CHROMATIC NUMBER TO THE TRANSFORMATION OF SOME GRAPHS

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ABSTRACT: Let G = (V,E) be a connected simple graph. A vertex coloring of a graph G is a function f:V(G) → C where C is a set of distinct colors. The vertex coloring is used some file transfer problem on computer networks. In this paper we have studied the chromatic number to the transformation of Fan graph, Ladder graph, Pan graph, Centipede graph also we established the color class and chromatic number to the transformation of Fan graph, Pan graph, Ladder graph and Centipede graphs.

Key Words: Fan graph, Ladder graph, Pan graph, Centipede graph, Transformation, Vertex Coloring, Chromatic number.

Introduction: 1.0
Graph coloring is one of the recent developing area in graph theory, graph coloring is a special case of graph labeling. Let G be a graph with vertex set V(G), proper vertex coloring of G is a labeling of the vertex set f:V(G) → {1,2,.....,k} where the labels are called colors and no two adjacent vertices share the same color. A graph that permits k-coloring is called k-colorable. The chromatic number \( \chi(G) \) of a graph G, is minimum number of colors needed for proper coloring.

In the year 2001, Wu and Meng introduced the transformation graph \( G^{xyz} \) of G. Since the set \{+,-\} has eight distinct three permutations, they introduce eight types of transformation graphs.

We shall investigate the transformation graph \( G^{xyz} \) of some graphs such as Fan, Ladder, Pan and Centipede graphs.

Definition: 1.1
A graph G is an ordered pair \((V(G),E(G))\) consisting of a non-empty set V(G) of vertices and a set E(G), disjoint from V(G), of edges, together with an incidence function \( \psi_G \) that associate with each edge of G is an unordered pair of vertices of G.

Definition: 1.2
A Fan graph \( F_n \) is a planar undirected graph with 2n+1 vertices and 3n edges. The Fan graph \( F_n \) can be constructed by joining n copies of the cycle \( C_3 \) with a common vertex.

Definition: 1.3
The Ladder graph \( L_n \) is a planar undirected graph with 2n vertices and 3n-2 edges. The Ladder graph can be obtained as the Cartesian product of \( P_n \) and \( P_2 \) (i.e., \( P_n \times P_2 \)).

Definition: 1.4
The n-pan graph is a graph obtained by joining a cycle graph \( C_{n-1} \) to a singleton graph \( K_1 \) with a bridge. The n- Pan graph is therefore isomorphic with the (n-1) - tadpole graph.

Definition: 1.5
The n-Centipede graph \( C_n \) is a tree with 2n-vertices and 2n-1 edges obtained by joining the bottoms of n copies of the path graph \( P_2 \) laid in a row with edges.

Definition: 1.6
Let \( G=(V(G), E(G))\) be a graph and \( x, y, z \) be three variables taking values + or -. The transformation graph \( G^{xyz} \) is the graph having \( V(G) \cup E(G) \) as the vertex set and for \( \alpha, \beta \in V(G) \cup E(G) \) and \( \beta \) are adjacent in \( G^{xyz} \) if and only if one of the following holds:

(i) \( \alpha, \beta \in V(G), \alpha and \beta \) are adjacent in G if \( x = +; \alpha and \beta \) are not adjacent in G if \( x = - \).

(ii) \( \alpha, \beta \in E(G), \alpha and \beta \) are adjacent in G if \( y = +; \alpha and \beta \) are not adjacent in G if \( y = - \).

(iii) \( \alpha \in V(G), \beta \in E(G), \alpha and \beta \) are incident in G if \( z = +; \alpha and \beta \) are not incident in G if \( z = - \).
Theorem: 1
Let $G = F_{2n+1}(n = 1, 2, \ldots)$ be the fan graph. If $n \geq 2$ then $\chi(G^+) = 3n$

Proof:
Let $G = F_{2n+1}$ be a fan graph.

To prove $\chi(G^+) = 3n$

Choose a vertex $v_0$ be the centre vertex which degree is $2n$.

The vertex set of $G$ is $V(G) = \{v_i / 0 \leq i \leq 2n\}$ and the edge set of $G$ is $E(G) = \{e_i / 1 \leq i \leq 2n - 1\}$.

