ON $g^{\#}\alpha$ CLOSED SETS IN SUPRA TOPOLOGICAL SPACES

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ABSTRACT: : *In this paper, we introduce and investigate a new class of sets called supra $g^{\#}\alpha$-closed sets,$g^{\#}\alpha$ –continuous function in supra topological space. We further discuss the concept of supra $g^{\#}\alpha$ –irresolute and we investigate several properties of the new notions*

Key Words: *Supra $g^{\#}\alpha$ -closed set,supra $g^{\#}\alpha$ –continuity and supra $g^{\#}\alpha$ –irresolute*

1. INTRODUCTION

In 1983 Mashhour et al [6] introduced Supra topological spaces and studied S- continuous maps and S^*-continuous maps. In 2008, Devi et al. [1] introduced and studied a class of sets called supra α-open and a class of maps called sα-continuous maps between topological spaces, respectively. In 2011, G.Ramkumar *et al* [9] Supra g -Closed set and supra g-continuity maps .Quite recently G. Ramkumar et al.[8] have introduced and studied a class of sets called supra $\widetilde{g}$ -closed.

In this paper, we introduce the concept $g^{\#}\alpha$ – closed sets , supra $g^{\#}\alpha$ - continuous functions , supra $g^{\#}\alpha$ -irresolute and investigated several properties for these classes of functions in supra topological spaces.

2. PRELIMINARIES

Definition: 2.1[6] A subfamily of μ of X is said to be a supra topology on X, if

(i) $X, \varphi \in \mu$

(ii) if $A_i \in \mu$ for all $i \in J$ then $\cup A_i \in \mu$.

The pair (X, μ) is called supra topological space. The elements of μ are called supra open sets in (X, μ) and complement of a supra open set is called a supra closed set.

Definition: 2.2[6](i) The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is defined as $cl^{\mu}(A) = \cap\{B : B$ is a supra closed set and $A \subseteq B\}$.

(ii) The supra interior of a set A is denoted by $int^{\mu}(A)$ and defined as $int^{\mu}(A) = \cup\{B$: B is a supra open set and $A \supseteq B\}$.

Definition: 2.3[6] Let (X, τ) be a topological spaces and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subset \mu$.

Definition: 2.4 [1] Let (X,μ) be a supra topological space. A Subset A of X is called supra α – open set if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$. The complement of supra α - open set is supra α - closed set.

Definition: 2.5 [9] Let (X, μ) be a supra topological space. A Subset A of X is called g-closed set if $cl^{\mu}(A) \subseteq U$,whenever $A \subseteq U$ and U is supra open set of X.

Definition: 2.6 [4] Let (X, μ) be a supra topological space .A Subset A of X is called gs -closed set if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open set of X

Definition: 2.7 [4] Let (X, μ) be a supra topological space.A Subset A of X is called αg-closed set if $\alpha cl^{\mu}(A) \subseteq U$,whenever $A \subseteq U$ and U is supra open set of X.

Definition: 2.8 [8] Let (X, μ) be a supra topological space. A subset A of X is called a supra $g^{\#}$ closed set if $cl^{\mu}(A) \subseteq U$,whenever $A \subseteq U$ and U is αg open set of X.

Definition: 2.9 Let (X, μ) and (Y, σ) be two topological spaces and $\tau \subset \mu$. A map f: $(X, \tau) \rightarrow (Y, \sigma)$ is called

(i) supra continuous [1] if the inverse image of each open set of Y is a supra open set in X.

(ii) supra α-continuous [1] if the inverse image of each open set of Y is a supra α -open set in X.

(iii) supra g-continuous [9] if the inverse image of each closed set of Y is a supra g-closed set in X.

(iv)Supra gs-continuous [2] if the inverse image of each closed set of Y is a supra gs-closed set in X.

(v) Supra αg-continuous [2]if the inverse image of each closed set of Y is a supra αg-closed set in X.

Definition: 2.11 Let (X, μ) and (Y, σ) be two topological spaces and $\tau \subset \mu$. A map f: $(X, \tau) \rightarrow (Y, \sigma)$ is called

(i) Supra closed [1] if the image of each closed set of X is a supra closed set in Y.

(ii) Supra α-closed [1] if the image of each closed set of X is a supra α -closed set in Y.

