CUBIC BI-IDEALS IN NEAR SUBTRACTION SEMIGROUPS

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ABSTRACT: : *Young Bae Jun made effort in defining a remarkable structure namely cubic structure. Concept of cubic sets which is inter linked with interval-valued fuzzy set and fuzzy set. Interval – valued fuzzy set is another generalization of fuzzy sets that was introduced by Lotfi Asker Zadeh. Motivated by the theory of cubic structure our aim in this paper is to introduce the notation of cubic bi-ideals in near subtraction semigroups .We show that every cubic bi –ideal in a regular near subtraction semigroup is a cubic sub near subtraction semigroup. Homomorphism of cubic bi-ideals of near subtraction semigroups, and family of cubic bi-ideals in intersection. We also provide some results and study their related properties with examples.*

Key Words: *Near subtraction semigroups, bi-ideals, fuzzy bi-ideals, interval valued fuzzy bi-ideals, cubic bi-ideals.*

1. Introduction

B.M.Schein [11] considered systems of the form (X; o;/), where X is a set of functions closed under the composition "o" of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction "/" (and hence (X;/) is a subtraction algebra in the sense of [1]). Y.B.Jun et al[5] introduced the notation of ideals in subtraction algebras and discussed the characterization of ideals. The concept of fuzzy set was first initiated by Zadeh [13]. Narayanan et al.[10] defined the concept of generalized fuzzy ideals of near rings. Mahalakshmi et al. [8] studied the notation of bi-ideals in near subtraction semigroups. Manikandan [9] studied fuzzy bi-ideals in near-rings. Dheena et al [4] discussed the fundamental properties related to ideals and sub subtraction semigroups. In 1975 Zadeh introduced the concept of interval-valued fuzzy subsets. Thillaigovindan et al [12] introduced interval valued fuzzy ideals of near rings. Jun et al. [7] introduced the notation of cubic sub sets. Jun et al. [6] introduced the notation of cubic sub groups. Chinnadurai et al [3] introduced the notation of cubic ring. Many researchers who are involved in studing, applying,refining and teaching fuzzy sets have successfully applied this theory in many different fields.

The purpose of this paper is to introduce the notation of cubic bi-ideals in near-subtraction semigroups and homomorphism in near-subtraction semigroups. We investigate some basic results, examples and properties.

2.Preliminaries

Definition:2.1. A nonempty set X together with binary operation ''$-$'' is said to be subtraction algebra if it satisfies the following conditions
(i) $x-(y-x) = x$.
(ii) $x-(x - y) = y-(y - x)$.
(iii) $(x - y) - z = (x - z)-y$.

Definition:2.2. A nonempty set X together with two binary operations "$-$" and "•" is said to be a subtraction semigroup if it satisfies the following conditions
(i) $(X,-)$ is a subtraction algebra.
(ii) $(X,\bullet)$ is a semigroup.
(iii) $x(y-z) = xy - xz$ and $(x-y)z= xz - yz$
for every $x, y, z \in X$.

Definition:2.3. A non empty set X together with two binary operations "$-$"and'‘•’ 'is said to be a near subtraction semigroup(right) if it satisfies the following conditions
(i) $(X, -)$ is a subtractionalgebra.
(ii) $(X, \bullet)$ is a semigroup.

(iii) $(x - y)\, z = xz - yz$
for every $x, y, z \in X$.

It is clear that $0x = 0$, for all $x \in X$. Similarly we can define a left near- subtraction semigroup. Here after a near–subtraction semigroup means only a right near-subtraction semigroup.

Definition:2.4. A nonempty subset S of a subtraction semigroup X is said to be a subalgebra of X, if $x-y \in S$, for all $x, y \in S$.

Definition:2.5. A nonempty subset S of a near - subtraction algebra X is said to be a near subtraction subsemigroup of X, if $x-y \in S$, $x\,y \in S$ for all $x, y \in X$.

Definition:2.6. Let $(X, -, .)$ be a near-subtraction semigroup. A nonempty subset I of X is called

(i) A left ideal if I is a subalgebra of $(X, -)$ and $xi - x(y - i) \in I$ for all $x, y \in X$ and $i \in I$.

(ii) A right ideal if I is a subalgebra of $(X, -)$ and $IX \subseteq I$.

(iii) If I is both a left and right ideal, it is called a two-sided ideal (simply, ideal) of X.

Definition:2.7. A near subtraction semigroup X is said to be Zero – symmetric if $x0 = 0$ for every $x \in X$.

Definition:2.8. A fuzzy subset μ of X is called fuzzy ideal of X if it satisfies the following conditions:

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(xi - x(y - i)) \geq \mu(i)$

(iii) $\mu(x\,y) \geq \mu(x)$ for all $x, y \in X$.

Definition: 2.9. A fuzzy subset μ of X is said to be a fuzzy subalgebra of X, if $x, y \in X$ implies $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

Definition: 2.10. A fuzzy subalgebra μ of X is called a fuzzy bi-ideal of X, if it satisfies the following conditions:

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x\,y\,z) \geq \min\{\mu(x), \mu(z)\}$

for all $x, y, z \in X$

Definition: 2.11. Let X be a non empty set. A mapping $\bar{\mu} : X \rightarrow D[0, 1]$ is called interval-valued fuzzy set(in short i-v), where D [0, 1] denote the family of all closed sub intervals of [0, 1] and $\bar{\mu}(x) = [\mu^-(x), \mu^+(x)]$ for all $x \in X$, where μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x) \leq \mu^+(x)$ for all $x \in X$.

Definition: 2.12. Let X be a non empty set. A cubic set $\mathcal{A}$ in X is a structure of the form $\mathcal{A} = \{< x, \bar{\mu}_A(x), \lambda(x) >: x \in X\}$ and denoted by $\mathcal{A} = < \bar{\mu}_A, \lambda >$, *where* $\bar{\mu}_A = [\mu^-_A, \mu^+_A]$ is an interval-valued fuzzy set (briefly, IVF) in X and λ is a fuzzy set in X.

Definition: 2.13. For any non-empty subset G of a set X, the characteristic cubic set of G is defined to be a structure $\chi_G(x) = < x, \bar{\mu}_{\chi G}(x), \gamma_{\chi G}(x) : x \in X >$ which is briefly denoted by $\chi_G(x) = < x, \bar{\mu}_{\chi G}(x), \gamma_{\chi G}(x) >$

where $\bar{\mu}_{\chi G}(x) = \begin{cases} [1, 1] \text{ if } x \in G & \text{and} \\ [0, 0] \text{ otherwise} \end{cases}$ $\quad \gamma_{\chi G}(x) = \begin{cases} 0 \text{ if } x \in G \\ 1 \text{ otherwise} \end{cases}$

3. Main Results:

In this section we introduced the concept of cubic bi-ideals in near-subtraction semigroups and discuss some of its properties.

