CHROMATIC NUMBER TO THE TRANSFORMATION OF SOME GRAPHS

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ABSTRACT: *Let $G = \{V, E\}$ be a connected simple graph. A vertex coloring of a graph G is a function $f: V(G) \to C$ where C is a set of distinct colors. The vertex coloring is used some file transfer problem on computer networks. In this paper we have studied the chromatic number to the transformationof Fan graph, Ladder graph, Pan graph, Centipede graph also we established the color class and chromatic number to the transformationof Fangraph,Pan graph, Ladder graph andCentipede graphs.*

Key Words: *Fan graph, Ladder graph, Pan graph, Centipede graph, Transformation, Vertex Coloring, Chromatic number.*

Introduction: 1.0

Graph coloring is one of the recent developing area in graph theory, graph coloring is a special case of graph labeling. Let G be a graph with vertex set V(G), proper vertex coloring of G is a labeling of the vertex set $f: V(G) \to \{1,2, \dots \dots \dots k\} where$ the labels are called colors and no two adjacent vertices share the same color. A graph that permits k-coloring is called k-colorable. The chromatic number χ (G) of a graph G, is minimum number of colors needed for proper coloring.

In the year 2001, Wu and Meng introduced the transformation graph G^{xyz} of G. Since the set {+,-} has eight distinct three permutations, they introduce eight types of transformation graphs.

We shall investigate the transformation graph G^{-+-} of some graphs such as Fan , Ladder , Pan and Centipede graphs .

Definition:1.1

A **graph** G is an ordered pair (V(G),E(G)) consisting of a non-empty set V(G) of vertices and a set E(G), disjoint from V(G), of edges, together with an incidence function ψ_G that associate with each edge of G is an unordered pair of vertices of G.

Definition: 1.2

A **Fan graph** F_n is a planar undirected graph with 2n+1 vertices and 3n edges . The Fan graph F_n can be constructed by joining n copies of the cycle C_3 with a common vertex .

Definition: 1.3

The **Ladder graph** L_n is a planar undirected graph with 2n vertices and 3n-2 edges . The Ladder graph can be obtained as the Cartesian product of P_n and P_2 (ie , $P_n x P_2$)

Definition: 1.4

The **n-pan graph** is a graph obtained by joining a cycle graph C_{n-1} to a singleton graph K_1 with a bridge. The n- Pan graph is therefore isomorphic with the (n-1) – tadpole graph.

Definition: 1.5

The **n-Centipede** ζ_n is a tree with 2n-vertices and 2n-1 edges obtained by joining the bottoms of n copies of the path graph P_2 laid in a row with edges.

Definition:1.6

Let $G = \big(V(G), E(G)\big)$ be a graph and x,y,z be three variables taking values $+$ or $-$. The **transformation graph** G^{xyz} is the graph having $V(G) \cup E(G)$ as the vertex set and for $\propto, \beta \in V(G) \cup E(G)$ $\propto and \beta$ are adjacent in G^{xyz} if and only if one of the following holds:

(i) $\propto, \beta \in V(G). \propto and \beta$ are adjacent in G if $x = +; \propto and \beta$ are not adjacent in G if $x = -$.

(ii) $\propto, \beta \in E(G). \propto and \beta$ are adjacent in G if $y = +; \propto and \beta$ are not adjacent in G if $y = -$.

(iii) $\propto \in V(G)$, $\beta \in E(G). \propto and \beta$ are incidentt in G if $z = +; \propto and \beta$ are not incident in G if $z = -$.

Theorem: 1

Let G= F_{2n+1}(n= 1, 2, …)be thefan graph , If n≥2 then $\chi(G^{-+-}) = 3n$

Proof:

LetG = F_{2n+1} be a fan graph

To prove $\chi(G^{-+-}) = 3n$

Choose a vertex v_0 be the centre vertex which degree is 2n.

The vertex set of G is V(G)=$\{v_i / 0 \leq i \leq 2n\}$and the edge set of G is E(G)=$\{e_i / 1 \leq i \leq 2n-1\}$.

