Parity Combination Cordial Labelling of Joins of H graph

[1]S.Sriram & [2]Dr.R.Govindarajan
[1]Assistant Professor, Department of Mathematics, Patrician College of Arts and Science, Adyar, Chennai,India
[2]Associate Professor and Head (Retd.),P.G and U.G Department of Mathematics, D.G.Vaishnav College, Arumbakkam, Chennai,India

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ABSTRACT: *: Let G= (V, E) be a graph with p vertices and q edges. An injective function* $f:V(G)\rightarrow\{1,2,3...p\}$ *is called an parity combination cordial labelling (PCC-Labeling) if for each edge* uv *assign the label* $\binom{x}{y}$ *or* $\binom{y}{x}$ *according as* $x \succ y$ or $y \succ x$ *and f is a one to one map and* $\left|e_f(0)-e_f(1)\right|\leq 1$ *where* $e_f(0)$ *denote the number of edges labelled with even numbers and* $e_f(1)$ *denote the number of edges labelled with odd numbers. A graph with a parity combination cordial labelling is called a parity combination cordial graph (PCC-graph). We in this paper study on the* H_n *graph for* $n\geq 3$ *by joining with similar* H_n *graph for* $n\geq 3$ *and we call the graph to be a join of* H_n *graph. We prove that join of* H_n *graph is parity combination cordial (PCC-graph)graph for n is odd and also obtain a formula for finding the sum of the edges of M joins of* H_n *graph.*

Key Words: H_n *graph, Joins of* H_n *graph, parity combination labelling of graph, Parity combination graph.*

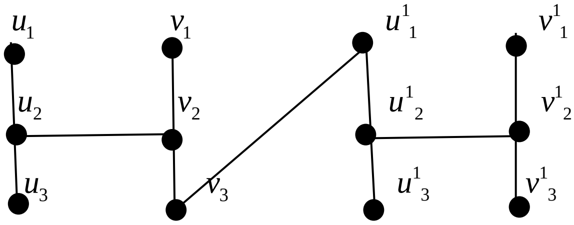
Preliminaries

Definition 2.1: The H graph of the path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1,v_2,...v_n$ and $u_1,u_2,...u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even.

Definition.2.2: An injective function $f:V(G)\rightarrow\{1,2,3...p\}$ is called an parity combination cordial labelling (PCC-Labeling) if for each edge uv assign the label $\binom{x}{y}$ or $\binom{y}{x}$ according as $x \succ y$ or $y \succ x$ and f is a one to one map and $\left|e_f(0)-e_f(1)\right|\leq 1$ where $e_f(0)$ denote the number of edges labelled with even numbers and $e_f(1)$ denote the number of edges labelled with odd numbers. A graph with a parity combination cordial labelling is called a parity combination cordial graph (PCC-graph).

We now join a H_n graph denoted by H_{n_1} with a H graph denoted by H_{n_2} by an edge where $n_1=n_2$ and we call it as 1-join of H_n graph

Figure.1: 1- Join of H_3 graph



Similarly we can construct M-joins of H graph where each of H graph denoted by H_{n_1} by an edge e_1 with H graph denoted by H_{n_2}, H graph denoted by H_{n_2} by an edge e_2 with H graph denoted by H_{n_3} and so on with H graph denoted by $H_{n_{M-1}}$ by an edge e_{M-1} with H graph denoted by H_{n_M} such that $n_1 = n_2 = \ldots = n_M$

Main Results

Theorem .3.1: 1-Join of H_n is parity combination cordial graph (PCC-graph) for $n \geq 3$ and when n is odd.

Proof: Consider the graph $G = 1 - Join\ of\ H_n$. And let us consider n is odd

We know the H graph of the path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots v_n$ and $u_1, u_2, \ldots u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even

We now join a H graph denoted by H_{n_1} with a H graph denoted by H_{n_2} by an edge where $n_1 = n_2$ which we call it as 1-join of H_n graph. Now the 1-Join of H_n graph consists of 4n vertices and 4n-1 edges.