Definition:3.1. A cubic set $\mathcal{A} = < \bar{\mu}, \omega >$ in X is a cubic subnear- subtraction semigroup of X if for all $x, y \in X$. If it satisfies the following conditions:

(i) $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and
$\omega(x - y) \leq \max\{\omega(x), \omega(y)\}$

(ii) $\bar{\mu}(x\,y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and
$\omega(x\,y) \leq \max\{\omega(x), \omega(y)\}$

Definition:3.2. A cubic set $\mathcal{A} = < \bar{\mu}, \omega >$ in X is a cubic bi-ideal of X if for all $x, y, z \in X$. If it satisfies the following conditions:

(i) $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and
$\omega(x - y) \leq \max\{\omega(x), \omega(y)\}$

(ii) $\bar{\mu}(x\,y\,z) \geq \min\{\bar{\mu}(x), \bar{\mu}(z)\}$ and
$\omega(x\,y\,z) \leq \max\{\omega(x), \omega(z)\}$

Example 3.3. Let X ={0,a,b,c} in which '–' and '•'are defined by

–	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	0
c	c	0	c	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	0	b	b
c	0	a	b	c

Then (X, –, .) is a near subtraction semigroup. Define a cubic set $\mathcal{A} = <\bar{\mu}, \omega>$ by $\bar{\mu}(0) = [0.9, 1]$, $\bar{\mu}$ (a) = [0.8, 0.9], $\bar{\mu}$ (b)= [0.7, 0.7] = $\bar{\mu}$ (c) is an interval- valued fuzzy bi-ideal of X and ω (0)= 0.3, ω (a) = 0.7, ω (b) = 0.9 = ω (c) is a fuzzy bi-ideal of X. Then $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic bi-ideal of X.

Definition :3.4. Let $\mathcal{A}_i = <\bar{\mu}_i, \omega_i>$ be cubic ideals of near- subtraction semigroups X_i for i = 1, 2, 3,….. n. Then the cubic direct product of $\mathcal{A}_i$, (i = 1, 2, 3,….. n) is a function $(\bar{\mu}_1 \times \bar{\mu}_2 \times, \ldots\ldots, \times \bar{\mu}_n) : R_1 \times R_2 \times \ldots\ldots \times R_n \to D[0, 1]$, $(\omega_1 \times \omega_2 \times, \ldots\ldots, \times \omega_n) : R_1 \times R_2 \times \ldots\ldots \times R_n \to D[0, 1]$ defined by
$(\bar{\mu}_1 \times \bar{\mu}_2 \times, \ldots\ldots, \times \bar{\mu}_n)(x_1, x_2, \ldots\ldots, x_n) = \min\{(\bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \ldots, \bar{\mu}_n(x_n)\}$ and
$(\omega_1 \times \omega_2 \times, \ldots\ldots, \times \omega_n)(x_1, x_2, \ldots\ldots, x_n) = \max\{(\omega_1(x_1), \omega_2(x_2), \ldots, \omega_n(x_n)\}$.

Definition :3.5. Let $\mathcal{A} = <\bar{\mu}, \omega>$ be a cubic set of X. Then the strongest cubic relation on X is a cubic relation α with respect to $\mathcal{A} = <\bar{\mu}, \omega>$ given by $\alpha(x, y) = \{((x, y), \beta(x, y), \gamma(x, y)) \mid x, y \in X\}$, where β is an interval-valued fuzzy relation with respect to $\bar{\mu}$ defined by $\beta(x, y) = \min\{(\bar{\mu}(x), \bar{\mu}(y)\}$ and γ is a fuzzy relation with respect to ω defined by $\gamma(x, y) = \max\{(\omega(x), \omega(y)\}$.

Theorem: 3.6. Every cubic bi-ideal in a regular near-subtraction semigroup X is a cubic sub near-subtraction semigroupof X.

Proof: Let $\mathcal{A} = <\bar{\mu}, \omega>$ be a cubic bi-ideral of X and a, b ∈ X. Since X is regular, there exist x ∈ X such that a = axa. Then

$$\begin{aligned}\bar{\mu}(ab) &= \bar{\mu}((axa)b)\\ &= \bar{\mu}(a(xa)b)\\ &\geq \min\{\bar{\mu}(a), \bar{\mu}(b)\} \text{ and}\end{aligned}$$

$$\begin{aligned}\omega(ab) &= \omega((axa)b)\\ &= \omega(a(xa)b)\\ &\leq \max\{\omega(a), \omega(b)\}\end{aligned}$$

Thus $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic sub near-subtraction semigroupof X.

Proposition 3.7. Let X be a strongly regular near-subtraction semigroup. If $\mathcal{A} = <\bar{\mu}, \omega>$ be a cubic bi-ideal of X. Then $\bar{\mu}(x) = \bar{\mu}(x^2)$ and $\omega(x) = \omega(x^2)$ for all x ∈ X.

Proof: Let $\mathcal{A} = <\bar{\mu}, \omega>$ be a cubic bi-ideal of X and x ∈ X. Since X is strongly regular, there exist y ∈ X such that $x = x^2 y x^2$. Then

$$\begin{aligned}\bar{\mu}(x) &= \bar{\mu}(x^2 y x^2)\\ &\geq \min\{\bar{\mu}(x^2), \bar{\mu}(x^2)\}\\ &= \bar{\mu}(x^2)\\ &\geq \min\{\bar{\mu}(x), \bar{\mu}(x)\}\\ &= \bar{\mu}(x) \quad \text{and}\end{aligned}$$

$$\begin{aligned}\omega(x) &= \omega(x^2 y x^2)\\ &\geq \min\{\omega(x^2), \omega(x^2)\}\\ &= \omega(x^2)\\ &\geq \min\{\omega(x), \omega(x)\}\\ &= \omega(x)\end{aligned}$$

Hence $\bar{\mu}(x) = \bar{\mu}(x^2)$ and $\omega(x) = \omega(x^2)$.

Theorem: 3.8. The direct product of cubic bi-ideals of near-subtraction semigroups is also a cubic bi-ideals of near-subtraction semigroup.