The adjacency of $G$ is:

Each $v_i \in V(G)$ is adjacent to $v_0$ and $v_{i+1}$.

$N(v_i) = \{v_0, v_{i+1} \text{ if } i \text{ is odd}\}$

Each edge $e_i \in E(G)$ is adjacent to each other edges except $e_{3i-1}$, and $e_{3i-1}$ is adjacent to $e_{3i-2}$ and $e_{3i}$.

Each $e_i \in E(G)$ is incident with $N(e_{3i-1}) = \{v_{2i-1}, v_{2i}\}$

$N(e_{3i-2}) = \{v_0, v_{2i-1}\}$

$N(e_{3i}) = \{v_0, v_{2i}\}$

The vertex set of $G^+$ is $V(G^+) = V(G) \cup E(G)$.

By the definition of the transformation $(G^+)$

The adjacency in $V(G^+)$ is as follows:

Those pair of vertices $(v_i, v_j)$ are not adjacent in $G$, are neighbouring vertices in $G^+$.

Those pair of edges $(e_i, e_j)$ which are connected in $G$, are neighbouring vertices in $G^+$.

In similar, the pair $(v_i, e_i)$ are not incident in $G$, are neighbouring vertices in $G^+$.

Now, the vertices of $V(G^+)$ can be classified as follows:

$C_i = \{v_{2i-1}, e_{3i-1}, v_{2i} / 1 \leq i \leq n\}$

$C_i = \{e_{3j-2}, e_{3j} if k \text{ is odd}\}$

where $n + 1 \leq j \leq 3n$

The elements of equation (1), are independent in $G^+$.

Hence, a particular colors $C_i$ can be given to all the vertices of equation (1).

The vertex $v_0$ is independent to $C_i$, so any color given to the elements of equation (1) can apply the vertex $v_0$.

The elements of equation (2), are independent in $G^+$.

Hence, a new colors $C_i$ except $C_0$, can be given to all the vertices of equation (2).

Therefore, the total numbers of color class is the minimum coloring number of the graph $G^+$:

That is, $\chi(G^+) = n + 3n - n = 3n$

Therefore, the chromatic number of $G^+$ is $3n$.

Hence, the proof.

Theorem: 2
Let $G = P_n (n \geq 5)$ be the Pan graph with $n$-vertices, then $\chi(G^+) = \left\lceil \frac{n}{2} \right\rceil + 1$.

Proof:
Let $G = P_n$ be the Pan graph with $n$ vertices in $G$.

To prove $\chi(G^+) = \left\lceil \frac{n}{2} \right\rceil + 1$

The vertex set of $G$ is $V(G) = \{v_i / 1 \leq i \leq n\}$ and the edge set of $G$ is $E(G) = \{e_i / 1 \leq i \leq n\}$

The adjacency of $G$ is as follows:

Adjacency between any two pair of vertices $(v_i, v_j) \in V(G)$,

$N(v_i) = \{v_{i-1}, v_{i+1}\}$

$N(v_2) = \{v_1, v_3, v_n\}$ and $N(v_n) = \{v_2, v_{n-1}\}$

Adjacency between any pair of edges $(e_i, e_j) \in E(G)$ is...
The definition of the transformation $G^+$ is $V(G^+) = V(G) \cup E(G)$.

The adjacency in $V(G^+)$ is as follows:

Those pair of vertices $(v_i, v_j)$ are not adjacent in $G$, are neighbouring vertices in $G^+$:

Those pair of edges $(e_i, e_j)$ which are adjacent in $G$, are neighbouring vertices in $G^+$.

In similar, the pair $(v_i, e_j)$ which are not incident in $G$, are neighbouring vertices in $G^+$.

Now, the vertices of $V(G)$ can be classified as follows:

$C_1 = \{e_{2j}, 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor \}$

$C_i = \{v_{2i-3}, e_{2i-3}, v_{2i-2} / 2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \}$

$C_k = \{v_n, e_n \text{ if } n \text{ is odd } \}
\{v_{n-1}, e_{n-1}, v_{n} \text{ if } n \text{ is even } \}$, where $k = \left\lceil \frac{n}{2} \right\rceil + 1$

The elements of $C_1$, are independent in $G^+$.