(iii) Supra g-closed [9] if the image of each closed set of X is a supra g-closed set in Y.
(iv)Supra gs-closed [2] if the image of each closed set of X is a supra gs-closed set in Y.
(v)Supra ?g -closed [2] if the image of each closed set of X is a supra ?g -closed set in Y.

Definition 2.12 Let (X, μ) and (Y, σ) be two topological spaces and $\tau \subset \mu$. A map f: $(X,\tau) \rightarrow (Y,\sigma)$ is called
(i) Supra irresolute [1] if $f^{-1}(v)$ is supra closed in X for every supra closed set V of Y.
(ii)Supra α -irresolute [1] if $f^{-1}(v)$ is supra a-closed in X for every supra α -closed set V of Y.
(iii) Supra ?g -irresolute [2] if $f^{-1}(v)$ is supra ?g-closed in X for every supra ?g -closed set V of Y.

3.BASIC PROPERTIES OF SUPRA $g^{\#}\alpha$ -CLOSED SETS

Definition: 3.1 Let (x, μ) be supra topological spaces. A Subset A of X is called supra $g^{\#}\alpha$ closed set if $\alpha cl^{\mu}(A) \subseteq U$,Whenever $A \subseteq U$ and U is supra g-open set of X.

Definition: 3.2 Let (x,μ) be supra topological spaces. A subset A of X is called supra g^{*} closed set if $cl^{\mu}(A) \subseteq U$, Whenever $A \subseteq U$ and U is supra g-open set of X.

Theorem: 3.3 Every supra closed set is supra α- closed.

Proof: Let A be supra closed in (X, μ) , $cl^{\mu}(A) = A$, then $cl^{\mu}(int^{\mu}(cl^{\mu}(A)) \subseteq cl^{\mu}(A) \subseteq A$. Hence $cl^{\mu}(int^{\mu}(cl^{\mu}(A)) \subseteq A$.Therefore A is supra α- closed.

Remark: 3.4 The converse of the above theorem need not be true as seen from the following example.

Example: 3.5 Let X= {a, b, c} and μ ={X, φ, {a},{b, c},{a.b}} Then the set {b} is supra α - closed set in (X, μ) but not supra closed.

Theorem: 3.6 Every supra semi closed set is supra α closed.

Proof: Let A be supra semi closed in (X, μ) .Therefore $int^{\mu}\left(cl^{\mu}(A)\right) = A$,
$cl^{\mu}(int^{\mu}(cl^{\mu}(A)) \subseteq int^{\mu}(cl^{\mu}(A) \subseteq A$ .Hence $cl^{\mu}(int^{\mu}(cl^{\mu}(A)) \subseteq A$.Therefore A is supra α closed

Remark: 3.7 The converse of the above theorem need not be true as seen from the following example.

Example: 3.8 Let X= {a, b, c} and μ ={X, φ, {a},{b, c},{a.b}} Then the set {b} is supra set in (X, μ) supra α - closed but not semi - closed.

Theorem: 3.9 Every supra closed set is supra $g^{\#}\alpha$- closed.

Proof: Let $A \subseteq U$ and U be supra g- open set in (X, μ) . Since A be supra closed in (X, μ) then
$cl^{\mu}(A) = A \subseteq U$.
We also know that $\alpha cl^{\mu}(A) \subseteq cl^{\mu}(A) \subseteq U$ which implies $\alpha cl^{\mu}(A) \subseteq U$ and U is supra g-open.
Therefore A is supra $g^{\#}\alpha$- closed.

Remark: 3.10 The converse of the above theorem need not be true as seen from the following example.

Example: 3.11 Let X= {a, b, c, d } and μ ={X, φ, {a},{a. c},{b, c},{a, b, c}}
Then the set {b, c} is supra $g^{\#}\alpha$ - closed set in (X, μ) but not supra closed.

Theorem 3.12 Every supra α-closed set is supra $g^{\#}\alpha$-closed

Proof: Let $A \subseteq U$ and U is supra g- open set in (X, μ). Since A is supra α-closed set then $\alpha cl^{\mu}(A) = A \subseteq U$.
Hence A is supra $g^{\#}\alpha$ closed

Remark: 3.13 The converse of the above theorem need not be true as seen from the following example.