Proof: $\mathcal{A}_i = <\bar{\mu}_i, \omega_i>$ be cubic ideals of near- subtraction semigroups X_i for i = 1, 2, 3,…n.

Let $x = (x_1, x_2, \ldots\ldots, x_n)$, $y = (y_1, y_2, \ldots\ldots, y_n)$ and $z = (z_1, z_2, \ldots\ldots, z_n) \in R_1 \times R_2 \times \ldots \times R_n$

(i) $\bar{\mu}_i(x - y) = \bar{\mu}_i((x_1, x_2, \ldots\ldots, x_n) - (y_1, y_2, \ldots\ldots, y_n))$

$= \bar{\mu}_i (x_1 - y_1, x_2 - y_2, \ldots, x_n - y_n)$
$= \min \{ \bar{\mu}_1(x_1 - y_1), \bar{\mu}_2(x_2 - y_2), \ldots, \bar{\mu}_n (x_n - y_n) \}$
$\geq \min \{ \min \{ \bar{\mu}_1(x_1), \bar{\mu}_1 (y_1)\}, \min \{ \bar{\mu}_2(x_2), \bar{\mu}_2(y_2)\} \ldots\ldots \{ \min \{ \bar{\mu}_n(x_n), \bar{\mu}_n(y_n)\}\}$
$= \min \{ \min \{ \bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \ldots, \bar{\mu}_n(x_n)\}, \min \{ \bar{\mu}_1(y_1), \bar{\mu}_2(y_2), \ldots \bar{\mu}_n(y_n)\}$
$= \min \{(\bar{\mu}_1 \times \bar{\mu}_2 \times , \ldots, \times \bar{\mu}_n) (x_1, x_2, \ldots, x_n), (\bar{\mu}_1 \times \bar{\mu}_2 \times, \ldots, \times \bar{\mu}_n) (y_1, y_2 , \ldots y_n)\}$
$\therefore$ $\bar{\mu}_i (x - y) \geq \min \{ \bar{\mu}_i(x), \bar{\mu}_i(y) \}$ and
$\omega_i (x - y) = \omega_i ((x_1, x_2, \ldots, x_n) - (y_1, y_2, \ldots, y_n))$
$= \omega_i (x_1 - y_1, x_2 - y_2, \ldots, x_n - y_n)$
$= \max\{\omega_1(x_1 - y_1), \omega_2 (x_2 - y_2), \ldots, \omega_n(x_n - y_n) \}$
$\leq \max\{\max \{ \omega_1 (x_1), \omega_1 (y_1)\}, \ldots \max\{ \omega_2 \ x_2), \omega_2 (y_2)\}, \ldots \{ \max\{ \omega_n (x_n), \omega_n (y_n)\}\}$
$= \max \{ \max\{ \omega_1 (x_1), \omega_2 (x_2), \ldots, \omega_n (x_n)\}, \max\{\omega_1 (y_1), \omega_2(y_2), \ldots, \omega_n (y_n)\}$
$= \max \{ (\omega_1 \times \omega_2 \times, \ldots, \times \omega_n) (x_1, x_2, \ldots, x_n), (\omega_1 \times \omega_2 \times, \ldots, \times \omega_n) (y_1, y_2 , \ldots y_n)\}$
$\therefore$ $\omega_i (x - y) \leq \max\{\omega_i (x), \omega_i (y) \}$
(ii) $\bar{\mu}_i (x y z) = \bar{\mu}_i ((x_1, x_2, \ldots, x_n) (y_1, y_2, \ldots, y_n) (z_1, z_2, \ldots, z_n))$
$= \bar{\mu}_i (x_1 y_1 z_1, x_2 y_2 z_2, \ldots, x_n y_n z_n)$
$= \min \{ \bar{\mu}_1(x_1 y_1 z_1), \bar{\mu}_2(x_2 y_2 z_2), \ldots, \bar{\mu}_n (x_n y_n z_n) \}$
$\geq \min \{ \min \{ \bar{\mu}_1(x_1), \bar{\mu}_1(z_1)\}, \min \{ \bar{\mu}_2(x_2), \bar{\mu}_2(z_2)\}, \ldots \{ \min \{ \bar{\mu}_n(x_n), \bar{\mu}_n(z_n)\}\}$
$= \min \{ \min \{ \bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \ldots, \bar{\mu}_n (x_n)\}, \min \{\bar{\mu}_1(z_1), \bar{\mu}_2(z_2), \ldots \bar{\mu}_n(z_n)\}$
$= \min \{(\bar{\mu}_1 \times \bar{\mu}_2 \times, \ldots, \times \bar{\mu}_n) (x_1, x_2, \ldots, x_n), (\bar{\mu}_1 \times \bar{\mu}_2 \times, \ldots, \times \bar{\mu}_n) (z_1, z_2 , \ldots z_n)\}$
$\therefore$ $\bar{\mu}_i (x y z) \geq \min \{ \bar{\mu}_i(x), \bar{\mu}_i(z) \}$ and
$\omega_i (x y z) = \omega_i ((x_1, x_2, \ldots, x_n) (y_1, y_2, \ldots, y_n) (z_1, z_2, \ldots, z_n))$
$= \omega_i (x_1 y_1 z_1 , x_2 y_2 z_2, \ldots, x_n y_n z_n)$
$= \max\{\omega_1(x_1 y_1 z_1), \omega_2 (x_2 y_2 z_2), \ldots, \omega_n(x_n y_n z_n) \}$
$\leq \max\{\max \{ \omega_1 (x_1), \omega_1 (z_1)\}, \max\{ \omega_2 (x_2), \omega_2 (z_2)\}, \ldots \{ \max\{ \omega_n (x_n), \omega_n (z_n)\}\}$
$= \max \{ \max\{ \omega_1 (x_1), \omega_2 (x_2), \ldots, \omega_n (x_n)\}, \max\{\omega_1 (z_1), \omega_2(z_2), \ldots, \omega_n (z_n)\}$
$= \max \{ (\omega_1 \times \omega_2 \times, \ldots, \times \omega_n) (x_1, x_2, \ldots, x_n), (\omega_1 \times \omega_2 \times, \ldots, \times \omega_n) (y_1, y_2 , \ldots y_n) \}$
$\therefore$ $\omega_i (x y z) \leq \max\{\omega_i (x), \omega_i (z) \}$

Hence, the direct product of cubic bi-ideals of near-subtraction semigroups is also a cubic bi-ideals of near-subtraction semigroup.

Theorem: 3.9. Let $\mathcal{A} = < \bar{\mu} , \omega >$ be a cubic bi-ideal of X, then the set $X_{\mathcal{A}} = \{ x \in X \mid \mathcal{A}(x) = \mathcal{A}(0) \}$ is a cubic bi-ideal of X.

Proof: Let $\mathcal{A} = < \bar{\mu} , \omega >$ be a cubic bi-ideal of X and x, y ∈ X, then $\mathcal{A}(x) = \mathcal{A}(0)$ and $\mathcal{A}(y) = \mathcal{A}(0)$. Suppose x, y, z ∈ $X_{\mathcal{A}}$.

Then $\bar{\mu} (x) = \bar{\mu} (y) = \bar{\mu} (z) = \bar{\mu} (0)$ and
$\omega(x) = \omega(y) = \omega(z) = \omega(0)$
Since, $\bar{\mu}$ is an i– v fuzzy bi-ideal of X,
(i) $\bar{\mu} (x - y) \geq \min \{\bar{\mu} (x), \bar{\mu} (y)\}$
$= \min \{ \bar{\mu} (0), \bar{\mu} (0)\} = \bar{\mu} (0)$ and
ω is a fuzzy bi-ideal of X
$\omega (x - y) \leq \max \{ \omega (x) , \omega (y) \}$
$= \max \{ \omega (0) , \omega (0) \} = \omega (0)$
Thus x – y ∈ $X_{\mathcal{A}}$.
(i) $\bar{\mu} (x y z) \geq \min \{\bar{\mu} (x), \bar{\mu} (z)\}$
$= \min \{ \bar{\mu} (0), \bar{\mu} (0)\} = \bar{\mu} (0)$ and
$\omega (x \ y z) \leq \max \{ \omega (x) , \omega (z) \}$
$= \max \{ \omega (0) , \omega (0) \} = \omega (0)$
Thus x y z ∈ $X_{\mathcal{A}}$.
Therefore, $X_{\mathcal{A}}$ is a cubic bi-ideal of X.

Theorem: 3.10. Let $\{ \mathcal{A}_i \} = < \bar{\mu}_i, \omega_i \mid i \in \wedge >$ be a family of cubic bi-ideals of X and then the cubic set $\underset{i\in\wedge}{\sqcap} \mathcal{A}_i = < \cap_{i\in\wedge} \bar{\mu}_i , \cup_{i \in \wedge} \omega_i >$ is also a cubic bi-ideal of X, where ∧ is any index set.

Proof: Let $\{ \mathcal{A}_i \} = < \bar{\mu}_i, \omega_i \mid i \in \wedge >$ be a family of cubic bi-ideals of X.

Let x, y, z ∈ X and $\bar{\mu} = \cap \bar{\mu}_i$; $\omega = \cup \omega_i$
$\bar{\mu} (x) = \cap \bar{\mu}_i (x) = (\inf \bar{\mu}_i) (x) = \inf \bar{\mu}_i (x)$
$\omega (x) = \cup \omega_i(x) = (\sup \omega_i) (x) = \sup \omega_i(x)$

$(i)\ \bar{\mu}(x - y) = \inf\{\bar{\mu}_i(x - y)\}$
$\geq \inf\{\min\{\bar{\mu}_i(x), \bar{\mu}_i(y)\}\}$
$= \min\{\inf \bar{\mu}_i(x), \inf \bar{\mu}_i(y)\}$
$= \min\{\cap \bar{\mu}_i(x), \cap \bar{\mu}_i(y)\}$
$\geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and
$\omega(x - y) = \sup \omega_i(x - y)$
$\leq \sup\{\max\{\omega_i(x), \omega_i(y)\}\}$
$= \max\{\sup \omega_i(x), \sup \omega_i(y)\}$
$= \max\{\cup \omega_i(x), \cup \omega_i(y)\}$
$\leq \max\{\omega(x), \omega(y)\}$
$(ii)\ \bar{\mu}(x\ y\ z) = \inf\{\bar{\mu}_i(x\,y\,z)\}$
$\geq \inf\{\min\{\bar{\mu}_i(x), \bar{\mu}_i(z)\}\}$
$= \min\{\inf \bar{\mu}_i(x), \inf \bar{\mu}_i(z)\}$
$= \min\{\cap \bar{\mu}_i(x), \cap \bar{\mu}_i(z)\}$
$\geq \min\{\bar{\mu}(x), \bar{\mu}(z)\}$ and
$\omega(x\,y\,z) = \sup \omega_i(x\,y\,z)$
$\leq \sup\{\max\{\omega_i(x), \omega_i(z)\}\}$
$= \max\{\sup \omega_i(x), \sup \omega_i(z)\}$
$= \max\{\cup \omega_i(x), \cup \omega_i(z)\}$
$\leq \max\{\omega(x), \omega(z)\}$

Hence, $\prod_{i\in\Lambda} \mathcal{A}_i = < \cap_{i\in\Lambda} \bar{\mu}_i , \cup_{i\in\Lambda} \omega_i >$ is a cubic bi-ideal of X.

Theorem: 3.11. Let $\mathcal{A} = <\bar{\mu}, \omega>$ be a cubic set of a near-subtraction semigroup X and $\alpha(x, y) = \{((x, y), \beta(x, y), \gamma(x, y)) \mid x, y \in X\}$ be a strongest cubic relation with respect to α. Then $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic bi-ideal of X if and only if α is a cubic bi-ideal of $X \times X$.

Proof: Assume that $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic bi-ideal of X. Let $x_1, x_2, y_1, y_2, z_1, z_2 \in X \times X$. Then $x = (x_1, x_2)$, $y = (y_1, y_2)$, $z = (z_1, z_2) \in X \times X$, We have