Hence, a particular color can be given to all the vertices of $C_1$, let it be $c_1$.

The elements of each $C_k$, are independent in $G^+$.

Since the elements belongs to each set of $C_k$, $2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$ are independent in $G^+$, we use $\left\lceil \frac{n}{2} \right\rceil$ different color other than $c_1$ to color the vertices of $C_k$ by using a single color to color all the vertices belongs to the same class.

In similar, the elements of $C_k$ is independent in $G^+$. Therefore, we use a new color $c_k$, $k=1, i$ to color the vertices of $C_k$.

$\chi(G^+) = 1 + \left\lceil \frac{n}{2} \right\rceil - 1 + 1$

Therefore, the chromatic number of $G^+$ is

$\chi(G^+) = \left\lceil \frac{n}{2} \right\rceil + 1$

Hence, the proof.

**Theorem 3**

Let $G = \phi_{2n}$ $(n = 1, 2, 3, \ldots)$ be the centipede graph, If $(n \geq 3)$ then

$\chi(G^+(-, -)) = n + 2$

Let $\zeta_{2n}$ be the centipede graph of $G$.

To prove $\chi(G^+(-, -)) = n + 2$

The vertex set of $\zeta_{2n}$ is $V(G) = \{v_{i}/1 \leq i \leq 2n\}$ and the edge set of $\zeta_{2n}$ is $E(G) = \{e_{i}/1 \leq i \leq 2n - 1\}$

The adjacency of $V(G)$ is,

$N(v_i) = \{v_{i-1}, v_{i+1}, v_{i+2} \text{ if } i \text{ is odd } \}
\{v_{i-1}\text{ if } i \text{ is even } \}$

Each $e_i \in E(G)$ is,

$N(e_i) = \{e_{i-2}, e_{i-1}, e_{i+1}, e_{i+2} \text{ if } i \text{ is odd } \}
\{e_{i-2}, e_{i-1}, e_{i+1}, e_{i+2} \text{ if } i \text{ is even } \}$

Each $e_i$ is incident with,

$N(e_i) = \{v_i, v_{i+1} \text{ if } i \text{ is odd } \}
\{v_{i-1}, v_{i+1} \text{ if } i \text{ is even } \}$

The vertex set of $G^+(-, -)$ is $V(G^+(-, -)) = V(\zeta_{2n}) \cup E(\zeta_{2n})$.

By the definition of the transformation $(G^+(-, -))$

The adjacency in $V(G^+(-, -))$ as follows:
Those pair of vertices \((v_i, v_j)\) are not adjacent in \(G\), are neighbouring vertices in \(G^{-+}\).

Those pair of edges \((e_i, e_j)\) which are adjacent in \(G\), are neighbouring vertices in \(G^{-+}\).

In similar, the pair \((v_i, e_i)\) which are not incident in \(G\), are neighbouring vertices in \(G^{-+}\).

Now, the vertices of \(V(G^{-+})\) can be classified as follows

\[C_1 = \left\{ \frac{e_{2j}}{1 \leq j \leq \frac{n}{2}} \right\} \] \hspace{1cm} (1)
\[C_2 = \left\{ \frac{e_{2k} / 1 \leq k \leq \frac{n}{2} - 1} \right\} \] \hspace{1cm} (2)
\[C_3 = \left\{ v_{2i-5}, v_{2i-5}, v_{2i-4} / 3 \leq i \leq n + 2 \right\} \] \hspace{1cm} (3)

The elements of equation (1), are independent in \(G^{-+}\),
Hence, a particular color \(C_1\) can apply all the vertices \(e_{2j}\)
The elements of equation (2), are independent in \(G^{-+}\),
Hence, a different color \(C_2\) (except \(C_1\)) to give the vertices \(e_{2k}\)
In similar, the elements of equation (3) are independent in \(G^{-+}\),
Therefore, the new colors \(C_i\) to apply all the vertices in equation (3).
Therefore, the total numbers of color class is the minimum coloring number of the graph \(G^{-+}\).
That is, \(\chi(G^{-+}) = 1 + 1 + n + 2 - 2 = n+2\)
Therefore, the chromatic number of is \(G^{-+}\),
\(\chi(G^{-+}) = n+2\)
Hence, the proof.

