Effect of ignoring singularities

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ABSTRACT *The integral of function is normally described as "area under curve". The integral may be singular or analytic. This paper considers two dimensional lamb problemfirstly and integral secondly which are evaluated by complex integration which further removes singularities of integral. Numerical integration methods are widely used for evaluating integrals viz., Romberg integration in which we ignore singularities of integral. Many computational methods and equations are implemented through Mat lab functions and accessories. This research activity contributes the difference between values of integral which are evaluated through thesemethods.*

Keywords: *Complex integration, Romberg integration, Mat lab.*

Introduction:
We consider a two dimensional problem and derive expressions for surface displacements arising from a force applied normal to the free surface along a line coincidewith the y axis. The displacements u and w are given by

$$u = \frac{\partial \phi}{\partial \phi} - \frac{\partial \psi}{\partial z}, \qquad w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, \tag{1}$$

where the functions ∂ and ψ are solution of wave equation

$$\nabla^2 \phi = \frac{1\partial^2 \phi}{\alpha^2 \partial t^2}, \tag{2}$$

$$\nabla^2 \psi = \frac{1\partial^2 \psi}{\beta^2 \partial t^2}, \tag{3}$$

respectively. The effect of periodic force applied perpendicular to the free surface is expressed by conditions

$$\left[p_{zx}\right]_{z=0} = 0, \left[p_{zz}\right]_{z=0} = Ze^{l}(wt - kx), \tag{4}$$

where the amplitude Z depends only on k.we assume potential are in the form

$$\phi - Ae^{-vz-lkx}, \psi = Be^{-v^1 z - lkx}, \tag{5}$$

which satisfy wave equation(2)and(3)respectively, provided that

$$v^2 = k^2 - k_\alpha^2,$$

$$v^{12} = k^2 - k_\beta^2,$$

$$k_\alpha = \frac{\omega}{\alpha},$$

$$k_\beta = \frac{\omega}{\beta},$$

and A , B are function of parameter k . As

$$\left[p_{zz}\right]_{z=0}=\mu(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z})=\mu(2\frac{\partial^2\phi}{\partial x\partial z}+\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}), \qquad (6)$$

$$\left[p_{zz}\right]_{z=0}=\lambda\theta+2\mu\frac{\partial w}{\partial x}=\lambda\nabla^2\phi+2\mu(\frac{\partial^2\phi}{\partial z^2}+\frac{\partial^2\psi}{\partial x\partial z}), \qquad (7)$$

Using (4),(5)

$$2Alk-(2K^2-k_\beta^2)B=0 \qquad (8)$$

$$(2k^2-k_\beta^2)A+2Bikv^1=\frac{Z(k)}{\mu} \qquad (9)$$

$$A=\frac{(2k^2-k_\beta^2)Z(k)}{F(k)\mu},B=\frac{2lkvZ(k)}{F(k)\mu}, \qquad (10)$$

Where

$$F(k)=(2k^2-k_\beta^2)^2-4k^2vv^1$$

is Rayleigh function. We superimpose infinite number of stress distributions of the form (1.4)such that resultant is a line source. For this ,we put $Z=\frac{-Qdk}{2\pi}$ in (1.4) and integratewith respect to k from $-\infty$ to $+\infty$ and obtain

$$\left[p_{zz}\right]_{z=0}=f(x)=\frac{-Q}{2\pi}\int_{-\infty}^{+\infty}e-^{lkx}dk, \qquad (11)$$

then if we put

$$\int_{-\infty}^{+\infty}f(\xi)e^{l\xi k}d\xi=-Q(k),^{lkx}$$

we obtain Fourier integral

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-lkx}dk\int_{-\infty}^{+\infty}f(\xi)e^{-l\xi k}d\xi,$$

representing the distribution of normal stresses.In order to obtain concentrated normal force at x = 0 assume that the normal force f(x) along the x axis vanishes everywhere

except at x = 0,where it approaches infinity in such a way that $\int_{-\infty}^{+\infty}f(\xi)d\xi-Qd$ is finite. With stress specified by (4)the displacement of any point in the surfacez= 0 written,using (1),(5),(10) as

$$u_0=\frac{iQ}{2\pi\mu}\int_{-\infty}^{+\infty}\frac{k(2k^2-k_\beta^2-2vv^1)e^{-lkx}dk}{F(k)}, \qquad (12)$$

$$w_0=\frac{-Q}{2\pi\mu}\int_{-\infty}^{+\infty}\frac{k_\beta^2ve^{-lkx}dk}{F(k)}, \qquad (13)$$

complex integration We replace variable of integration k by complex variable ξ and use contour integration in ξ plane. We evaluate integrals of the form

$$\int \phi(\xi)d\xi = \int \frac{\xi(2\xi^2 - k_\beta^2 - 2\sqrt{\xi^2 - k_\alpha^2}\sqrt{\xi^2 - k_\beta^2})e^{-l\xi x}d\xi}{(2\xi - k_\beta^2)^2 - 4\xi^2\sqrt{\xi^2 - k_\alpha^2}\sqrt{\xi^2 -}k_\beta^2} \qquad (14)$$

$$\int \psi(\xi)d\xi = \int \frac{k_\beta^2\sqrt{\xi^2 - k_\alpha^2}e^{-l\xi x}d\xi}{(2\xi - k_\beta^2)^2 - 4\xi^2\sqrt{\xi^2 - k_\alpha^2}\sqrt{\xi^2 -}k_\beta^2} \qquad (15)$$

branch point and branch cut Since ξ a is complex variable , ξ = k +i τ ,the integrand ϕ and ψ has real poles $(\pm k, 0)$ which are determined by zeros of
F (ξ). Branch points are given by $(\pm k_\alpha, 0)$, $(\pm k_\beta, 0)$. We make cut according to Re v $\geq$ 0 and Re v' $\geq$ 0. For complex $w = s - i\sigma$

$$k_\alpha = \frac{(s - i\sigma)}{\alpha}$$

$$k_\beta = \frac{(s - i\sigma)}{\beta}$$

Re v=0,requires that

$$k^2 - \tau^2 + ik\tau - \frac{s^2 - \sigma^2 - 2is\sigma}{\alpha^2}$$

be real and negative or

$$k\tau = \frac{-s\sigma}{\alpha^2}$$

and

$$k^2 + \tau^2 < \frac{s^2 - \sigma^2}{\alpha^2} \qquad (16)$$

For a real !, σ = 0 these conditions are of the form

$$k\tau = 0$$

$$k^2 + \tau^2 < \frac{s^2}{\alpha^2}$$

These figure show the branch point and branch cut for Re!

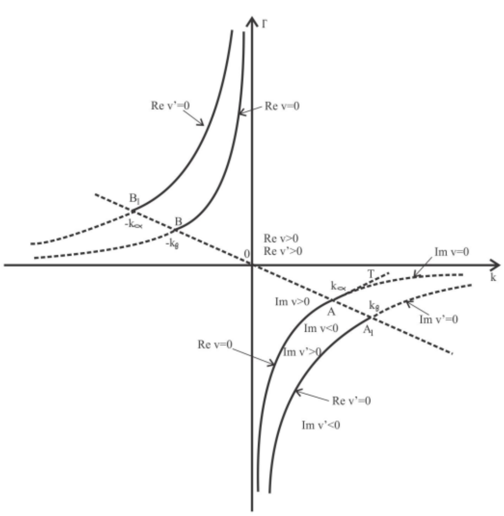


Figure 1: branch point and cut for Re! >0

0.1 residues

We take contour in lower semi-infinite plane The integral (14),becomes

$$\int \phi(\xi)d\xi = \int_M^N + \int_N^H + \int_{L\alpha} + \int_{L\beta} + \int_G^M = 2\pi l \sum \text{Res}$$

On the infinite arcs NH and GM, exp($(-l\xi x)$ makes integrals zero, due to contour in lower half space. Therefore the integral

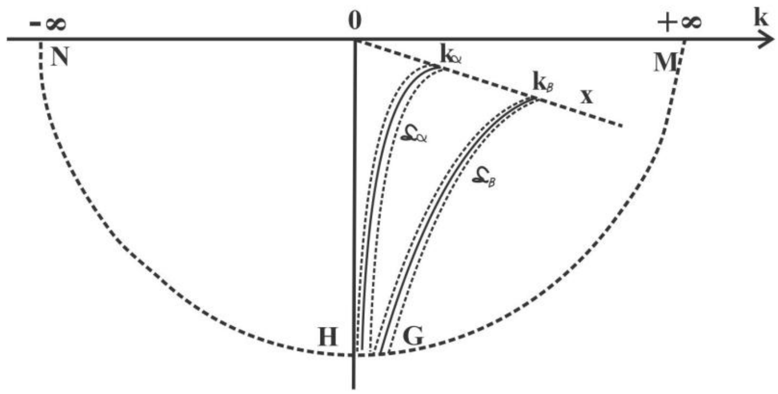


Figure 2: contour in lower semi infinite plane

along real axis becomes

$$\int_{-\infty}^{+\infty} \phi(k)dk = \int_{L\alpha} + \int_{L\beta} - 2\pi l \sum \text{Re}\, s$$