$(i)\ \beta(x - y) = \beta((x_1, x_2) - (y_1, y_2))$
$= \beta(x_1 - y_1, x_2 - y_2)$
$= \min\{\bar{\mu}(x_1 - y_1), \bar{\mu}(x_2 - y_2)\}$
$\geq \min\{\min\{\bar{\mu}(x_1), \bar{\mu}(y_1)\}, \min\{\bar{\mu}(x_2), \bar{\mu}(y_2)\}\}$
$= \min\{\min\{\bar{\mu}(x_1), \bar{\mu}(x_2)\}, \min\{\bar{\mu}(y_1), \bar{\mu}(y_2)\}\}$
$= \min\{\beta(x_1, x_2), \beta(y_1, y_2)\}$
$= \min\{\beta(x), \beta(y)\}$
$\gamma(x - y) = \gamma((x_1, x_2) - (y_1, y_2))$
$= \gamma(x_1 - y_1, x_2 - y_2)$
$= \max\{\omega(x_1 - y_1), \omega(x_2 - y_2)\}$
$\leq \max\{\max\{\omega(x_1), \omega(y_1)\}, \max\{\omega(x_2), \omega(y_2)\}\}$
$= \max\{\max\{\omega(x_1), \omega(x_2)\}, \max\{\omega(y_1), \omega(y_2)\}\}$
$= \max\{\gamma(x_1, x_2), \gamma(y_1, y_2)\}$
$= \max\{\gamma(x), \gamma(y)\}$
$(ii\ \ \beta(x\,y\,z) = \beta((x_1, x_2)(y_1, y_2)(z_1, z_2))$
$= \beta(x_1 y_1 z_1, x_2 y_2 z_2)$
$= \min\{\bar{\mu}(x_1 y_1 z_1), \bar{\mu}(x_2 y_2 z_2)\}$
$\geq \min\{\min\{\bar{\mu}(x_1), \bar{\mu}(z_1)\}, \min\{\bar{\mu}(x_2), \bar{\mu}(z_2)\}\}$
$= \min\{\min\{\bar{\mu}(x_1), \bar{\mu}(x_2)\}, \min\{\bar{\mu}(z_1), \bar{\mu}(z_2)\}\}$
$= \min\{\beta(x_1, x_2), \beta(z_1, z_2)\}$
$= \min\{\beta(x), \beta(z)\}$
$\gamma(x\,y\,z) = \gamma((x_1, x_2)(y_1, y_2)(z_1, z_2))$
$= \gamma(x_1 y_1 z_1, x_2 y_2 z_2)$
$= \max\{\omega(x_1 y_1 z_1), \omega(x_2 y_2 z_2)\}$
$\leq \max\{\max\{\omega(x_1), \omega(z_1)\}, \max\{\omega(x_2), \omega(z_2)\}\}$
$= \max\{\max\{\omega(x_1), \omega(x_2)\}, \max\{\omega(z_1), \omega(z_2)\}\}$
$= \max\{\gamma(x_1, x_2), \gamma(z_1, z_2)\}$
$= \max\{\gamma(x), \gamma(z)\}$

Hence α is a cubic bi-ideal of X × X.

Conversely, assume that α is a cubic bi-ideal of X × X.

Then x= (x_1, x_2), y = (y_1, y_2), z = $(z_1, z_2) \in X \times X$.

(i) $\min\{\bar{\mu}(x_1 - y_1), \bar{\mu}(x_2 - y_2)\} = \beta(x_1 - y_1,\ x_2 - y_2)$
$= \beta((x_1, x_2) - (y_1, y_2))$
$= \beta(x - y)$
$\geq \min\{\beta(x), \beta(y)\}$
$= \min\{\beta(x_1, x_2), \beta(y_1, y_2)\}$
$= \min\{\min\{\bar{\mu}(x_1), \bar{\mu}(x_2)\}, \min\{\bar{\mu}(y_1), \bar{\mu}(y_2)\}\}$

If $\bar{\mu}(x_1 - y_1) \leq \bar{\mu}(x_2 - y_2)$, then $\bar{\mu}(x_1) \leq \bar{\mu}(x_2)$ and $\bar{\mu}(y_1) \leq \bar{\mu}(y_2)$, we get
$\bar{\mu}(x_1 - y_1) \geq \min\{\bar{\mu}(x_1), \bar{\mu}(y_1)\}$.

$\max\{\omega(x_1 - y_1), \omega(x_2 - y_2)\} = \gamma(x_1 - y_1, x_2 - y_2)$
$= \gamma((x_1, x_2) - (y_1, y_2))$
$= \gamma(x - y)$
$\geq \max\{\gamma(x), \gamma(y)\}$
$= \max\{\gamma(x_1, x_2), \gamma(y_1, y_2)\}$
$= \max\{\max\{\omega(x_1), \omega(x_2)\}, \max\{\omega(y_1), \omega(y_2)\}\}$

If $\omega(x_1 - y_1) \geq \omega(x_2 - y_2)$, then $\omega(x_1) \geq \omega(x_2)$ and $\omega(y_1) \geq \omega(y_2)$, we get
$\omega(x_1 - y_1) \leq \max\{\omega(x_1), \omega(y_1)\}$.

(ii) $\min\{\bar{\mu}(x_1 y_1 z_1), \bar{\mu}(x_2 y_2 z_2)\} = \beta(x_1 y_1 z_1,\ x_2 y_2 z_2)$
$= \beta((x_1, x_2)(y_1, y_2)(z_1, z_2))$
$= \beta(x y z)$
$\geq \min\{\beta(x), \beta(z)\}$
$= \min\{\beta(x_1, x_2), \beta(z_1, z_2)\}$
$= \min\{\min\{\bar{\mu}(x_1), \bar{\mu}(z_1)\}, \min\{\bar{\mu}(x_2), \bar{\mu}(z_2)\}\}$

If $\bar{\mu}(x_1 y_1 z_1) \leq \bar{\mu}(x_2 y_2 z_2)$, then $\bar{\mu}(x_1) \leq \bar{\mu}(x_2)$ and $\bar{\mu}(y_1) \leq \bar{\mu}(y_2)$ and $\bar{\mu}(z_1) \leq \bar{\mu}(z_2)$, we get
$\bar{\mu}(x_1 y_1 z_1) \geq \min\{\bar{\mu}(x_1), \bar{\mu}(z_1)\}$.

$\max\{\omega(x_1 y_1 z_1), \omega(x_2 y_2 z_2)\} = \gamma(x_1 y_1 z_1,\ x_2 y_2 z_2)$
$= \gamma((x_1, x_2)(y_1, y_2)(z_1, z_2))$
$= \gamma(x y z)$
$\geq \max\{\gamma(x), \gamma(z)\}$
$= \max\{\gamma(x_1, x_2), \gamma(z_1, z_2)\}$
$= \max\{\max\{\omega(x_1), \omega(x_2)\}, \max\{\omega(z_1), \omega(z_2)\}\}$

If $\omega(x_1 y_1 z_1) \geq \omega(x_2 y_2 z_2)$, then $\omega(x_1) \geq \omega(x_2)$ and $\omega(y_1) \geq \omega(y_2)$ and $\omega(z_1) \geq \omega(z_2)$, we get
$\omega(x_1 y_1 z_1) \leq \max\{\omega(x_1), \omega(z_1)\}$.