Now let us suppose that the vertices of 1-Join of H_n denoted by

$$V(G) = \{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n\} \cup \{u^1_1, u^1_2, \ldots u^1_n, v^1_1, v^1_2, \ldots v^1_n\}$$

$$E(G) = \{u_i u_{i+1}, 1 \leq i \leq n-1\} \cup \{v_i v_{i+1}, 1 \leq i \leq n-1\}$$

$$\cup \{u^1_i u^1_{i+1}, 1 \leq i \leq n-1\} \cup \{v^1_i v^1_{i+1}, 1 \leq i \leq n-1\}$$

$$\cup \left\{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right\} \cup \left\{u^1_{\frac{n+1}{2}}, v^1_{\frac{n+1}{2}}\right\} \cup \left\{v_n u^1_{n+1}\right\} \text{, for n is odd}$$

$$E(G) = \{u_i u_{i+1}, 1 \leq i \leq n-1\} \cup \{v_i v_{i+1}, 1 \leq i \leq n-1\}$$

$$\cup \{u^1_i u^1_{i+1}, 1 \leq i \leq n-1\} \cup \{v^1_i v^1_{i+1}, 1 \leq i \leq n-1\}$$

$$\cup \left\{u_{\frac{n}{2}} v_{\frac{n}{2}+1}\right\} \cup \left\{u^1_{\frac{n}{2}}, v^1_{\frac{n}{2}+1}\right\} \cup \left\{v_n u^1_{n+1}\right\} \text{, for n is even}$$

Now let us label the vertices as follows

$f(u_i) = i$ for $1 \leq i \leq n$

$f\left(v_{i-n}\right)=i$ for $n+1\le i\le 2n$

$f\left(u^1{}_{i-2n}\right)=i$ for $2n+1\le i\le 3n$

$f\left(v^1{}_{i-3n}\right)=i$ for $3n+1\le i\le 4n$

Then the induced edge labelling is obtained as

$$f^*\left(u_iu_{i+1}\right)=\begin{pmatrix} f\left(u_{i+1}\right) \\ f\left(u_i\right)\end{pmatrix}=f\left(u_{i+1}\right)=i+1$$

$$f^*\left(v_iv_{i+1}\right)=\begin{pmatrix} f\left(v_{i+1}\right) \\ f\left(v_i\right)\end{pmatrix}=f\left(v_{i+1}\right)=i+1$$

$$f^*\left(u^1{}_iu^1{}_{i+1}\right)=\begin{pmatrix} f\left(u^1{}_{i+1}\right) \\ f\left(u^1{}_i\right)\end{pmatrix}=f\left(u^1{}_{i+1}\right)=i+1$$

$$f^*\left(v^1{}_iv^1{}_{i+1}\right)=\begin{pmatrix} f\left(v^1{}_{i+1}\right) \\ f\left(v^1{}_i\right)\end{pmatrix}=f\left(v_{i+1}\right)=i+1$$

$f^*\left(u_iv_i\right)=\begin{pmatrix} f\left(u_i\right) \\ f\left(v_i\right)\end{pmatrix}=even$ for n is odd and $i=\frac{n+1}{2}$, and $f\left(u_i\right)=i$, $f\left(v_i\right)=i+n$

$f^*\left(u_iv_i\right)=\begin{pmatrix} f\left(u_i\right) \\ f\left(v_i\right)\end{pmatrix}=odd$ or even for n is even and $i=\frac{n}{2}$, and $f\left(u_i\right)=i$, $f\left(v_i\right)=i+n$