The adjacency of G is,

Each $v_i \in V(G)$is adjacent to,

$N(v_i) = \begin{cases} v_0, v_{i+1} & if\ i\ is\ odd \\ v_0, v_{i-1} & if\ i\ is\ even \end{cases}$

Each $e_i \in E(G)$ is adjacent to each other edges except e_{3i-1},

and e_{3i-1} is adjacent to e_{3i-2} and e_{3i} .

Each $e_i \in E(G)$ is incident with

$N(e_{3i-1}) = \{v_{2i-1}, v_{2i}\}$

$N(e_{3i-2}) = \{v_0, v_{2i-1}\}$

$N(e_{3i}) = \{v_0, v_{2i}\}$

The vertex set of G^{-+-} is$V(G^{-+-}) = V(G) \cup E(G)$

By the definition of the transformation (G^{-+-})

The adjacency in $V(G^{-+-})$ is as follows:

Those pair of vertices (v_i, v_j) are not adjacent in G, areneighbouring vertices in G^{-+-}.

Those pair of edges(e_i, e_j) which are connected in G, areneighbouring vertices inG^{-+-}.

In similar, the pair(v_i, e_i) are not incident in G , are neighbouring vertices in G^{-+-} .

Now, the vertices of $V(G^{-+-})$ can be classified as follows:

$C_i = \{v_{2i-1}, e_{3i-1}, v_{2i} / 1 \leq i \leq n\}$ ……..(1)

$C_j = \begin{cases} e_{3k-2} & if\ k\ is\ odd \\ e_{3k} & if\ k\ is\ even \end{cases}$, where $n+1 \leq j \leq 3n$ ……..(2)

The elements of equation (1), are independent in G^{-+-}.

Hence, a particular colors C_i, can be given to all the verticesofequation(1).

The vertexv_0 is independent to C_i, so any color given to the elements of equation (1) can apply the vertex v_0.

The elements of equation (2), are independent in G^{-+-}.

Hence, a new colors C_j(except C_i), can be given to all the verticesof equation(2) .

Therefore, the total numbers of color class is the minimum coloring number of the graph G^{-+-}.

That is, $\chi(G^{-+-}) = n + 3n - n$

$= 3n$

Therefore, the chromatic number of is G^{-+-},

$\chi(G^{-+-}) \Rightarrow n$

Hence, the proof.

Theorem:2

LetG=P_n(n≥5)be the Pangraph with n-vertices, then$\chi(G^{-+-}) = \left\lceil \frac{n}{2} \right\rceil + 1$.

Proof:

LetP_n be the Pan graph with 'n' vertices in G

To prove $\chi(G^{-+-}) = \left\lceil \frac{n}{2} \right\rceil + 1$

The vertex set of G is V(G) =$\{v_i / 1 \leq i \leq n\}$ and the edge set of Gis E(G)=$\{e_i / 1 \leq i \leq n\}$

The adjacency of G is as follows:

Adjecency between any two pair of vertices$(v_i, v_j) \in V(G)$,

$N(v_i) = \{v_{i-1}, v_{i+1}\}$

$N(v_2) = \{v_1, v_3, v_n\}$ and $N(v_n) = \{v_2, v_{n-1}\}$

Adjacency between any pair of edges $(e_i, e_j) \in E(G)$ is

$N(e_i)=\{ e_{i-1}, e_{i+1}\}$

$N(e_1) = \{e_2, e_n\}$ and $N(e_2) = \{e_1, e_3, e_n\}$

$N(e_n) = \{e_1, e_2, e_{n-1}\}$

Adjecency between any two pairs$(e_i, v_j) \in V(G)$ is,

$N(e_n) = \{v_2, v_n\}$

The vertex set of G^{-+-} is $V(G^{-+-}) = V(G) \cup E(G)$.

By the definition of the transformation (G^{-+-}).

The adjacency in $V(G^{-+-})$ is as follows:

Those pair of vertices (v_i, v_j) are not adjacent in G, areneighbouring vertices in G^{-+-}.

Those pair of edges (e_i, e_j) which are adjacent in G, areneighbouring vertices in G^{-+-}.

In similar, the pair (v_i, e_i) which are not incident in G, areneighbouring vertices in G^{-+-}.