Hence $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a cubic bi-ideal of X.

Theorem: 3.12. If $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a cubic set of a near-subtraction semigroup X. Then $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic bi-ideal of X if and only if the cubic level set $\cup(\mathcal{A}; \tilde{t}, n)$ is a bi-ideal of X, when it is non-empty.

Proof: Assume that $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic bi-ideal of X.

Let x, y, z $\in \cup(\mathcal{A}; \tilde{t}, n)$ for all $\tilde{t} \in D[0, 1]$ and $n \in [0, 1]$.

Then $\bar{\mu}(x) \geq \tilde{t}$, $\bar{\mu}(y) \geq \tilde{t}$, $\bar{\mu}(z) \geq \tilde{t}$ and $\omega(x) \leq n$, $\omega(y) \leq n$, $\omega(z) \leq n$.

Now suppose x, y $\in \cup(\mathcal{A}; \tilde{t}, n)$ then by definition of cubic bi-ideal

$\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} \geq \min\{\tilde{t}, \tilde{t}\} \geq \tilde{t}$ and

$\omega(x - y) \leq \max\{\omega(x), \omega(y)\} \leq \max\{n, n\} \leq n$.

Hence $x - y \in \cup(\mathcal{A}; \tilde{t}, n)$.

Suppose x, z $\in \cup(\mathcal{A}; \tilde{t}, n)$ and $y \in X$ then

$\bar{\mu}(x y z) \geq \min\{\bar{\mu}(x), \bar{\mu}(z)\} \geq \min\{\tilde{t}, \tilde{t}\} \geq \tilde{t}$ and

$\omega(x y z) \leq \max\{\omega(x), \omega(y)\} \leq \max\{n, n\} \leq n$.

Hence $x\ y\ z \in \cup(\mathcal{A}; \tilde{t}, n)$.

Therefore $\cup(\mathcal{A}; \tilde{t}, n)$ is a bi-ideal of X.

Conversely, let $\tilde{t} \in D[0, 1]$ and $n \in [0, 1]$ be such that $\cup(\mathcal{A}; \tilde{t}, n) \neq \emptyset$ and $\cup(\mathcal{A}; \tilde{t}, n)$ is a bi-ideal of X.

Suppose we assume that

$\bar{\mu}(x - y) \ngeq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ (or)

$\omega(x - y) \nleq \max\{\omega(x), \omega(y)\}$

If $\bar{\mu}(x - y) \ngeq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$, then there exist $\tilde{t}_1 \in D[0, 1]$ such that

$\bar{\mu}(x - y) < \tilde{t}_1 < \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ hence x, y $\in \cup(\mathcal{A}; \tilde{t}_1, n, \max\{\omega(x), \omega(y)\})$, but

$x - y \notin \cup(\mathcal{A}; \tilde{t}_1, n, \max\{\omega(x), \omega(y)\})$

This is a contraction.

If ω (x – y) $\nleq$ max { ω (x) , ω (y) }, then there exist $n_1 \in$ [0, 1] such that

ω (x – y) > n_1 > max { ω (x) , ω (y) } hence x, y $\in \cup$ ($\mathcal{A}$; min {$\bar{\mu}$ (x), $\bar{\mu}$ (y)}, n_1), but

x – y $\notin \cup$ ($\mathcal{A}$; min {$\bar{\mu}$ (x), $\bar{\mu}$ (y)}, n_1)

This is a contraction.

Hence $\bar{\mu}$ (x – y) $\geq$ min {$\bar{\mu}$ (x), $\bar{\mu}$ (y)} and

ω (x – y) $\leq$ max { ω (x) , ω (y) }

Suppose we assume that

$\bar{\mu}$ (x y z) $\ngeq$ min {$\bar{\mu}$ (x), $\bar{\mu}$ (z)} (or)

ω (x y z) $\nleq$ max { ω (x) , ω (z) }

If $\bar{\mu}$ (x y z) $\ngeq$ min {$\bar{\mu}$ (x), $\bar{\mu}$ (z)}, then there exist $\tilde{t}_1 \in$ D [0, 1] such that

$\bar{\mu}$ (x y z) $< \tilde{t}_1 <$ min {$\bar{\mu}$ (x), $\bar{\mu}$ (z)} hence x, z $\in \cup$ ($\mathcal{A}$; $\tilde{t}_1$, n , max { ω (x) , ω (z) }), but

x y z $\notin \cup$ ($\mathcal{A}$; $\tilde{t}_1$, n , max { ω (x) , ω (z) })

This is a contraction.

If ω (x y z) $\nleq$ max { ω (x) , ω (z) }, then there exist $n_1 \in$ [0, 1] such that

ω (x y z) > n_1 > max { ω (x) , ω (z) } hence x, z $\in \cup$ ($\mathcal{A}$; min {$\bar{\mu}$ (x), $\bar{\mu}$ (z)}, n_1), but

x y z $\notin \cup$ ($\mathcal{A}$; min {$\bar{\mu}$ (x), $\bar{\mu}$ (z)}, n_1)

This is a contraction.

Hence $\bar{\mu}$ (x y z) $\geq$ min {$\bar{\mu}$ (x), $\bar{\mu}$ (z)} and

ω (x y z) $\leq$ max { ω (x) , ω (z) }

Therefore $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic bi-ideal of X.

Theorem: 3.13. Let H be a non empty subset of X. Then H is a bi-ideal of X if and only if the characteristic cubic set $\chi_H = < \bar{\mu}_{\chi_H}, \omega_{\chi_H} >$ of H in X is a cubic bi-ideal of X.

Proof: Assume that H is a bi-ideal of X. Let x, y $\in$ X. Suppose that

$\bar{\mu}_{\chi_H}$ (x – y) $<$ min {$\bar{\mu}_{\chi_H}$ (x), $\bar{\mu}_{\chi_H}$ (y)} and ω_{χ_H} (x – y) $>$ max {ω_{χ_H} (x), ω_{χ_H} (y)}.

It follows that

$\bar{\mu}_{\chi_H}$ (x – y) = $\bar{0}$, min {$\bar{\mu}_{\chi_H}$ (x), $\bar{\mu}_{\chi_H}$ (y)} = $\bar{1}$ and

ω_{χ_H} (x – y) = 1, max {ω_{χ_H} (x), ω_{χ_H} (y)} = 0.