**Theorem 4**

Let \(G = L_{2n}\) (n = 1, 2, ..) be a ladder graph, If (n ≥ 4) then \(\chi (G^{+}) = n + 2\).

**Proof:**

Let \(G = L_{2n}\) (n = 1, 2, ..) be a ladder graph
To prove \(\chi (G^{+}) = n + 2\)
The vertex set of \(G\) is \(V(G) = \{v_i / 1 \leq i \leq 2n\}\) and the edge set of \(G\) is
\(E(G) = \{e_i / 1 \leq i \leq 2n - 1\}\)
The adjacency of \(G\) is,
Each \(v_i \in V(G)\) is adjacent to,
\[N(v_i) = \left\{ \begin{array}{ll}
v_{i-2}, v_{i+1}, v_{i+2} & \text{if } i \text{ is odd} \\
v_{i-2}, v_{i-1}, v_{i+2} & \text{if } i \text{ is even} \\
\end{array} \right\} \]
Each \(e_i \in E(G)\) is adjacent to,
\[N(e_{3i}) = \left\{ e_{3i-3}, e_{3i-2}, e_{3i-1}, e_{3i+1}, e_{3i+2} \right\} \]
\[N(e_{3i-1}) = \left\{ e_{3i-4}, e_{3i-2}, e_{3i+1}, e_{3i+2} \right\} \]
\[N(e_{3i-2}) = \left\{ e_{3i-4}, e_{3i-3}, e_{3i-1}, e_{3i} \right\} \]
Each \(e_i \in E(G)\) is incident with
\[N(e_{3i-2}) = \left\{ v_{2i-1}, v_{2i+1} \right\} \]
\[N(e_{3i-1}) = \left\{ v_{2i-1}, v_{2i+1} \right\} \]
\[N(e_{3i}) = \left\{ v_{2i-1}, v_{2i+1} \right\} \]
The vertex set of \(G^{+}\) is \(V(G^{+}) = V(G) \cup E(G)\)
By the definition of the transformation (\(G^{+}\))
The adjacency in \(V(G^{+})\) is as follows:

Those pair of vertices \((v_i, v_j)\) are not adjacent in \(G\), are neighbouring vertices in \(G^{+}\)

Those pair of vertices \((e_i, e_j)\) which are connected in \(G\), are neighbouring vertices in \(G^{+}\)

In similar, the pair \((v_i, e_i)\) are not incident in \(G\), are neighbouring vertices in \(G^{+}\)

Now, the vertices of \(V(G^{+})\) can be classified as follows:
\[C_i = \left\{ v_{2i-1}, v_{2i} , e_{3i-2} / 1 \leq i \leq n \right\} \] \hspace{1cm} (1)
\[C_k = \left\{ e_{3j-1} , \right\} \] \hspace{1cm} if \(j\) is odd
\[C_k = \left\{ e_{3j-3} , \right\} \] \hspace{1cm} if \(j\) is even

where \(k = n+1\) \hspace{1cm} (2)
\[ C_L = \begin{cases} e_{3j-2} & \text{if } j \text{ is odd} \\ e_{3j} & \text{if } j \text{ is even} \end{cases} \text{where } L = n + 2 \quad \ldots \ldots (3) \]

The elements of equation (1) are independent in \( G^{-+} \).

Hence, a different color \( C_i \) can be given to all the vertices of equation (1).

The elements of equation (2) are independent in \( G^{+} \).

Hence, a particular color \( C_k \) can be given to all the vertices of equation (2).

In similar, all the elements of equation (3) are independent in \( G^{-+} \).

Therefore, we use a new color \( C_k \) to apply the vertex of equation (3).

Therefore, the total numbers of color class is the minimum coloring number of the graph \( G^{-+} \).

That is, \( \chi (G^{-+}) = n + 1 + 1 \)

\[ = n + 2 \]

Therefore, the chromatic number of \( G^{-+} \) is \( n + 2 \).

Hence, the proof.

References:


