Analysis of MIMO system for trade off between transmit diversity and spectral efficiency

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ABSTRACT

The rise of the Internet and multimedia communications demands higher transmission speeds and higher bandwidths. In wireless communications, the radio channel is a hostile transmission medium due to the fainty generated by the multichannel propagation, which reduces the quality and reliability of the channel and consequently the speed of communication. On the other hand, the radio spectrum is a limited and saturated resource that prevents free bandwidth allocations. This paper explores the STBC and SM techniques using different equalization ZF, MMSE and ML. MRC and EG are used at receiver end as combing the diversified information. The comparative analysis has been explored in this paper. Further VBLAST technique is developed for higher spectral efficiency. Performance evaluation claims lesser BER when higher transmit diversity.

Keywords: VBLAST, ZF, MMSE, ML, BER, STBC, SM.

Introduction

Multiple transmitter and multi-receiver systems (MIMO systems) can overcome these limitations and meet the demand for high data rates. These are its features that MIMO systems are presented as optional or recommended in the main standards and projects that are currently developed as 3GPP - HSDPA, IEEE 802.11n, IEEE 802.16, IEEE 802.20 or IS-856 (evolution of cdma2000).


[3] Establish the basis of MIMO systems and their ability to increase spectral efficiency. Spatial diversity [4] is a widely used technique at the receiving end to improve channel performance, as it increases the resulting signal-to-noise ratio. At the transmitting end there are also precedents of diversity; [5] proposes a system of diversity in which a symbol is transmitted by an antenna and later the same symbol will be transmitted by the other at the following time; Jakes [6] proposes a system of selection of the transmitting antenna in function of the signal-to-noise ratio in the receiver. The spatial-temporal coding, in its two main versions, of STBC blocks [7] and Trellis STTC [8], aims to improve the reliability and quality of the link, reducing the bit error rate (BER). In [9] he proposed a coding block space-time scheme for 2 × 1 and 2 × 2 systems. This system was extended to M × N systems by Tarokh [10].

The performance of the MIMO channel is measured through the capacity of the channel, i.e. the maximum spectral efficiency offered by the channel. [11] Quantified the capacity of the channel from the capacity expressions proposed by Shannon for SISO channels; the MIMO channel capacity will only depend on the signal-to-noise ratio in the receiver and the channel matrix, regardless of the transmission or coding scheme used. [12]

MIMO systems

In a MIMO system of M transmitting antennas and N receiving antennas, M × N system, MN subchannels are generated between the transmitting array and the receiving array. MIMO channel is expressed in matrix form as:

\[
H(t) = \begin{bmatrix}
    h_{11}(t) & h_{12}(t) & \ldots & h_{1N}(t) \\
    h_{21}(t) & h_{22}(t) & \ldots & h_{2N}(t) \\
    \vdots     & \vdots     & \ddots & \vdots     \\
    h_{M1}(t) & h_{M2}(t) & \ldots & h_{MN}(t)
\end{bmatrix}
\] (1)

Each of the elements \(h_{ij}(t)\) represents the channel generated between the transmitting antenna \(j,\) with \(j = 1, \ldots, M\) and the receiving antenna \(i,\) with \(i = 1, \ldots, N.\) The MIMO channel described by expression (1) is represented in Figure 1 in which the M transmitting antennas and the receiving N antennas can be seen and as MN subchannels.
Bases of MIMO systems
The high transmission speeds of MIMO systems depend on several factors that either improve the transmission schemes, or improve the reliability of the link. The second factors improve the channel characteristics by minimizing the probability of error and improving the signal-to-noise ratio which allows the use of higher transmission rates through higher coding schemes, increase range or reduce transmitted power.

Spatial Multiplexing Gain
Mathematically the number of equivalent sub channels K is given by the number of nonzero singular values of the H-channel matrix, i.e., the range of the H-matrix, which is bounded by the number of transmitters or receivers (K = rank (H) ≤ min (M, N).