Example: 3.14 Let X= {a, b, c ,d } and μ ={X, φ, {a},{a.c},{b,c},{a,b,c}}
Then the set {a ,b, d} is supra $g^{\#}\alpha$ - closed set in (X, μ) but not supra α -closed.

Theorem 3.15 Every supra g^{*} closed set is supra $g^{\#}\alpha$ closed

Proof: Let $A \subseteq U$ and U is supra g- open set in (X, μ). Since A is supra g^{*} closed set, $cl^{\mu}(A) = A \subseteq U$. We know that
$\alpha cl^{\mu}(A) \subseteq cl^{\mu}(A) = A \subseteq U$. Therefore $\alpha cl^{\mu}(A) \subseteq U$. Hence A is supra $g^{\#}\alpha$ closed.

Remark: 3.16 The converse of the above theorem need not be true as seen from the following example.

Example 3.17 Let X = {a, b, c, d} with μ ={X, φ, {a},{a.c},{b,c},{a,b,c}}. It is evident that {b, c} is supra $g^{\#}\alpha$ - closed set but not supra g^{*} - closed set.

Theorem 3.18 Every supra $g^{\#}\alpha$-closed set is supra gs -closed

Proof: Let $A \subseteq U$ and U is supra open set in (X, μ). Since A is supra $g^{\#}\alpha$ - closed set, we know the result every supra open set is supra g open set and $\alpha cl^{\mu}(A) \subseteq Scl^{\mu}(A) \subseteq U$

Remark: 3.19 The converse of the above theorem need not be true as seen from the following example.

Example 3.20 Let X = {a, b, c, d} with μ ={X, φ, {a},{a.c},{b,c},{a,b,c}}.It is evident that {b, c} is supra $g^{\#}\alpha$ - closed set but not supra gs - closed set.

Theorem: 3.21 The union of two supra $g^{\#}\alpha$ - closed set is supra $g^{\#}\alpha$ - closed set.

Proof: Let A and B two supra $g^{\#}\alpha$ - closed set. Let $A \cup B \subseteq G$, where G is supra open.

Since A and B are supra $g^{\#}\alpha$ -closed sets. Therefore $(\alpha cl^{\mu}(A) \cup \alpha cl^{\mu}(B)) \subseteq G$. Thus $\alpha cl^{\mu}(A \cup B) \subseteq G$. Hence A∪B is supra $g^{\#}\alpha$ -closed set.

Theorem 3.22 Let A be supra $g^{\#}\alpha$ -closed set of (X,μ). Then $\alpha cl^{\mu}(A) - A$ does not contain any non empty supra g-closed set.

Proof: Necessity Let A be supra $g^{\#}\alpha$ -closed set. suppose F≠ φ is a supra g - closed set of $\alpha cl^{\mu}(A) - A$. Then $F \subseteq \alpha cl^{\mu}(A) - A$ implies $F \subseteq \alpha cl^{\mu}(A)$ and A^c. This implies $A \subseteq F^c$. Since A is supra $g^{\#}\alpha$ -closed set, $\alpha cl^{\mu}(A) \subseteq U^c$ .Consequently, $F \subseteq [\alpha cl^{\mu}(A)]^c$.Hence $F \subseteq \alpha cl^{\mu}(A) \cap [\alpha cl^{\mu}(A)]^c = \varphi$. Therefore F is empty, a contradition.

Sufficiency: Suppose A ⊆U and that U is supra g - open. If $\alpha cl^{\mu}(A) \not\subset U$. Then $\alpha cl^{\mu}(A) \cap U^c$ is a not empty supra g - closed subset of $\alpha cl^{\mu}(A)$ - A.

Hence $\alpha cl^{\mu}(A) \cap U^c = \varphi$ and $\alpha cl^{\mu}(A) \subseteq U$ .Therefore A is supra $g^{\#}\alpha$ - closed.

Theorem: 3.23 If A is supra $g^{\#}\alpha$ -closed set in a supra topological space (X,μ) and $A \subseteq B \subseteq \alpha cl^{\mu}(A)$ then B is also supra $g^{\#}\alpha$ - closed set.

