STUDY OF EOQ MODEL WITH QUANTITY INCENTIVE STRATEGY FOR DETERIORATING ITEMS AND PARTIAL BACKLOGGING

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ABSTRACT *The objective of this paper is to develop an inventory model for deteriorating products with quantity discount and partial backlogging to determine the optimal ordering quantity for the retailer optimizing the total cost or profit of the associated model.*

Keywords: *Ordering Quantity, Retailer, Inventory, Profit.*

1. Introduction

Inventory or stock is the goods and materials that business holds for the ultimate goals to have a purpose of resale. Inventory management is a discipline primarily about specifying the shape and placement of stocked goods. It is required at different locations within a facility or within many locations of a supply network to precede the regular and planned course of production and stock of materials. **Chang et al.** proposes a multi-supplier multi-product inventory model in which the suppliers have unlimited production capacity, allow delayed payment. The retailer can delay payment until after they have sold all the units of the purchased product. The retailer's warehouse is limited, but the surplus can be stored in a rented warehouse at a higher holding cost. The demand over a finite planning horizon is known. This model aims to choose the best set of suppliers and seeks to determine the economic order quantity allocated to each supplier.**Chang and Dye** introduced an inventory model for deteriorating items allowing the partial backlogging of unsatisfied demand in their model. It is more reasonable to assume that the backlogging rate is dependent on the length of the waiting time for the next replenishment. **Hsu et al.** presented an inventory model for deteriorating items with expiration date and uncertain lead-time having price sensitive demand. We considered an inventory model for seasonal products with Weibull rate of deterioration having two potential markets, say, primary and alternate. To handle the inventory up to the next season will result an increase in total cost. So it is a favourable task to transfer the remaining stock to the alternate market even at a slightly differ in selling price.**Papa Christos and Skouri** generalized the model of Wee (1999) by time dependent rate of backlogging. Quantity discount schemes for the unit cost, partial backlogging at a fixed rate, deterioration of stock in time and demand rate being a linear function of the selling price. In this article we generalize the work of Wee (1999). More specifically, we consider a model where the demand rate is a convex decreasing function of the selling price and the backlogging rate is a time-dependent function, which ensures that the rate of backlogged demand increases as the waiting time to the following replenishment point decreases.**Singh et al.** considered a soft computing based inventory model with deterioration and price dependent demand. This deals with two warehouse inventory model of determining the optimal replenishment policy for non-instantaneous deteriorating items partial backlogging and stock-dependent demand. In the model, shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment.

Wee presented a joint pricing and replenishment inventory policy for deteriorating items with constant rate of deterioration and in this paper, a mathematical model is developed to formulate optimal pricing and ordering policies for retailer when demand is dependent on selling price and stock, and the supplier offers two progressive credit periods to settle the account.

Wu et al. developed an inventory model for the determination of optimal replenishment policy for non-instantaneous deteriorating items and stock dependent demand. In this model, backlogging rate is considered as a function of waiting time. **Singh and Singh** considered a production inventory model with variable demand rate for deteriorating items under permissible delay in payments. For today's high competitive market, when immensity purchasing of inventory becomes convenient or obligatory, to discover an alternative market in order to maximize the revenue earned is a tradition. In this paper, we considered an inventory model for seasonal products with Weibull rate of deterioration having two potential markets, say, primary and alternate. To handle the inventory up to the next season will result an increase in total cost.

2.Assumptions and notations

Assumptions

1. Demand rate for the products is taken as a function of selling price.
2. The products considered in this model are deteriorating in nature and rate of deterioration is a linear function of time.
3. The shortages are followed and partially backlogged.
4. The backlogging rate is considered as a constant fraction of occurring shortages.
5. During a given cycle the deteriorated items are neither repaired nor replaced.
6. Lead time is not considered in this in model.