Diversity Gain
It is observed that the diversity gain for an outage probability of 1% is higher than the probability of outage 5%.
Thus, the probability of error falls according to SNR-d while in a SISO system it falls according to SNR-1 [32]. Therefore, the diversity gain is limited to the degree of spatial diversity offered by the channel. Ideally, the order of diversity of an M × M system M is of MN, then dmax = MN.

Array Gain
Andersen defines the array gain as the average value of the power received in a MIMO system with respect to the power received in a SISO system [33, 34]. To perform the combination it is necessary to know the instantaneous channel status (CSI) at the corresponding end to obtain the weights [13].

Commitment Solution
If a high diversity gain is pursued, part of the gain is sacrificed by multiplexing and vice versa, a compromise between diversity gain and gain by multiplexing being possible. For information blocks of size Nb> M + N-1 the curve that defines the diversity gain d as a function of the gain by multiplexing k is given by the following expression:
This translates into a compromise between error probability and data rate.

\[ d(k) = (M - k)(N - k) \]  

(4)

This relationship does not depend on the size of the coding block provided that \( N_b > N + M - 1 \) is maintained. When coding on blocks of greater length does not obtain greater gain in diversity.

**Spatial diversity in reception**

Recruitment diversity is a technique commonly used and extensively studied since the 1950s [14]. If the separation between receiving antennas is sufficient, the fading suffered by each of the channels will be independent, it being very likely that both signals received will not simultaneously fade.

**Selection Combining**

A variant of this method [15] selects the signal with the highest SNR and maintains it, without further probing, until the ratio falls below a certain threshold; at this time the antenna with the best received SNR is selected again.

\[ \text{Selection max}_{i\{s_i+N\}} \]

A system in an incorrect Rayleigh channel with two branches of diversity introduces an improvement in the SNR with respect to the non-diversity case of 10 dB with a probability of 99%.

**Spatial Diversity in Transmission**

On the one hand, the transmitted signals are spatially combined prior to reaching the receiver, therefore a transmission and reception processing is required to be able to separate the received signals and achieve diversity gain.

The impulse response of the channel is:

\[ h[k] = h_1\delta[k] + h_2\delta[k-1] \]  

(5)

The channel between the transmitting antenna \( i \) and the receiver being \( h_i \). A maximum likelihood detector allows maximum gain for diversity (order 2).

**Space Time Block Coding (STBC)**

This coding proposed by Alamouti for \( 2 \times 1 \) and \( 2 \times 2 \) systems and generalized by Tarokh to \( M \times N \) systems, is a block spacetime coding that reduces the complexity of STTC coding.

**Vertical Encoding VE (V-BLAST)**

In this case the temporal coding, symbol mapping and interleaver are applied to the entire bitstream to be then multiplexed by the transmitting \( M \) antennas.
Combination of HE and VE (D-BLAST)

Combinations of both horizontal and vertical schemes can be applied to take advantage of both. One is the D-BLAST (Diagonal-BLAST) architecture. Equivalent model of the MIMO channel here we present the equivalent model of the MIMO channel obtained from the physical interpretation of the channel matrix. The response to the matrix impulse of a MIMO channel $M \times N$, where $M$ is the number of transmitting antennas and $N$ the number of receiving antennas is represented in (6). If a vector $s = [s_1, s_2 \ldots s_M]^T$ is transmitted, the signal received by the receiver array is:

$$r = Hs + n$$

(6)

Where $n$ is a column vector $N \times 1$ whose elements are Gaussian white noise of zero mean and unit variance. By performing the decomposition in singular values of $H$ (SVD), expression (6) yields:

$$r = UDV^H s + n$$

(7)

Where $D$ is a diagonal and positive matrix of the same order as $H$ ($N$ rows $\times$ $M$ columns), $U$ and $V$ are matrices of order $N \times N$ and $M \times M$, respectively and satisfy that $UU^H = I$, $V^HV = I$. The columns of the matrix $V$ represent the own input vectors of $H$ and the columns of $U$ the proper eigenvectors of $H$.