This implies that x, y $\in$ H but x – y $\notin$ H.

This is a contraction.

Suppose that

$\bar{\mu}_{\chi_H}$ (x y z) $<$ min {$\bar{\mu}_{\chi_H}$ (x), $\bar{\mu}_{\chi_H}$ (z)} and ω_{χ_H} (x y z) $>$ max {ω_{χ_H} (x), ω_{χ_H} (z)}.

It follows that

$\bar{\mu}_{\chi_H}$ (x y z) = $\bar{0}$, min {$\bar{\mu}_{\chi_H}$ (x), $\bar{\mu}_{\chi_H}$ (z)} = $\bar{1}$ and

ω_{χ_H} (x y z) = 1, max {ω_{χ_H} (x), ω_{χ_H} (z)} = 0.

This implies that x, z $\in$ H but x y z $\notin$ H.

This is a contraction.

Hence $\chi_H = < \bar{\mu}_{\chi_H}, \omega_{\chi_H}$(x) $>$ is a cubic bi-ideal of X.

Conversely, assume that $\chi_H = < \bar{\mu}_{\chi_H}, \omega_{\chi_H} >$ is a cubic bi-ideal of X.

Let x, y $\in$ H then $\bar{\mu}_{\chi_H}$ (x) = $\bar{\mu}_{\chi_H}$ (y) = $\bar{1}$ and

ω_{χ_H}(x) = ω_{χ_H}(y) = 0, since χ_H is a cubic bi-ideal of X.

$\bar{\mu}_{\chi_H}$ (x – y) $\geq$ min {$\bar{\mu}_{\chi_H}$ (x), $\bar{\mu}_{\chi_H}$ (y)} = $\bar{1}$ and

ω_{χ_H} (x – y) $\leq$ max {ω_{χ_H} (x), ω_{χ_H} (y)} = 0.

This implies that x – y $\in$ H.

Let x, z $\in$ H and y $\in X$ then $\bar{\mu}_{\chi_H}$ (x) = $\bar{\mu}_{\chi_H}$ (z) = $\bar{1}$ and

ω_{χ_H}(x) = ω_{χ_H}(z) = 0, since χ_H is a cubic bi-ideal of X.

$\bar{\mu}_{\chi_H}$ (x y z) $\geq$ min {$\bar{\mu}_{\chi_H}$ (x), $\bar{\mu}_{\chi_H}$ (z)} = $\bar{1}$ and

ω_{χ_H} (x y z) $\leq$ max {ω_{χ_H} (x), ω_{χ_H} (z)} = 0.

This implies that x y z $\in$ H.

Hence H is a bi-ideal of X.

Theorem: 3.14. If $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic bi-ideal of X, then $\mathcal{A}^c = < (\bar{\mu})^c, (\omega)^c >$ is also a cubic bi-ideal of X.

Proof: Let x, y $\in$ X and $\mathcal{A} = <\bar{\mu}, \gamma>$ is a cubic bi-ideal of X, then

(i) $(\bar{\mu})^c$ (x – y) = 1 – $\bar{\mu}$ (x – y)

$\leq 1 - \min\{\bar{\mu}(x), \bar{\mu}(y)\}$
$= \max\{1 - \bar{\mu}(x), 1 - \bar{\mu}(y)\}$
$= \max\{(\bar{\mu})^c(x), (\bar{\mu})^c(y)\}$ and
$(\omega)^c(x - y) = 1 - \omega(x - y)$
$\geq 1 - \max\{\omega(x), \omega(y)\}$
$= \min\{1 - \omega(x), 1 - \omega(y)\}$
$= \min\{(\omega)^c(x), (\omega)^c(y)\}$
(ii) $(\bar{\mu})^c(xyz) = 1 - \bar{\mu}(xyz)$
$\leq 1 - \min\{\bar{\mu}(x), \bar{\mu}(z)\}$
$= \max\{1 - \bar{\mu}(x), 1 - \bar{\mu}(z)\}$
$= \max\{(\bar{\mu})^c(x), (\bar{\mu})^c(z)\}$ and
$(\omega)^c(xyz) = 1 - \omega(xyz)$
$\geq 1 - \max\{\omega(x), \omega(z)\}$
$= \min\{1 - \omega(x), 1 - \omega(z)\}$
$= \min\{(\omega)^c(x), (\omega)^c(z)\}$
Therefore, $\mathcal{A}^c = <(\bar{\mu})^c, (\omega)^c>$ is a cubic bi-ideal of X.

4.Homomorphism of cubic bi-ideals in near subtraction semigroups:

Definition: 4.1. Let X and Y be two near subtraction semigroups. A map f: X → Y is called near subtraction semigroup homomorphism. If it satisfies the following conditions:
(i) $f(x - y) = f(x) - f(y)$
(ii) $f(xy) = f(x) f(y)$ for all x, y ∈ X.

Definition: 4.2. Let X and Y given classical sets. A mapping f : X → Y induces two mappings C_f : C (X) → C(Y), $\mathcal{A}_1 \rightarrow C_f(\mathcal{A}_1)$ and ${C_f}^{-1}$: C(Y) → C(X), $\mathcal{A}_2 \rightarrow {C_f}^{-1}(\mathcal{A}_2)$ where the mapping C_f is called cubic transformation and ${C_f}^{-1}$ is called inverse cubic transformation.

Definition: 4.3. A cubic set $\mathcal{A} = <\bar{\mu}, \lambda>$ in X has the cubic property if for any subset T of X there exist $x_0 \in T$ such that $\bar{\mu}(x_0) = \sup_{x \in T} \bar{\mu}(x)$ and $\lambda(x_0) = \inf_{x \in T} \lambda(x)$.