Now, the vertices of V(G) can be classified as follows:

$C_1 = \left\{e_{2j}, 1 \le j \le \left\lceil \frac{n}{2} \right\rceil\right\}$

$C_i = \{v_{2i-3}, e_{2i-3}, v_{2i-2} \ / \ 2 \le i \le \left\lceil \frac{n}{2} \right\rceil\}$

$C_k = \begin{Bmatrix} v_n, e_n if\ n\ is odd \\ v_{n-1}, e_{n-1}, v_n if n is even \end{Bmatrix}$, where $k = \left\lceil \frac{n}{2} \right\rceil + 1$

The elements of C_1, are independent in G^{-+-}.

Hence, a particular color can be given to all the vertices of C_1, let it be c_1.

The elements of each C_i, are independent in G^{-+-}.

Since the elements belongs to each set of C_i, $2 \le i \le \left\lceil \frac{n}{2} \right\rceil$ are independent in G^{-+-}, we use $\left\lceil \frac{n}{2} \right\rceil$ different color $c_i$other than c_1 to color the vertices of $C_i$by using a single color to color all the vertices belongs to the same class.

In similar, the elements of c_k is independent in G^{-+-}. Therefore, we use a new color c_k, k=1, i to color the vertices of C_k.

$\chi(G^{-+-}) = 1 + \left\lceil \frac{n}{2} \right\rceil - 1 + 1$

$= \left\lceil \frac{n}{2} \right\rceil + 1$

Therefore, the chromatic number of is G^{-+-},

$\chi(G^{-+-}) = \left\lceil \frac{n}{2} \right\rceil + 1$

Hence, the proof.

Theorem : 3

Let$G=\zeta\phi_2n$ (n=1,2,3,...) be the centipede graph,$If\ (n \ge 3)$ then

Proof:

$\chi G\hat{}(- + -))= n+2$

Let ζ_{2n} be the centipede graph of G ,

To prove $\chi(G^{-+-}) = n + 2$

The vertex set of$\zeta_{2n}$is$V(G)=\{v_i/1 \le i \le 2n\}$ and the edge set of $\zeta_{2n}$is

$$E(G) = \{e_i / 1 \le i \le 2n-1\}$$

The adjacency of V(G) is,

$$N(v_i) = \begin{Bmatrix} v_{i-1}, & v_{i+1}, & v_{i+2} & if'i'isodd \\ & v_{i-1} & if'i'iseven & \end{Bmatrix}$$

Each $e_i \in E(G) is$,

$$N(e_i) = \begin{Bmatrix} & e_{i+1} & if'i'isodd & \\ e_{i-2}, e_{i-1}, & e_{i+1}, e_{i+2} & & if'i'iseven \end{Bmatrix}$$

Each e_i is incident with,

$$N(e_i) = \begin{Bmatrix} v_i, & v_{i+1} if'i'isodd \\ v_{i-1}, & v_{i+1} if'i'iseven \end{Bmatrix}$$

The vertex set of G^(- + -) is V(G^(- + -)) = $V(\zeta_{2n}) \cup E(\zeta_{2n})$.

By the definition of the transformation (G^{-+-})

The adjacency in $V(G^{-+-})$as follows:

Those pair of vertices (v_i, v_j) are not adjacent in G , are neighbouring vertices inG^{-+-}.

Those pair of edges (e_i, e_j) which are adjacent in G, are neighbouring vertices in G^{-+-}.

In similar, the pair (v_i, e_i) which are not incident in G, are neighbouring vertices inG^{-+-}.