Proof: Let U be supra g - open set in (X,μ) such that B⊆ U. Since A⊆ B ⇒ A⊆U and since A is supra $g^{\#}\alpha$ - closed set in (X, μ) $\alpha cl^{\mu}(A) \subseteq U$, since $B \subseteq \alpha cl^{\mu}(A)$. Then $\alpha cl^{\mu}(B) \subseteq U$. Therefore B is also supra $g^{\#}\alpha$ - closed set in (X,μ)

Theorem: 3.24 Let A be supra $g^{\#}\alpha$ - closed set then A is supra $g^{\#}\alpha$ - closed iff $\alpha cl^{\mu}(A)$-A is supra g - closed .

Proof: Let A be supra $g^{\#}\alpha$ - closed set.If A is supra $g^{\#}\alpha$ - closed, we have $\alpha cl^{\mu}(A)-A = \varphi$, which is g - closed. Conversely, let $\alpha cl^{\mu}(A)$-A is $g^{\#}\alpha$ - closed. Then by the theorem 3.12, $\alpha cl^{\mu}(A)$ - A does not contain any non empty supra g - closed and $\alpha cl^{\mu}(A)-A=\varphi$. Hence A is supra $g^{\#}\alpha$ - closed.

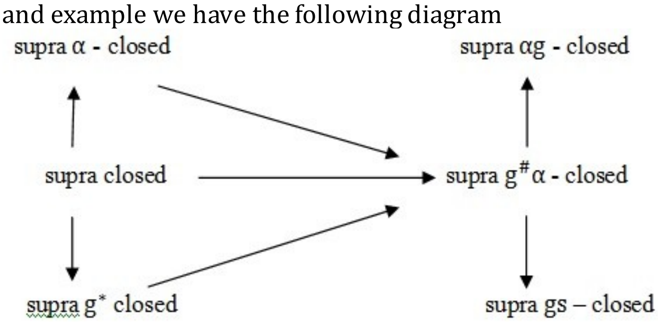
Theorem: 3.25 If B is supra g - open and supra $g^{\#}\alpha$ - closed set in X, then B is supra g - closed.

Proof: Since B is supra g - open and supra $g^{\#}\alpha$ - closed then $\alpha cl^{\mu}(B) \subseteq B$, but $B \subseteq \alpha cl^{\mu}(B)$. Therefore $B = \alpha cl^{\mu}(B)$.Hence B is g - closed.

Theorem: 3.26 Let A be supra -open and supra $g^{\#}\alpha$ - closed set. Then A∩F is g- closed whenever F is supra g- closed.

Proof: Let A be supra gs -open and supra $g^{\#}\alpha$ - closed set then $\alpha cl^{\mu}(A) \subseteq A$ and also $A \subseteq \alpha cl^{\mu}(A)$. Therefore $\alpha cl^{\mu}(A) = A$. Hence A is supra g-closed. Since F is supra g-closed. Therefore A∩ F is supra g-closed in X. Hence A∩F is supra g - closed in X.

From the above theorem and example we have the following diagram



4. SUPRA $g^{\#}\alpha$ CONTINUOUS FUNCTIONS

Definition: 4.1 (i) A function f: (X, τ) → (Y, σ) is called supra $g^{\#}\alpha$ - Continuous if $f^{-1}(A)$ is supra $g^{\#}\alpha$- supra closed in (X, τ) for every supra closed set A of (Y, σ).

(ii) A function f: (X, τ) → (Y, σ) is called supra g^* - Continuous if $f^{-1}(A)$ is supra $g^{\#}\alpha$- supra closed in (X, τ) for every supra closed set A of (Y, σ).

Definition: 4.2 A function f: (X, τ) → (Y, σ) is called $g^{\#}\alpha$ - irresolute if $f^{-1}(A)$ is supra $g^{\#}\alpha$ -closed in (X,μ) for every supra $g^{\#}\alpha$ – supra closed set A of (Y, σ).

Theorem: 4.3 Every supra continuous function is supra $g^{\#}\alpha$ - continuous.

Proof: Let f: (X, τ) → (Y, σ) be a supra continuous function .Let A is supra closed set in (Y, σ). Since f is supra continuous, then $f^{-1}(A)$ is supra closed in (X, τ).We know that every supra closed set is supra $g^{\#}\alpha$ - closed in (X , τ). Therefore $f^{-1}(A)$ is supra $g^{\#}\alpha$ - closed set in (X ,τ).Hence f is supra $g^{\#}\alpha$ continuous function.