Notations

$c_i$purchasing cost per unit
p selling price per unit
$q_i$ordering quantity
θdeterioration coefficient,$\theta <<< 1$
αpositive demand coefficient
βdemand coefficient,$\beta > 1$
T cycle length
v the time at which inventory level becomes zero
Q_1initial inventory level at the beginning of each cycle
Q_2backordered quantity
h holding cost per unit
s shortage cost per unit
l lost sale cost per unit
Oordering cost per order
$R(v,p)$sales revenue per replenishment cycle

3. Mathematical modelling

The differential equations showing the fluctuations of inventory with time t are shown as below:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) == -\frac{\alpha}{p^{\beta}} o \le t \le v, \quad (1)$$

$$\frac{dI_2(t)}{dt} = -\frac{\alpha}{p^{\beta}} v \le t \le T. \quad (2)$$

With boundary condition:

$$I_1(v) = I_2(v) = o \quad (3)$$

The solutions of (1) and (2) equations are given by:

$$I_1(t) = -\frac{ce^{-t\theta}(e^{t\theta} - e^{T\theta})}{\theta} o \le t \le v \quad (4)$$

$$I_2(t) = -ct + cTv \le t \le T \quad (5)$$

Where $c = \frac{\alpha}{p^{\beta}}$ is any constant.

Now if $F(v,p)$ is the unit time profit function then:

$$F(v,p) = \frac{1}{T}[Sales\ revenue - purchasing\ cost - deterioration\ cost - \quad shortage\ cost - lost\ sale\ cost - holding\ cost - ordering\ cost]$$

(6)

Sales Revenue

Sales revenue= $(Q_1 + Q_2).p$

Where

$$Q_1 = -\frac{c(1-e^{T\theta})}{\theta}$$

$Q_{2=}cT.$

Therefore, Sales revenue is

$R(v,p) = \left\{cT - \frac{c(1-e^{T\theta})}{\theta}\right\}.p$ (7)

Purchasing cost

Now the purchasing cost of the system is given by

$P.C = \left\{cT - \frac{c(1-e^{T\theta})}{\theta}\right\}.c_i$(8)

Deterioration cost

The cost of deterioration for the system is given by

$D.C = \{I_1(o) - \int_0^v c\,dt\} = \left\{-c(-o+v) - \frac{c(1-e^{T\theta})}{\theta}\right\}.c_i$ (9)

Holding cost

The holding cost is given by

$H.C = h\int_o^v I_1(t)dt = \frac{ch(e^{(-o+T)\theta} - e^{(T-v)\theta} + (o-v)\theta)}{\theta^2}$(10)

Shortage cost

The shortage cost is given by

$S.C = s\int_v^T cdt = cs(T-v)$(11)

Lost Sale Cost

The lost sale cost for the system can be calculated as

$L.S.C = l\int_v^T (1-\eta).c\,dt = l(T-v)(1-\eta)c$(12)

Ordering Cost

The ordering cost per order

$O.C = O$(13)

Therefore, the unit profit for the system can be calculated as follows

$$F(v,p) = \frac{1}{T}\left[\left\{cT - \frac{c(1-e^{T\theta})}{\theta}\right\}.p - \left\{cT - \frac{c(1-e^{T\theta})}{\theta}\right\}.c_i - \left\{-c(-o+v) - \frac{c(1-e^{T\theta})}{\theta}\right\}.c_i - \frac{ch\left(e^{(-o+T)\theta} - e^{(T-v)\theta} + (o-v)\theta\right)}{\theta^2} - cs(T-v) - l(T-v)(1-\eta).c - O\right]$$

(14)

3. Solution procedure

The unit time profit is a function of two variables that are v and p. We obtain a partial derivative with respect to v and p to find out the optimal solution i.e.

$\frac{\partial F(v,p)}{\partial v} = 0$and $\frac{\partial F(v,p)}{\partial p} = 0.$

Therefore, we find the optimal values of v and p after solving these equations simultaneously.

4. CONCLUTION

The study has presented an economy order quantity model by considering joint pricing and replenishment policy. Since selling price is an important factor affecting the demand. Demand is considered as a function of selling price. In this model, deterioration rate is considered has constant to make this study realistic. Shortage cost is partially backlogged the presented model. The retailer accepts the policy of quantity and price. The presented model can be extended for various demand patterns and deterioration rate.

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