Now, the vertices of V(G^{-+-}) can be classified as follows

$C_1 = \{e_{2j} \ / 1 \leq j \leq \lfloor \frac{n}{2} \rfloor\}$..... (1)

$C_2 = \{e_{2k} / 1 \leq k \leq \lceil \frac{n}{2} \rceil - 1\}$.....(2)

$C_i = \{v_{2i-5}, e_{2i-5}, v_{2i-4} \ / 3 \leq i \leq n+2\}$.....(3)

The elements of equation (1), are independent in G^{-+-},
Hence, a particular colorC_1 can apply all the vertices e_{2j}
The elements of equation (2) ,are independent in G^{-+-},
Hence, a different color C_2(except C_1) to give the vertices e_{2k}
In similar, the elements of equation (3) ,are independent in G^{-+-},
Therefore, the new colors C_i , to apply all the vertices in equation (3).
Therefore, the total numbers of color class is the minimum coloring number of the graphG^{-+-}.
That is,$\chi G^{-+-}) = 1 + 1 + n + 2 - 2$
$= n+2$
Therefore, the chromatic number of isG^{-+-},
$\chi\ (G^{-+-}) = n+2$
Hence, the proof.

Theorem: 4

Let $G = L_{2n}$(n= 1, 2, ..n) be a ladder graph, If (n≥ 4) then $\chi\ (G^{-+-}) = n + 2$.

Proof:

Let $G = L_{2n}$(n= 1, 2, ..) be a ladder graph

To prove $\chi\ (G^{-+-}) = n + 2$

The vertex set of G is V(G) ={v_i /1≤ $i \leq 2n$}and the edge set of G is
E(G) ={e_i /1 ≤ $i \leq 2n-1$}
The adjacency of G is,
Each $v_i \in V(G)$is adjacent to,

$$N(v_i) = \begin{Bmatrix} v_{i-2}, v_{i+1}, v_{i+2} & if\ i\ is\ odd \\ v_{i-2}, v_{i-1}, v_{i+2} & if\ i\ is\ even \end{Bmatrix}$$

Each $e_i \in E(G)$ is adjacent to,

$$N(e_{3i}) = \{e_{3i-3}, e_{3i-2}, e_{3i+1}, e_{3i+3}\}$$
$$N(e_{3i-1}) = \{e_{3i-4}, e_{3i-2}, e_{3i+1}, e_{3i+2}\}$$
$$N(e_{3i-2}) = \{e_{3i-4}, e_{3i-3}, e_{3i-1}, e_{3i}\}$$

Each $e_i \in E(G)$ is incident with

$$N(e_{3i-2}) = \{v_{2i}, v_{2i-1}\}$$
$$N(e_{3i-1}) = \{v_{2i-1}, v_{2i+1}\}$$
$$N(e_{3i}) = \{v_{2i}, v_{2i+2}\}$$

The vertex set of G^{-+-} is V(G^{-+-}) = $V(G) \cup E(G)$
By the definition of the transformation (G^{-+-})
The adjacency in V(G^{-+-}) is as follows:

Those pair of vertices (v_i, v_j) are not adjacent in G, are neighbouring vertices in G^{-+-}

Those pair of vertices (e_i, e_j) which are connected in G , are neighbouring vertices in G^{-+-}

In similar, the pair(v_i, e_i) are not incident in G, are neighbouring vertices in G^{-+-}
Now, the vertices of V(G^{-+-}) can be classified as follows:

$C_i = \{v_{2i-1}, v_{2i}, e_{3i-2} \ /1 \leq i \leq n\}$ ..... (1)

$C_k = \begin{Bmatrix} e_{3j-1}, & if\ j\ is\ odd \\ e_{3j-3}, & if\ j\ is\ even \end{Bmatrix}$ where k= n+1 .....(2)

$C_L = \begin{cases} e_{3j-2} , & if\ j\ is odd \\ e_{3j} , & if\ j\ is even \end{cases}$ where L= n+2 ……..(3)

The elements of equation (1), are independent in G^{-+-}.

Hence, a different colors C_i can be given to all the vertices of equation (1).

The elements of equation (2), are independent in G^{-+-}.

Hence, a particular color C_k can be given to all the vertices of equation (2).

In similar, all the elements of equation (3),are independent in G^{-+-}.

Therefore, we use a new color $C_{k,}$ to apply the vertex of equation (3).

Therefore, the total numbers of color class is the minimum coloring number of the graph G^{-+-}.

That is, $\chi(G^{-+-}) = n+1+1$

$= n+2$

Therefore, the chromatic number of is G^{-+-},

$\chi(G^{-+-}) \Rightarrow +2$

Hence, the proof.

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