Remark: 4.4 The converse of the above theorem need not be true as seen from the following example.

Example: 4.5 Let X ={a, b, c}, τ = {X, ϕ ,{a},{b},{a,b},{{a,c} } and σ ={Y,ϕ,{a}{a,b}{b,c}}.

Let f :(X, τ) → (Y, σ) be a function defined by f(a) = b, f(b) = a, f(c) = c. Let f^{-1}(({b,c}) ={a, c} is supra $g^{\#}\alpha$ - closed but not supra closed. Then f is supra $g^{\#}\alpha$ -continuous but not supra continuous.

Theorem: 4.6 Every supra α continuous function is supra $g^{\#}\alpha$ – continuous

Proof: Let f: (X, τ) → (Y, σ) be a supra continuous .Let A is supra closed in (Y, σ). Since f is supra α continuous, then $f^{-1}(A)$ is supra closed in (X, τ). We know that every supra α closed set is supra $g^{\#}\alpha$ - closed in (X ,τ) Therefore $f^{-1}(A)$ is supra closed and it is supra $g^{\#}\alpha$ - closed in (X, μ). Hence f is supra $g^{\#}\alpha$ - continuous.

Remark: 4.7 The converse of the above theorem need not be true as seen from the following example.

Example: 4.8 Let X ={a, b, c}, τ = {X, {a},{b},{a,b},{{a,c} } and σ ={Y,ϕ,{a}{a,b}{b,c}}.

Let f :(X, τ) → (Y, σ) be a function defined by f(a) = b, f(b) = a, f(c) = c. Let $f^{-1}(\{b, c\})$={a, c} is supra $g^{\#}\alpha$ - closed but not supra closed. Then f is supra $g^{\#}\alpha$ -continuous but not supra continuous.

Theorem: 4.9 Every supra g^* continuous function is supra $g^{\#}\alpha$ - continuous

Proof: Let f: (X, τ) → (Y, σ) be a supra continuous .Let A is supra closed in (Y, σ). Since f is supra α continuous, then $f^{-1}(A)$ is supra closed in (X, τ). We know that every supra g^* closed set is supra $g^{\#}\alpha$ - closed in (X , τ). Therefore $f^{-1}(A)$ is supra closed and it is supra $g^{\#}\alpha$ - closed in (X, μ). Hence f is supra $g^{\#}\alpha$ - continuous.

Remark: 4.10 The converse of the above theorem need not be true as seen from the following example.

Example: 4.11 Let X ={a, b, c}, τ = {X, {a},{b},{a,b},{{a,c} } and σ ={Y,ϕ,{a}{a,b}{b,c}}.

Let f : (X, τ) → (Y, σ) be a function defined by f(a) = c, f(b) = a, f(c) = b. Let $f^{-1}((\{b,c\})$ ={a,b} is supra $g^{\#}\alpha$ -closed but not g^* supra closed. Then f is supra $g^{\#}\alpha$ -continuous but not supra continuous.

Theorem: 4.12 Every supra $g^{\#}\alpha$ continuous function is supra αg - continuous

Proof: Let f: (X, τ) → (Y, σ) be a supra continuous. Let A is supra closed in (Y, σ). Since f is supra $g^{\#}\alpha$ continuous, then $f^{-1}(A)$ is supra closed in (X, τ). We know that every supra $g^{\#}\alpha$ closed set is supra αg - closed in (X ,τ). Therefore $f^{-1}(A)$ is supra closed and it is supra $g^{\#}\alpha$ - closed in (X, τ). Hence f is supra $g^{\#}\alpha$ - continuous.

Remark: 4.13 The converse of the above theorem need not be true as seen from the following examples.

Example: 4.16 Let X=Y ={a, b, c}, τ = {X, {a},{b},{a,b},{{a,c} } and σ ={Y,ϕ,{a}{a,b}{b,c}}.

Let f: (X, τ) → (Y, σ) be a function defined by f(a) = c, f(b) = a, f(c) = b.Let $f^{-1}((\{b,c\})$ = {a, b} which is $g^{\#}\alpha$ - continuous but not αg continuous.