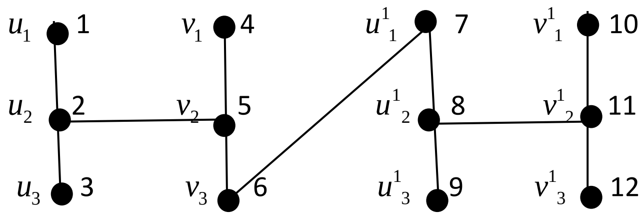
$$f^*\left(v_nu^1{}_1\right)=\begin{pmatrix} f\left(v_n\right) \\ f\left(u^1{}_1\right)\end{pmatrix}=\text{odd}$$

We find from the above labelling procedure the induced edge labelling of 1-Join of H_n Graph a parity combination cordial labelling graph as the number of edges labelled with even numbers is defined as $e_f(0)$ and number of edges labelled with odd numbers is defined as $e_f(1)$ such that $\left|e_f(1)-e_f(0)\right|\le 1$.

Observation.1: We find from the above labelling procedure that the induced edge labelling for $f^*\left(u_iv_i\right)=\begin{pmatrix} f\left(v_i\right) \\ f\left(u_i\right)\end{pmatrix}=even$ and $f^*\left(u_2{}^1v_2{}^1\right)=\begin{pmatrix} f\left(v_2{}^1\right) \\ f\left(u_2{}^1\right)\end{pmatrix}$=odd alternatively. When n=3.

This is explained in the following example

Figure.2: Parity Combination cordial labelling (PCC-Labelling) of 1- join of H_3 graph



From the above labelling procedure for n=3 i.e. H_3 graph
We have

$$f^*(u_2v_2) = \binom{f(v_2)}{f(u_2)} = \binom{5}{2} = 10$$

$$f^*(u_2^1v_2^1) = \binom{f(v_2^1)}{f(u_2^1)} = \binom{11}{8} = 165$$

$$f^*(u_2^2v_2^2) = \binom{f(v_2^2)}{f(u_2^2)} = \binom{17}{14} = \binom{17}{3} = 680$$

Hence for n=3 for M-Joins of H_3 graph the induced edge labels are even and odd alternatively.

Now let us obtain the sum of the edges of M-Join of H_n graph. For this let us suppose consider 1-join of H_3 graph. Let us adopt the procedure to label the vertices as in theorem.3.1 so as to find the sum of the edges of 1- join of H_3 graph.
We find that 1- join of H_3 graph consists of 12 vertices and 11 edges. Now the sum of the edges of 1- join of H_3 graph is computed for parity combination cordial labelling as 2,3,5,6, 7,8,9,10,11,12,165 and whose sum = 2+3+5+6+7+8+9+10)11+12+165=238
We try to find the sum of the edges of H_3 graph in a different way as given below

We define the formula for 1- Join of H_n graph as follows for n is odd.

$$S_{2i} = S_{2i-2} + 2S_1 + (4i-3)n(V_1)n(E_1) + \binom{f\left(u_{\frac{n+1}{2}}\right)}{f\left(v_{\frac{n+1}{2}}\right)} + \binom{f\left(u^1_{\frac{n+1}{2}}\right)}{f\left(v^1_{\frac{n+1}{2}}\right)} + \binom{f(v_n)}{f(u^1_1)}$$

Now substituting in the formula for 1- join of H_3 graph by considering i=2 we have

$$S_4 = S_2 + 2S_1 + 5(3)(2) + \begin{pmatrix} f(u_2) \\ f(v_2) \end{pmatrix} + \begin{pmatrix} f(u^1{}_2) \\ f(v^1{}_2) \end{pmatrix} + \begin{pmatrix} f(v_{\ 3}) \\ f(u^1{}_1) \end{pmatrix}$$

From the labelling procedure we know

$$S_2 = 16;\ S_1 = 5;\ \begin{pmatrix} f(u_2) \\ f(v_2) \end{pmatrix} + \begin{pmatrix} f(u^1{}_2) \\ f(v^1{}_2) \end{pmatrix} + \begin{pmatrix} f(v_{\ 3}) \\ f(u^1{}_1) \end{pmatrix} = 10 + 165 + 7 = 182$$

Hence $S_4 = 16 + 2(5) + 30 + 182$

Therefore we have $S_4 = 16 + 10 + 30 + 10 + 165 + 7 = 238$.

