A COMPARTIVE STUDY ON PRODUCTION PLANNING USING FFLPP AND FFGLP

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Received: March 04, 2019 Accepted: April 09, 2019

ABSTRACT: : *In this paper, a Linear Programming problem is executed in a factory for a consumer product production model and is investigated through a fully fuzzy linear programming model and is compared with fully fuzzy goal programming problem.*

Key Words: *FFLPP, Goal programming, Decision making, Compensatory operator.*

I. Introduction

Decision making helps executives for determining the best alternative in any decision activity which comprise of different procedures and criteria [2]. Fuzzy set theory is an optimum approach to cope with the uncertainties in decision problems modelled with linear programming. Linear programming is one of the most frequently applied operations research techniques. It is useful to develop new approaches in real world problems within the framework of linear programming. In the real world situations, a linear programming model involves a number of parameters whose values are assigned by experts. However, both experts and decision makers do not precisely know the value of those parameters. Therefore, the knowledge of experts on parameters is considered as fuzzy data [5].Linear programming technique is applicable only when there is a single goal (objective function) such as maximizing the profit or minimizing the cost or loss. There are some situations where the system may have multiple goals. In these situations fuzzy goal programming problems are used to find an optimum solution.

II. Literature Review

In fuzzy decision making problems, the concept of Decision Making was proposed by Bellman and Zadeh (1970) [3]. This concept was adapted to problems of mathematical programming by Tanaka et.al. (1984)[10]. Zimmermann (1983) presented a fuzzy approach to multi-objective linear programming problems. It is concerned with the optimization of a linear function while satisfying a set of linear equality and/or inequality constraints or restrictions. In the present practical situations, the available information in the system under consideration are not exact, therefore fuzzy linear programming was introduced. Bahaguney et.al. [2] proposed a fuzzy linear programming approach for determining the production amounts in food industry. Effati and Abbasiyan [4] proposed solving fuzzy linear programming problems with piecewise linear membership function. Jayalakshmi and Pandian [5] proposed a new method for finding an optimal fuzzy solution for fully fuzzy linear programming problems. Karpagam and Sumathi [6] proposed a new approach to solve fuzzy linear programming problems by ranking function. Nasseri [8] proposed a new method for solving fuzzy linear programming by solving linear programming. Rajarajeswari and SahayaSudha [9] proposed solving a fully fuzzy linear programming problem by ranking. Babita Mishra and singh.S.R [1] proposed solving a fully fuzzy multi-objective programming problem using its equivalent weighted goal programming problem. Mukesh Kumar Sinha et.al. [7] proposed fuzzy muli-objective linear programming approach for solving problem of food industry.

III. Preliminaries

3.1Fuzzy set

Let X denote a universal set. Then the membership function $\mu_{\tilde{A}}$ by which a fuzzy set A is usually defined by,

$$\mu_{\tilde{A}}: X \to [0,1]$$

3.2 Membership Function

The characteristic function $\mu_{\tilde{A}}$ of a crisp set $A \subset X$ assigns a value either 0 or 1 to each member in X. This function can be generalised to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range,

$$\mu_{\tilde{A}}: X \rightarrow [0,1]$$

The assigned value indicate the membership function and the set,

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x));\ x \in X\}$$

Defined by $\mu_{\tilde{A}}(x)$ for x Є X is called fuzzy set.

IV. Mathematical Formulation For Fflpp

A formulation of the fuzzy linear programming problem is the flexible approach in which the decision variables are crisp numbers while the right hand side coefficients are characterized by uncertainties. Another formulation of fuzzy linear programming is the Possibilistic Approach. In general, Possibilistic Programming deals with problems where the coefficients of decision variables are obtained as fuzzy numbers. However, this means that in an uncertainty environment a crisp decision is made to meet some decision criteria. Furthermore, it was proposed that the coefficient of decision variables should be fuzzy numbers.

The fuzzy LPP is defined as:

Max $\tilde{z} = \tilde{c}^T x$

subject to,

$\tilde{A}x \lesssim \tilde{b}$

$x \geq 0$

The solution of this problem is to find the possibility distribution of the optimal objective function Z, where objective function is fuzzy or right-hand side and left-hand side coefficients of constraints are fuzzy numbers.

4.1 Proposed Method

Step 1:

A linear programming problem with a fuzzy objective function and the fuzzy inequalities is indicated as follows:

$$\text{Max } Z = \lambda$$

subject to,

$$C^T x \gtrsim b_0,$$

$$(Ax)_i \lesssim b_i,$$

$x \geq 0 \quad i = 1,2,\ldots,m$, where C^T denote the fuzzy number.

Step 2:

To write a general formulation, inequality is converted to a matrix form as:

$$-C^T x \lesssim -b_0$$

in which

$$b = \begin{bmatrix} -b_0 \\ b_i \end{bmatrix}$$

Step 3:

The degree of violation is represented by membership function as:

$$\mu_0(x) = \begin{cases} 0 & \text{if } cx \leq b_0 - d_0 \\ 1 - \frac{b_0 - cx}{d_0} & \text{if } b_0 - d_0 \leq cx \leq b_0 \\ 1 & \text{if } cx \leq b_0 \end{cases}$$

$$\mu_i(x) = \begin{cases} 0 & \text{if } (Ax_i) \geq b_i + d_i \\ 1 - \frac{(Ax)_i - b_i}{d_i} & \text{if } b_i \leq (Ax)_i \leq b_i + d_i \\ 1 & \text{if } (Ax)_i \leq b_i \end{cases}$$

Step 4:

This problem can be transformed by introducing the auxiliary variable λ as follows:

$$\mu_0(x) \geq \lambda$$
$$\mu_i(x) \geq \lambda$$
$$\lambda \in [0, 1]$$

Step 5:

This problem with membership functions of fuzzy objective function and fuzzy constraints as follows:

$$\text{Max } Z = \lambda$$
$$\text{subject to,}$$
$$1 - \frac{b_0 - cx}{d_0} \geq \lambda,$$
$$1 - \frac{(\mathrm{Ax})_i - b_i}{d_i} \geq \lambda, \forall\, i$$
$$\text{where } \lambda \in [0, 1],$$
$$x \geq 0$$

Step 6:

After further simplification, fuzzy linear programming model is obtained as follows:

$$\text{Max } Z = \lambda$$
$$\text{subject to,}$$
$$C^T x - \lambda d_0 \geq b_0 - d_0\ ,$$
$$(Ax)_i + \lambda d_j \leq b_j + d_j, \text{ for all i}$$
$$\lambda \in [0,1],\ x \geq 0$$

4.2 NUMERICAL EXAMPLE

A consumer product manufacturer produces different types of products. Since the expected profit and the demand of the different product types are uncertain the problem is built as fuzzy linear programming model in order to schedule the production per month for each product type for maximizing the profit.

Table 4.1: The details of the production in a food industry

Variables									
Variable name	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Unit profits (per tonne)	1509	1509	503	1509	1006	503	1509	1006	503
Expected demands (tonne per month)	25	100	150	30	150	200	30	250	80
Tolerances for demands (tonne per month)	5	50	40	5	40	50	5	40	20
Labour usage (hour per tonne)	0.333	0.333	0.333	0.283	0.283	0.283	0.3	0.3	0.3
Expected profit	12,00,000								
Tolerance for profit	2,00,000								
Monthly production capacity(tonne)	1150								
Monthly labour capacity(hour)	364								

Let $X_1, X_2, .., X_9$ denote the products of the food industry. In the above problem the constraint condition of the left hand side is written as,

$C^T x = 1509x_1+1509x_2+503x_3+1509x_4+1006x_5+503x_6+1509x_7+1006x_8+503x_9$

and b_0=12, 00,000

$$C^T x \gtreqqless b_0$$

(i.e.) $1509x_1+1509x_2+503x_3+1509x_4+1006x_5+503x_6+1509x_7+1006x_8+503x_9 \gtreqqless 12{,}00{,}000$

Now the second constraint condition is defined as

$$C^T x - \lambda d_0 \geq b_0 - d_0$$

(i.e.) $1509x_1+1509x_2+503x_3+1509x_4+1006x_5+$
$503x_6+1509x_7+1006x_8+503x_9 - 2,00,000\lambda \gtreqqless 12,00,000 - 2,00,000$

(i.e.) $1509x_1+1509x_2+503x_3+1509x_4+1006x_5+$
$503x_6+1509x_7+1006x_8+503x_9 - 2,00,000\lambda \gtreqqless 10,00,000$

Similarly the other constraint conditions can be defined as

$$(Ax)_i + \lambda d_j \leq b_j + d_j, \text{ for all } i$$

Therefore the fuzzy linear programming model is written as

$$\text{Max } Z = \lambda$$
$$\text{subject to}$$
$$x_1 \tilde{\leq} 25 + 5$$
$$x_2 \tilde{\leq} 100 + 50$$
$$x_3 \tilde{\leq} 150 + 40$$
$$x_4 \tilde{\leq} 30 + 5$$
$$x_5 \tilde{\leq} 150 + 40$$
$$x_6 \tilde{\leq} 200 + 50$$
$$x_7 \tilde{\leq} 30 + 5$$
$$x_8 \tilde{\leq} 250 + 40$$
$$x_9 \tilde{\leq} 80 + 20$$
$$\lambda \in [0, 1]$$
$$x_i \geq 0, i = 1,2,3,\ldots\ldots,9$$

where $b_0 = 12{,}00{,}000$, $b_1 = 25$, $b_2 = 100$, $b_3 = 150$, $b_4 = 30$, $b_5 = 150$, $b_6 = 200$, $b_7 = 30$, $b_8 = 250$, $b_9 = 80$, $d_0 = 2{,}00{,}000$, $d_1 = 5$, $d_2 = 50$, $d_3 = 40$, $d_4 = 5$, $d_5 = 40$, $d_6 = 50$, $d_7 = 5$, $d_8 = 40$, $d_9 = 20$.

Solution:

Fuzzy linear programming problem has been solved by TORA software. Results of the solution are given in table 1.8. As can be seen from the solution, the factory should produce 28.48 tonnes for x_1, 134.82 tonnes for x_2, 177.85 tonnes for x_3, 33.48 tonnes for x_4, 177.85 tonnes for x_5, 234.82 tonnes for x_6, 33.48 tonnes for x_7, and 277.85 tonnes for x_8 and 93.93 tonnes for x_9.

Table 4.2 Solution of the problem

Variable	Value
x1: x1	28.48
x2: x2	134.82
x3: x3	177.85
x4: x4	33.48
x5: x5	177.85
x6: x6	234.82
x7: x7	33.48
x8: x8	277.85
x9: x9	93.93
x10: lamda	0.30

Total profit of the factory can be calculated as follows:

$(1509 \times 28.48) + (1509 \times 134.82) + (503 \times 177.85) + (1509 \times 33.48) + (1006 \times 177.85) + (503 \times 234.82) + (1509 \times 33.48) + (1006 \times 277.85) + (503 \times 93.93)$

$$\boxed{\textbf{Max } \lambda = \textbf{10, 40,716.34}}$$

V. Mathematical Formulation for Fuzzy Goal Programming problem

Step: 1

$$\text{Determine } x = (x_1, x_2, x_n)$$
$$\text{which satisfies,}$$
$$f_i(x) \tilde{\leq} p_i, \quad i = 1, 2, k$$
$$g_i(x) \tilde{\leq} b_j, \quad j = 1, 2, m$$
$$x \geq 0$$

$\sim$ sign stands for fuzzification.

Step: 2

P_i replaced by $p_i + \alpha_i$ and b_j is replaced by $b_j + \beta_j$ (α_i and β_j) are tolerance limits for i^{th} fuzzy goal $f_i(x) \tilde{\leq} p_i$ the linear membership function is defined as:

$$\mu_{f_i}(x) = \frac{(p_i + \alpha_i) - f_i(x)}{\alpha_i}, \quad i = 1, 2, m$$

$$\mu_{g_j}(x) = \frac{(b_i + B_i) - g_i(x)}{\beta_i}, \quad j = 1, 2, m$$

Step: 3

Using compensatory operator canonical model is given as:

$$\text{Maximize } V(\mu) = (\mu f_1(x))^2+(\mu f_2(x))^2+(\mu f_3(x))^2+\ldots+(\mu g_m(x))^2$$

subject to

$$\mu_{f_i}(x) = \frac{(p_i+\alpha_i)- f_i(x)}{\alpha_i} \quad i = 1, 2, k$$

$$\mu_{g_j}(x) = \frac{(b_j+\beta_j)- g_j(x)}{\beta_j} \quad j = 1, 2, m$$

$$\mu_{f_i}(x) \geq 0 \text{ , i = 1, 2, k}$$

$$\mu_{g_j}(x) \geq 0\text{, j = 1, 2, m}$$

$$x\geq 0$$

Step: 4

V (μ) is replaced by W (μ) such that,

$$W(\mu) = \sum_{i=1}^{k} \mu_{f_i}(x) + \sum_{j=1}^{m} \mu_{g_j}(x)$$

VI FUZZY GOAL PROGRAMMING

The problem given in 1.6 is converted into a goal programming model as:

$1509x_1+1509x_2+503x_3+1509x_4+1006x_5+503x_6+1509x_7+1006x_8+503x_9 =1200000$(goal 1)

$25x_1+100x_2+150x_3+30x_4+150x_5+200x_6+30x_7+250x_8+80x_9 =1150$(goal 2)

$$x_1 \lesssim 25$$
$$x_2 \lesssim 100$$
$$x_3 \lesssim 150$$
$$x_4 \lesssim 30$$
$$x_5 \lesssim 150$$
$$x_6 \lesssim 200$$
$$x_7 \lesssim 30$$
$$x_8 \lesssim 250$$
$$x_9 \lesssim 80$$
$$x_i \geq 0\text{, where } i=1,2,3\ldots\ldots 9$$

The earlier step is converted into a Crisp model as:

$$\text{Max } (\mu_1)^2+(\mu_2)^2+(\mu_3)^2+\ldots\ldots,(\mu_{11})^2$$

subject to

$$\mu_1 = \frac{12,00,000-(1509x_1+1509x_2+503x_{3+}1509x_4+1006x_5+503x_6+1509x_7+1006x_8+503x_9)}{10,000}$$

$$\mu_2 = \frac{1150-(25x_1+100x_2+150x_{3+}30x_4+150x_5+200x_6+30x_7+250x_8+80)}{150} \quad ,$$

Similarly,

$$\mu_3 = \frac{25-x_1}{5} \text{ , } \mu 4 = \frac{100-x_2}{50} \text{ , } \mu_5 = \frac{150-x_3}{40} \text{ , } \mu_6 = \frac{30-x_4}{5} \text{ ,} \mu_7 = \frac{150-x_5}{40} \text{ ,}$$

$$\mu_8 = \frac{200-x_6}{50} \text{ , } \mu_9 = \frac{30-x_7}{5} \text{ ,} \mu_{10} = \frac{250-x_8}{40} \text{ , } \mu_{11} = \frac{80-x_9}{20} \text{ ,}$$

$$\mu_1,\mu_2,\mu_3,\ldots\ldots, \mu_{11}>0$$

$$\text{Min } Z = d_1+d_3+0d_2+0d_4$$

subject to

$$x_1 \lesssim 25$$
$$x_2 \lesssim 100$$
$$x_3 \lesssim 150$$
$$x_4 \lesssim 30$$
$$x_5 \lesssim 150$$
$$x_6 \lesssim 200$$
$$x_7 \lesssim 30$$
$$x_8 \lesssim 250$$
$$x_9 \lesssim 80$$

$1509x_1+1509x_2+503x_3+1509x_4+1006x_5+503x_6+1509x_7+1006x_8+503x_9 +d_1 - d_2=1200000$

$25x_1+100x_2+150x_3+30x_4+150+200x_6+30x_7+250x_8+80x_9 +d_3 - d_4=1150$

Table 5.1 Solution of the problem

Variable	Value
x1: x1	25.00
x2: x2	100.00
x3: x3	150.00
x4: x4	30.00
x5: x5	150.00
x6: x6	200.00
x7: x7	30.00
x8: x8	250.00
x9: x9	80.00
x10: d1	302145.00
x11: d2	0.00
x12: d3	0.00
x13: d4	165175.00

Substitute these values in $\mu_1, \mu_2, \ldots, \mu_{11}$ equation. We have,
$\mu_1 = 30.2145$
$\mu_2 = -\,1101.1666$
$\mu_1\ \mu_2\ \mu_3, \ldots\ldots\ldots, \mu_{11} = 0$
Max $Z = (\mu_1)^2+(\mu_2)^2+(\mu_3)^2+\ldots,(\mu_{11})^2$
Max $Z = (30.2145)^2+(-\,1101.1666)^2$

Max Z = 12,13,480

Table 5.2 Comparison b/w FLPP & FGPP

Variable	FLPP	FGPP
X_1	28.48	25
X_2	134.82	100
X_3	177.85	150
X_4	33.48	30
X_5	177.85	150
X_6	234.82	200
X_7	33.48	30
X_8	277.85	250
X_9	93.93	80
Max Z	**10, 40,716**	**12,13,480**

VII.CONCLUSION

In this decision making process of the factory, a consumer product production planning is solved by using fuzzy linear programming and fuzzy goal programming. This problem has fuzziness in both objective function and constraints. Hence by comparing the production process using FFLP and FFGP, we assume and plan through Goal programming method which gives higher optimism and leverage for maximum profit in FFGP.

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