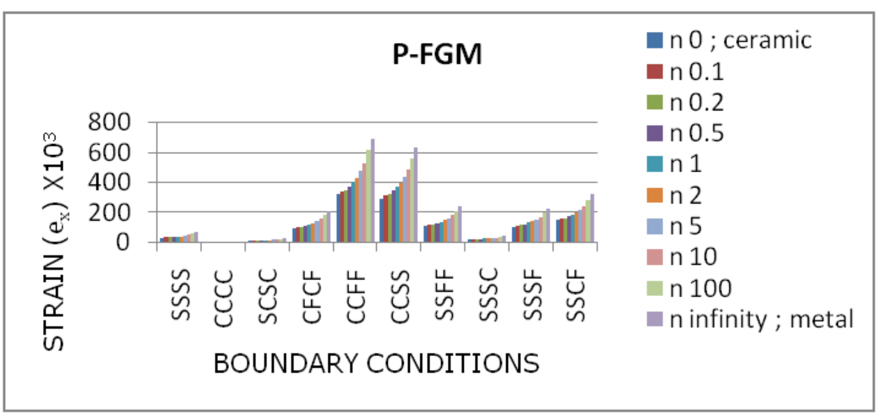

Fig.7: Effect of boundary conditions on strain (e_x) of a square plate under point load for P-FGM

Fig. 7 and Fig. 8 show the variation of Strain (e_x) for various boundary conditions of a square plate under point load for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM have been presented.

Comparing Fig. 6 and Fig. 7, one can conclude the following:

(i) The isotropic ceramic plate has the minimum strain (e_x) for all the boundary conditions considered here, and the isotropic metallic has the maximum Strain (e_x).The Strain (e_x) increases with increasing value of volume fraction exponent 'n'.

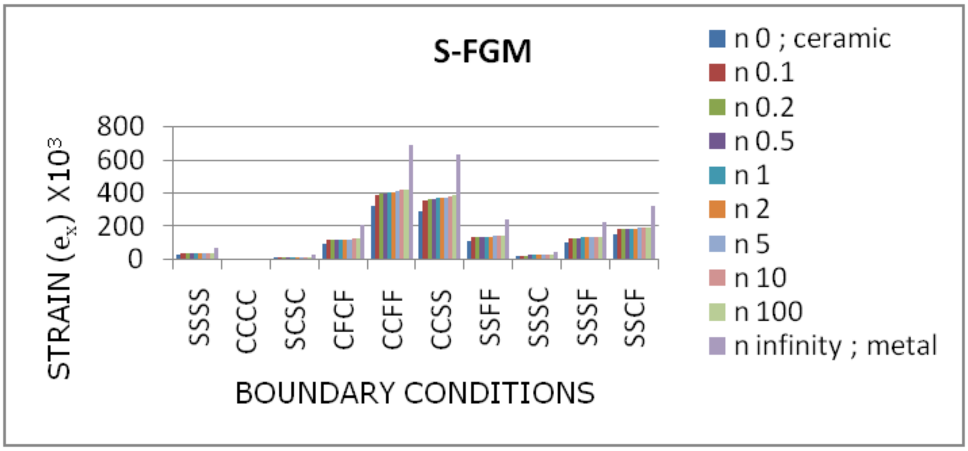

Fig.8: Effect of boundary conditions on strain (e_x) of a square plate under point load for S-FGM

(ii) In case of P-FGM, increase in strain is approximately proportional to the volume fraction exponent, whereas in case of S-FGM such trend is not seen, it increases slowly with increasing value of 'n' but at 'n' equal to infinity i.e. pure metal, sudden jump in the strain is observed. The maximum Strain (e_x) is obtained for clamped - free (CCFF) boundary conditions and its value for pure metal and pure ceramic plate is 0.69 and 0.32 respectively. The minimum strain (e_x) is obtained in the case of CCCC boundary condition amongst for all the cases considered here.

e. Shear Strain (e_{xy})

Fig. 9 and Fig. 10 show the variation of shear strain (e_{xy}) for various boundary conditions of a square plate under point load for P-FGM and S-FGM respectively. The comparison of results for various values of volume fraction exponent 'n' for P-FGM and S-FGM have been presented.

The effect of boundary conditions on shear strain (e_{xy}) is given below:

The isotropic ceramic plate has the lowest shear strain (e_{xy}) for all the boundary conditions considered here, whereas the isotropic metallic plate has the largest shear strain (e_{xy}). It is also found that the maximum shear strain (e_{xy}) is obtained for simply supported - free (SSFF) boundary conditions and while it is minimum for clamped (CCCC) boundary condition.

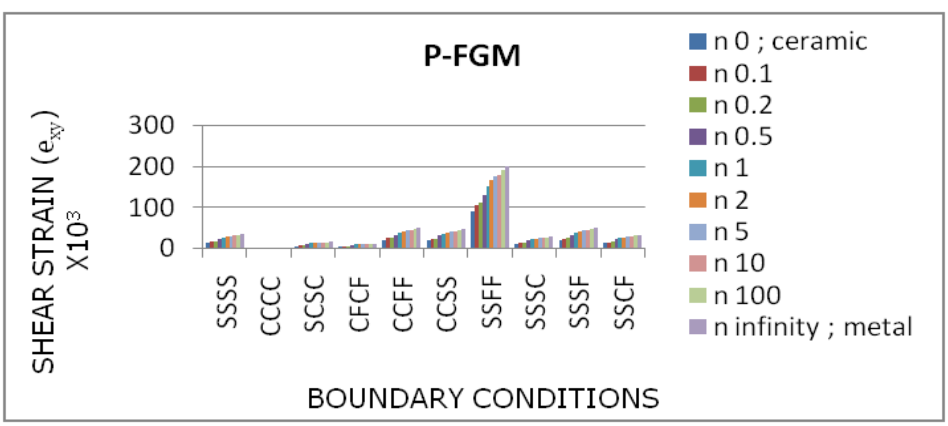

Fig. 9: Effect of boundary conditions on shear strain (e_{xy}) of a square plate under point load for P-FGM

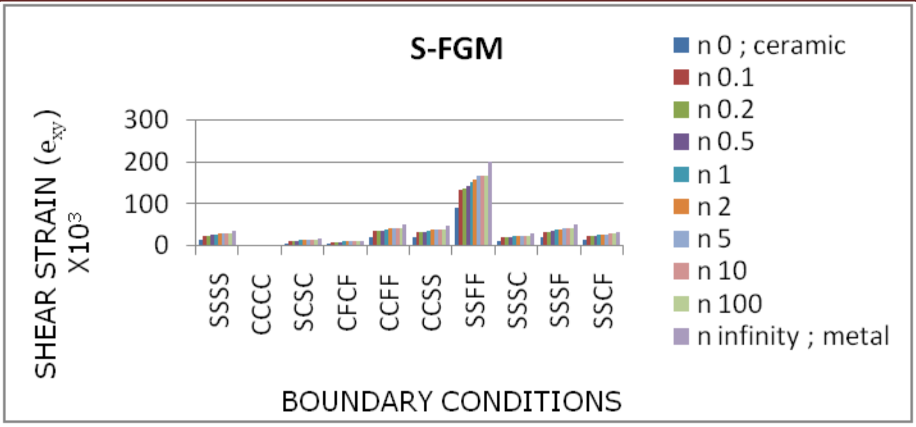

Fig.10: Effect of boundary conditions on shear strain (e_{xy}) of a square plate under point load for S-FGM

4. Conclusion

The behaviour of FGM plate under mechanical load environment was studied. The work includes parametric study performed by varying volume fraction distribution, types of load and boundary conditions. The close investigation of the various graphs for various non-dimensional parametersreveals the following:

(a) As the volume fraction exponent 'n' increases the non-dimensional deflection increases. This is due to the fact that the modulus of elasticity and thereby bending stiffness is the maximum for ceramic plate, while minimum for metallic plate, and degrades continuously as 'n'increases.

(b) The value of non-dimensional deflection, tensile stress, shear stress, strain and shear strain for the FGM's are in between that of ceramic and metal.

(c) The values of non-dimensional characteristics are quite highwhen the FGM plate is subjected to point load as compared with that of UDL since the load is uniformly distributed and whereas the point load is concentrated at a point.

(d) The maximum deflection occurs for simply supported - free (SSFF) boundary condition and minimum for clamped (CCCC) boundary condition among all the cases considered here.

(e) The maximum tensile stress occurs for clamped - free (CCFF) boundary conditions and the second highest value of tensile stress is obtained for clamped-simply supported (CCSS) boundary condition. The minimum is obtained for simply supported – clamped (SCSC) boundary condition amongst all the cases considered here.The maximum non-dimensional shear stress ($\overline{\sigma_{xy}}$)is obtained for simply supported - free (SSFF) boundary conditions while its value is minimum for clamped (CCCC) boundary condition among all the cases considered here.

(f) The strain (e_x) increases with increasing value of volume fraction exponent 'n'. This is due to the fact that the bending stiffness is the maximum for ceramic plate, while minimum for metallic plate, and degrades continuously as 'n' increases.

References

1. Cheng Z.Q, Batra R.C. (2000). Three-Dimensional Thermoelastic Deformations of Functionally Graded Elliptic Plate. Composites: Part B, 31: 97–106.
2. Qian L.F., Batra R.C. and Chen L.M. (2004). Static And Dynamic Deformations Of Thick Functionally Graded Elastic Plates By Using Higher Order Shear And Normal Deformable Plate Theory And Meshless Local Petrov-Galerkin Method. Composite Part B, 35: 685–697.
3. Shyang-Ho C. and Yen-Ling C. (2006). Mechanical Behavior of Functionally Graded Material Plates Under Transverse Load—Part II: Numerical Results. Int J of Solids and Structures, 43: 3675–3691.
4. Bhandari M. and Purohit K. (2014). Static Response of Functionally Graded Material Plate under Transverse Load for Varying Aspect Ratio. International Journal of Metals.http://dx.doi.org/10.1155/2014/980563.
5. Dai K.Y., Liu G.R., Han X. and Lim K.M. (2005).Thermo mechanical Analysis of Functionally Graded Material Plates Using Element-Free Galerkin Method. Computers and Structures, 83: 1487–1502.
6. Alshorbagy E., Alieldin S.S., Shaat M. and Mahmoud F.F. (2013). Finite Element Analysis of The Deformation Of Functionally Graded Plates Under Thermomechanical Loads. Hindawi Publishing Corporation Mathematical Problems in Engg.2013: 1-14.
7. Reddy J.N. (1998). Thermomechanical Behavior of Functionally Graded Materials. Final Report for Afosr Grant F49620-95-1-0342, Cml Report 98-01.
8. Dai K.Y., Liu G.R., Han X. and Lim K.M. (2005).Thermo mechanical Analysis of Functionally Graded Material Plates Using Element-Free Galerkin Method. Computers and Structures. 83: 1487–1502.

9. Sharma K., Kumar D. and Gite A. (2016). Thermo-mechanical buckling analysis of FGM plate using generalized plate theory. AIP Conference Proceedings 1728.https://doi.org/10.1063/1.4946236.
10. Bhandari M. and Purohit K. (2015). Response of Functionally Graded Material Plate under Thermomechanical Load Subjected to Various Boundary Conditions. International Journal of Metals. http://dx.doi.org/10.1155/2015/416824.
11. Craig S.C., Phillip W.Y., Jacob A., Marek J.P., Steven M.A. and Brett A.B. (2002). Higher Order Theory - Structural/Micro Analysis Code (Hot-Smac) Software for Thermo-Mechanical Analysis of FGMs. Collier Research Corporation, 02:1-8.
12. Ki-Hoon S. (2006).Fea Based Design of Heterogeneous Objects. Int Design Engg. Technical Conferences & Computers and Information in Engg. Conference, Philadelphia, Pennsylvania, Usa, 06: 10-13.
13. Yasser M.S. and Naotake N. (2008). Numerical Evaluation of The Thermo mechanical Effective Properties of a Functionally Graded Material Using the Homogenization Method. Int J of Solids and Structures, 45: 3494–3506.
14. Moita J., Araújo A., Correia V.Soares C. and Herskovits J.(2018). Buckling and nonlinear response of functionally graded plates under thermo-mechanical loading. Composite Structures, 202:719-730.
15. Reddy J.N. (2000). Analysis of Functionally Graded Plates.Int J Numer. Meth. Engg, 47: 663-684.