Theorem: 4.14 Every supra $g^{\#}\alpha$ continuous function is supra gs - continuous

Proof: Let f: (X, τ) → (Y, σ) be a supra continuous. Let A is supra closed in (Y, σ). Since f is supra $g^{\#}\alpha$ continuous, then $f^{-1}(A)$ is supra closed in (X, τ). We know that every supra $g^{\#}\alpha$ closed set is gs – closed in (X ,τ).Therefore $f^{-1}(A)$ is supra closed and it is supra gs - closed in (X, μ).Hence f is supra gs - continuous.

Remark: 4.15 The converse of the above theorem need not be true as seen from the following examples.

Example: 4.16 Let X=Y ={a, b, c}, τ = {X, {a},{b},{a,b},{{a,c} } and σ ={Y,ϕ,{a}{a,b}{b,c}}.

Let f: (X, τ) → (Y, σ) be a function defined by f(a) = c, f(b) = a, f(c) = b.Let $f^{-1}((\{b,c\})$ = {a, b} which is $g^{\#}\alpha$ - continuous but not gs - continuous

Theorem: 4.17 Every supra $g^{\#}\alpha$ irresolute is supra $g^{\#}\alpha$ continuous

Proof: Let f: (X, τ) → (Y, σ) be a supra $g^{\#}\alpha$ irresolute.Let A is supra closed in (Y, σ). Since A is supra $g^{\#}\alpha$ closed set, then A is supra $g^{\#}\alpha$ irresolute, $f^{-1}(A)$ is a supra $g^{\#}\alpha$ closed set in (X, τ) .Therefore f is supra $g^{\#}\alpha$ continuous

Remark: 4.18 The converse of the above theorem need not be true as seen from the following examples.

Example: 4.19 Let X=Y={a, b, c}, τ = {X, ϕ ,{a},{b},{a,b},{{a,c} } and σ ={Y,ϕ,{a}{a,b}{b,c}}.

Let f: (X, τ) → (Y, σ) be any two function defined by f(a) = c, f(b) = b, f(c) = a. Then f is supra $g^{\#}\alpha$ - continuous. Since f−1{a, b} = {c, b} is not supra $g^{\#}\alpha$ closed in (X, τ).Therefore f is not supra $g^{\#}\alpha$ -irresolute.

Theorem: 4.20 Every supra $g^{\#}\alpha$ irresolute is supra gs continuous

Proof: Let f: (X, τ) → (Y, σ) be a supra $g^{\#}\alpha$ irresolute and A is supra closed in (Y, σ). Since A is supra $g^{\#}\alpha$ closed set, then A is supra $g^{\#}\alpha$ irresolute, $f^{-1}(A)$ is a supra $g^{\#}\alpha$ closed set in (X, τ) .Therefore f is supra $g^{\#}\alpha$ continuous

Remark: 4.21 The converse of the above theorem need not be true as seen from the following examples.

Example: 4.22 Let X=Y={a, b, c}, τ = {X, ϕ ,{a},{b},{a,b},{{a,c} } and σ ={Y,ϕ,{a}{a,b}{b,c}}.

Let f: (X, τ) → (Y, σ) be any two function defined by f(a) = c, f(b) = b, f(c) = a. Then f is supra $g^{\#}\alpha$ - continuous. Since $f^{-1}((\{b,c\})$ = {a, b} is not supra $g^{\#}\alpha$ closed in (X, τ).Therefore f is not supra $g^{\#}\alpha$ - irresolute.

Theorem: 3.23 Every supra $g^{\#}\alpha$ irresolute is supra gs irresolute

Proof: Let f: (X, τ) → (Y, σ) be a supra $g^{\#}\alpha$ irresolute and A is supra closed in (Y, σ). Since A is supra $g^{\#}\alpha$ closed set, then A is supra $g^{\#}\alpha$ irresolute, $f^{-1}(A)$ is a supra $g^{\#}\alpha$ closed set in X.Therefore f is supra $g^{\#}\alpha$

closed set is supra gs closed. Therefore f is supra gs irresolute.

Remark: 3.24 The converse of the above theorem need not be true as seen from the following examples.