Three new vectors of received signal, transmitted signal and noise can be defined by the expressions presented in (8) [37].

$$r' = U^H r, s' = V^H s, n' = U^H n$$

(8)

The three vectors verify that the traces of their covariance matrices are equal to the corresponding original vectors $r$, $s$ and $n$.

$$r' = Ds' + n'$$

(9)

The range of the matrix $H$ indicates the number of unique values other than zero, ie degrees of freedom. For an $M \times N$ system the range of $H$ will check (10).

$$\text{range}(H) = K \leq \min\{M, N\}$$

(10)

From (9) and the range definition of the matrix, expressions (11) and (12) can be written for each received signal depending on the number of transmitting elements $M$ and $N$ receptors.

- If $N > M$ (receiver number > number of transmitters)

$$r_i' = \lambda_i x_i + n_i', i = 1,2, \ldots, K$$

(11)

$$r_i' = 0, i = K + 1, K + 2, \ldots, M$$

In the above expression $\lambda_i$ represents the $i$-th eigenvalue of the matrix $HH^H$, and its positive square root is the $i$-th singular value of the matrix $H$.

Equation (11) shows that only the first $K$ received equivalent signals $r_i'$, contribute to the MIMO system. If $N < M$ (receiver number < number of transmitters)

$$r_i' = \lambda_i x_i' + n_i', i = 1,2, \ldots, N$$

(12)

In this case only the transmitted signals equivalent to $s_i'$ with $i = 1, \ldots, K$ contribute to the system, while the rest, $M$-$K$ sub channels, consumes power unnecessarily. For a $5 \times 3$ MIMO channel the expression (9) is expressed as (13).

$$\begin{bmatrix}
r_1 \\
r_2 \\
r_3
\end{bmatrix} =
\begin{bmatrix}
\sqrt{\lambda_1} & 0 & 0 \\
0 & \sqrt{\lambda_2} & 0 \\
0 & 0 & \sqrt{\lambda_3}
\end{bmatrix}
\begin{bmatrix}
s_1 \\
s_2 \\
s_3
\end{bmatrix}$$

(13)

Figure 6 graphically depicts the equivalent MIMO channel described in this section. According to this model, the MIMO channel can be decomposed into $K$ parallel and independent equivalent sub channels with power gain $\lambda_i$, with $i = 1, \ldots, K$. 
Figure 5. MIMO channel equivalent

\[ K = \text{rank}(H) \leq \min(N, M). \lambda_1 > \lambda_2 > ... > \lambda_K > 0. \] [35]

The concept of effective degrees of freedom \( K_0 \) (EDOF) [26, 43] is defined as the number of equivalent subchannels that actually contribute to the channel. \( K_0 \) can be obtained as the variation of the capacity by increasing the power transmitted by a factor \( 2^\delta \) according to the expression (14).

\[
EDOF = K_0 = \frac{d}{d\delta} C\left(2^\delta P_{\text{tx}}\right)_{\delta=0}
\] (14)

Thus, if \( K_0 \) is low due to high correlation or a low SNR.

### Simulation And Results

![BER for BPSK modulation with 2x2 MIMO and MMSE equalizer (Rayleigh channel)](image)

Ber for BPSK modulation with 2x2 MIMO and MMSE equalizer (Rayleigh channel)
BER for BPSK modulation with 2x2 MIMO and ML equalizer (Rayleigh channel)

- theory (nTx=1, nRx=1)
- theory (nTx=1, nRx=2, MRC)
- sim (nTx=2, nRx=2, ML)

Eb/No (dB) vs. Bit Error Rate (BER)

- STBC 2x2 coded
- 1x1 coded
- STBC 2x1 coded

Eb/No vs. BER

Convolution

- STBC 2x2
- 1x1
- STBC 2x1

Eb/No vs. BER
Conclusion
This paper analysed the performance of STBC, SM with different modulation scheme BPSK, QPSK, 16QAM and 64 QAM. The system model is evaluated with ZF (zero forcing), MMSE (minimum mean square error) and ML (Maximum likelihood) techniques. The likelihood receiver gives better BER. BPSK modulation has lesser BER than QPSK, 16 QAM and 64 QAM respectively. The BER is lesser when transmit antenna goes high but the spectral efficiency is low in that case. To improve better spectral efficiency Vblast technique is used in Spatial Multiplexing techniques.

Reference