Where $S_1, S_2, S_3, S_4 \ldots$ are sum of each of the path graph of H_n.

Note.1 : When we calculate $S_2, S_3, S_4 \ldots$ using the general formula given in theorem.3.2 we will not include the join of H_n graph while computation given in the formula and the edge connecting between the vertices $f\left(u_{\frac{n+1}{2}}\right), f\left(v_{\frac{n+1}{2}}\right)$ and the edges connecting between the vertices $f\left(u^M{}_{\frac{n+1}{2}}\right), f\left(v^M{}_{\frac{n+1}{2}}\right)$ for the corresponding joins when n is odd and the edge connecting between the vertices $f\left(u_{\frac{n}{2}}\right), f\left(v_{\frac{n}{2}}\right)$ and the edges connecting between the vertices $f\left(u^M{}_{\frac{n}{2}}\right), f\left(v^M{}_{\frac{n}{2}}\right)$ for the corresponding joins when n is even.

Note.2: We here consider $n(V_1) = n$ and $n(E_1) = n - 1$ for M-Join of H_n graph.

Theorem.3.2: For a M-Join of H_n graph the sum of the edges is given by the formula

$$S_{2i} = S_{2i-2} + 2S_1 + (4i-3)n(V_1)n(E_1) + \begin{pmatrix} f\left(u_{\frac{n+1}{2}}\right) \\ f\left(v_{\frac{n+1}{2}}\right) \end{pmatrix} + \sum_{i=1}^{M} \begin{pmatrix} f\left(u^i{}_{\frac{n+1}{2}}\right) \\ f\left(v^i{}_{\frac{n+1}{2}}\right) \end{pmatrix} + \begin{pmatrix} f(v_{\ n}) \\ f(u^1{}_1) \end{pmatrix} + \sum_{i=1}^{M-2} \begin{pmatrix} f(v^i{}_n) \\ f(u^{i+1}{}_1) \end{pmatrix}$$

When n is odd

$$S_{2i} = S_{2i-2} + 2S_1 + (4i-3)n(V_1)n(E_1) + \begin{pmatrix} f\left(u_{\frac{n}{2}}\right) \\ f\left(v_{\frac{n}{2}}\right) \end{pmatrix} + \sum_{i=1}^{M} \begin{pmatrix} f\left(u^i{}_{\frac{n}{2}}\right) \\ f\left(v^i{}_{\frac{n}{2}}\right) \end{pmatrix} + \begin{pmatrix} f(v_{\ n}) \\ f(u^1{}_1) \end{pmatrix} + \sum_{i=1}^{M-2} \begin{pmatrix} f(v^i{}_n) \\ f(u^{i+1}{}_1) \end{pmatrix}$$

When n is even.

Proof : Given for a M-join of H_n graph the sum of the edges given by the formula can be proved by fixing the value of i=2,3,4,… and fixing the value of M, the number of joins according as 1,2,3… for the corresponding sum of the edges represented by $S_4, S_6, S_8 \ldots$ respectively. We understand from the notion of construction of the formula that S_2 is the representation of sum of the edges of single H_n graph and S_4 is the representation of the sum of the edges of 1-join of H_n graph and S_6 is the representation of sum of the edges of 2-join of H_n graph and so on. To find the sum of the edges of single H_n graph namely S_2 we ignore the computation of the above said edges in Note.1. Similarly we calculate for $S_4, S_6, S_8 \ldots$ and correspondingly according to value of n whether it is odd or even and the number of joins we compute by substituting the values for i the sum of the edges of M-Joins of H_n graph.

Results

In this paper we have constructed M-joins of H_n and proved that it is Parity combination cordial labelling graph when n is odd and have found out formula to find the sum of the edges of M-join of H_n graph.

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