Example: 4.25 Let X=Y={a, b, c}, τ = {X, ϕ ,{a},{b},{a,b},{{a,c} } and σ ={Y,ϕ,{a}{a,b}{b,c}}.

Let f: (X, τ) $\rightarrow$ (Y, σ) be any two function defined by f(a) = c, f(b) = b, f(c) = a. Then f is supra $g^{\#}\alpha$ -continuous. Since $f^{-1}((\{b,c\})$ = {a, b} is not supra $g^{\#}\alpha$ closed in (X, τ).Therefore f is not supra $g^{\#}\alpha$ -irresolute.

Theorem: 3.26 Let f : (X, τ) $\rightarrow$ (Y, σ) and

g : (Y, σ) $\rightarrow$ (Z, γ) be any two function then

(i) gof is supra $g^{\#}\alpha$ -continuous if g is supra continuous if f is supra $g^{\#}\alpha$ -continuous.

(ii) gof is $g^{\#}\alpha$ - irresolute if g is $g^{\#}\alpha$ - irresolute and f is supra $g^{\#}\alpha$ - irresolute.

(iii) gof is supra $g^{\#}\alpha$ -continuous if g is supra $g^{\#}\alpha$ -continuous and f is supra $g^{\#}\alpha$ - irresolute.

Proof: (i) Let A be supra closed in (Z, γ). Then , $g^{-1}(A)$ is supra closed in (Y, σ). Since g is supra continuous, then $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is supra $g^{\#}\alpha$ -closed in (X, τ). Hence gof is supra $g^{\#}\alpha$ -continuous.

(ii) Let A be $g^{\#}\alpha$ -closed in (Z, γ).Then $g^{-1}(A)$ is supra $g^{\#}\alpha$ -closed in (Y, σ). Since g is supra $g^{\#}\alpha$ -irresolute, then $f^{-1}(g^{-1}(A))$ =$(g \circ f)^{-1}(A)$ is supra $g^{\#}\alpha$ -closed in (X, τ). Hence gof is supra $g^{\#}\alpha$ -irresolute.

(iii) Let A be supra closed in (Z, γ).Then , $g^{-1}(A)$ is supra $g^{\#}\alpha$ -closed in (Y, σ). Since g is supra $g^{\#}\alpha$ -continuous, then $f^{-1}(g^{-1}(A))$ =$(g \circ f)^{-1}(A)$ is supra $g^{\#}\alpha$ -closed in (X, τ). Hence gof is supra $g^{\#}\alpha$ -continuous.

Remark: 3.26 The composition of two supra $g^{\#}\alpha$ -continuous function need not supra $g^{\#}\alpha$ - continuous and it is shown by the following example.

Example: 3.27 Let X ={a, b, c, d} ,τ = {X, , ϕ , {a}{b}{a, b}} and σ = {X, , ϕ , {a}{c}{a, c}}

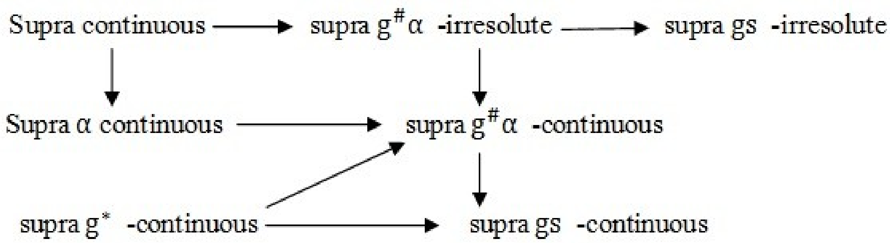
Let f: (X, τ) $\rightarrow$ (X, τ) be a function defined by f(a) = b, f(b) = c, f(c) = d and f(d)=a.

Let g: (X, τ) $\rightarrow$ (X, σ) be a function defined by g(a) = b, g(b) = c,g(c) =d and g(d)=a.

Then f and g are supra $g^{\#}\alpha$ -continuous, since {b, c, d} is supra closed in (X, σ),

$(g \circ f)^{-1}$\{b, c, d\} ={a, b, d} which is not $g^{\#}\alpha$ -closed in (X, τ).

Therefore gof is not supra $g^{\#}\alpha$ - continuous.



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