Definition: 4.4. Let f be a mapping from a set X to a set Y and $\mathcal{A} = <\bar{\mu}, \lambda>$ be a cubic set of X then the image of P $C_f(\mathcal{A}) = <C_f(\bar{\mu}), C_f(\lambda)>$ is a cubic set of Y is defined by

$$C_f(\mathcal{A})(x) = \begin{cases} C_f(\bar{\mu})(y) = \begin{cases} \sup_{y=f(x)} \bar{\mu}(x) & \text{if } f^{-1}(y) \neq \emptyset \\ [0,0] & \text{otherwise} \end{cases} \\ C_f(\lambda)(y) = \begin{cases} \inf_{y=f(x)} \lambda(x) & \text{if } f^{-1}(y) \neq \emptyset/ \\ 1 & \text{otherwise} \end{cases} \end{cases}$$

for all y ∈Y and let f be a mapping from a set X to Y and $\mathcal{A} = <\bar{\mu}, \lambda>$ be a cubic set of Y then the pre image of Y ${C_f}^{-1}(\mathcal{A}) = <{C_f}^{-1}(\bar{\mu}), {C_f}^{-1}(\lambda)>$ is a cubic set of X is defined by

$${C_f}^{-1}(\mathcal{A}) = \begin{cases} {C_f}^{-1}(\bar{\mu}(x)) = \bar{\mu}(f(x)) \\ {C_f}^{-1}(\lambda(x)) = \lambda(f(x)) \quad \text{for all } x \in X. \end{cases}$$

Theorem: 4.5. Let f : X → X_1 be a homomorphism of near subtraction semigroup and ${C_f}^{-1}$: C (X_1) → C(X) be the inverse cubic transformation induced by f. If $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic bi-ideal of X_1 by the cubic property then ${C_f}^{-1}(\mathcal{A}) = <{C_f}^{-1}(\bar{\mu}), {C_f}^{-1}(\omega)>$ is a cubic bi-ideal of X.

Proof: Let $\mathcal{A} = <\bar{\mu}, \omega>$ is a cubic bi-ideal of X_1.
For all x, y, z ∈ X then
(i) ${C_f}^{-1}(\bar{\mu}(x - y)) = \bar{\mu}(f(x - y))$

$= \bar{\mu}\ (f(x) - f(y))$

$\geq \min\{\bar{\mu}\ (f(x),\ \bar{\mu}\ (f(y))\}$

$C_f^{-1}(\bar{\mu}(x-y)) \geq \min\{C_f^{-1}(\bar{\mu}(x)),\ C_f^{-1}(\bar{\mu}(y))\}$ and

$C_f^{-1}(\omega(x-y)) = \omega(f(x-y))$

$= \omega(f(x) - f(y))$

$\leq \max\{\omega(f(x)),\ \omega(f(y))\}$

$C_f^{-1}(\omega(x-y)) \leq \max\{C_f^{-1}(\omega(x)),\ C_f^{-1}(\omega(y))\}$

(ii) $C_f^{-1}(\bar{\mu}(x y z)) = \bar{\mu}\ (f(x y z))$

$= \bar{\mu}\ (f(x) f(y) f(z))$

$\geq \min\{\bar{\mu}\ (f(x)),\ \bar{\mu}\ (f(z))\}$

$C_f^{-1}(\bar{\mu}(x y z)) \geq \min\{C_f^{-1}(\bar{\mu}(x)),\ C_f^{-1}(\bar{\mu}(z))\}$ and

$C_f^{-1}(\omega(x y z)) = \omega(f(x y z))$

$= \omega(f(x) f(y) f(z))$

$\leq \max\{\omega(f(x)),\ \omega(f(z))\}$

$C_f^{-1}(\omega(x y z)) \leq \max\{C_f^{-1}(\omega(x)),\ C_f^{-1}(\omega(z))\}$

Hence, $C_f^{-1}(\mathcal{A}) = < C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) >$ is a cubic bi-ideal of X.

Theorem: 4.6. Let f : X→ X_1 be an onto near subtraction semigroup homomorphism, and let C_f: C (X) → C(X_1) be the cubic transformation induced by f. If $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic bi-ideal of X by the cubic property, then $C_f(\mathcal{A})$ is a cubic bi-ideal of X_1.

Proof: Let $\mathcal{A} = < \bar{\mu}, \omega >$ is a cubic bi-ideal of X.

Since $(C_f(\bar{\mu}))(x') = \sup_{f(x)=x'} \bar{\mu}(x)$ and

$(C_f(\omega))(x') = \inf_{f(x)=x'} \omega(x)$ for all x' ∈ X_1.

So, $C_f(\mathcal{A}) = < C_f(\bar{\mu}), C_f(\omega) >$ is non empty.

Let x', y', z' ∈ X_1. Then we have

(i) $C_f(\bar{\mu}))(x' - y') = \sup_{f(a)=x'-y'} \bar{\mu}(a)$

$\geq \sup_{f(x)=x',\ f(y)=y'} \bar{\mu}(x - y)$

$\geq \sup_{f(x)=x',\ f(y)=y'} \min\{\bar{\mu}(x),\ \bar{\mu}(y)\}$

$= \min\{\sup_{f(x)=x'} \bar{\mu}(x),\ \sup_{f(y)=y'} \bar{\mu}(y)\}$

$= \min\{C_f(\bar{\mu})(x'),\ C_f(\bar{\mu})(y')\}$ and

$C_f(\omega))(x' - y') = \inf_{f(a)=x'-y'} \omega(a)$

$\leq \inf_{f(x)=x',\ f(y)=y'} \omega(x - y)$

$\leq \inf_{f(x)=x',\ f(y)=y'} \max\{\omega(x),\ \omega(y)\}$

$= \max\{\inf_{f(x)=x'} \omega(x),\ \inf_{f(y)=y'} \omega(y)\}$

$= \max\{C_f(\omega)(x'),\ C_f(\omega)(y')\}$

(ii) $C_f(\bar{\mu}))(x' y' z') = \sup_{f(a)=x' y' z'} \bar{\mu}(a)$

$\geq \sup_{f(x)=x',\ f(y)=y',\ f(z)=z'} \bar{\mu}(x y z)$

$\geq \sup_{f(x)=x',\ f(z)=z'} \min\{\bar{\mu}(x),\ \bar{\mu}(z)\}$

$= \min\{\sup_{f(x)=x'} \bar{\mu}(x),\ \sup_{f(z)=z'} \bar{\mu}(z)\}$

$= \min\{C_f(\bar{\mu})(x'),\ C_f(\bar{\mu})(z')\}$ and

$C_f(\omega))(x' y' z') = \inf_{f(a)=x' y' z'} \omega(a)$

$\leq \inf_{f(x)=x',\ f(y)=y',\ f(z)=z'} \omega(x y z)$

$\leq \inf_{f(x)=x',\ f(z)=z'} \max\{\omega(x),\ \omega(z)\}$

$= \max\{\inf_{f(x)=x'} \omega(x),\ \inf_{f(z)=z'} \omega(z)\}$

$= \max\{C_f(\omega)(x'),\ C_f(\omega)(z')\}$

Hence, $C_f(\mathcal{A}) = < C_f(\bar{\mu}), C_f(\omega) >$ is a cubic bi-ideal of X_1.

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