Since there is only one pole κ and integrals along the loops L_α and L_β are branch line integrals. Therefore by equation (14),(15) we have

$$\int_{-\infty}^{+\infty} \frac{k(2K^2 - K_\beta^2 - 2vv^1)E^{-lkx} dk}{F(k)} = 2\pi l H e^{-lkx} + \int_{L\alpha} \phi(\xi)d\xi + \int_{L\beta} \phi(\xi)d\xi \qquad (17)$$

$$\int_{-\infty}^{+\infty} \frac{K_\beta^2 ve -^{-lkx}\ dk}{F(k)} = 2\pi l K e^{-lkx} + \int_{L\alpha} \psi(\xi)d\xi + \int_{L\beta} \psi(\xi)d\xi \qquad (18)$$

where

$$H = \frac{-k(2k^2 - k_\beta^2) - 2\sqrt{k^2 - k_\alpha^2}\sqrt{k^2 - k_\beta^2}}{F^1(k)} \qquad (19)$$

$$K = \frac{-k_\beta^2 \sqrt{k^2 - k_\beta^2}}{F^1(k)} \qquad (20)$$

Using these results for displacement (12),(13) ,we have

$$u_0 = \frac{-QH}{\mu} \exp[wt - kx] + \ldots \qquad (21)$$

$$w_0 = \frac{-lQK}{\mu}\exp[i(wt-kx)]+\ldots \tag{22}$$

Branch line integral for real !,the two loops L_α and L_β degenerate into one L with branch point k_α and k_β on real axis. As integral (14)

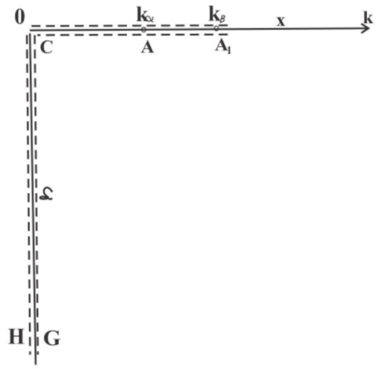


Figure 3: Loop formed by contraction of L_α and L_β for !real

$$\int_L \frac{\xi(2\xi^2-k_\beta^2-2vv^1)e^{-i\xi x}d\xi}{(2\xi^2-k_\beta^2)^2-4\xi^2vv^1}$$

, integrand is function of vv^1. On the part of loop HOA the product 4 vv^1 has the same value as on GCA and crossponding parts of integral along L cancel each other. Therefore branch line integral of (14) reduces to form

$$I_1 = \int_{k\alpha}^{k\beta}[\frac{2k^2-k_\beta^2-2vv_1^1}{(2k^2-k_\beta^2)^2-4k^2vv_1^1}-\frac{2k^2-k_\beta^2-2vv_{11}^1}{(2k^2-k_\beta^2)^2-4k^2vv_{11}^1}e^{-lkx}kdk] \tag{23}$$

where $=\sqrt{k^2-k_\alpha^2}$ is real positive.

$v_1^1 = \operatorname{Im} v^1 \geq 0$ infirst quadrant.

$v_{11}^1 = \operatorname{Im} v^1 > 0$ in fourth quadrant.

since $v_{11}^1 = v_1^1$ near OA1..above equation become

$$I_1 = 4k_{k\alpha}^{k\beta}\int_{k\alpha}^{k\beta}\frac{(2k^2-k_\beta^2)vv_1^1k}{(2k^2-k\beta^2)^4-16k^4v^2v_1^12}e^{-lkx}dk$$

The integrals of type (15) cannot reduced similarly because the factor v alone and parts along HOA and GCA not cancel by each other. Therefore

$$I_2 = 2k_\beta^2\int_\infty^0\frac{l\sqrt{\tau^2+k_\alpha^2}}{F(-i\tau)}e^{-\tau x}d(-l\tau)+8k_\beta^2\int_{k\alpha}^{k\beta}\frac{k^2v^2v_1^1}{(2k^2-k_\beta^2)^4-16k^4v^2v_1^1\ 2}e^{-lkx}dk$$

$$+2k_\beta^2\int_0^{k\alpha}\frac{v_1}{F(k)}e^{-lkx}dk$$

if we put v = v0 then equation(22),(23) reduces to form

$$\int_{L\alpha} = \int_{-\infty}^{0} [\frac{2\xi^2 - k_\beta^2 - 2v, v_1^1}{(2\xi^2 - k_\beta^2)^2 - 4\xi^2 v_1 v_1^1} - \frac{2\xi^2 - k_\beta^2 + 2v, v_1^1}{(2\xi^2 - k_\beta^2)^2 + 4\xi^2 v^1 v_1^1}] e^{-l\xi x} kd\xi$$

$$+\int_0^{k\alpha} \frac{2k^2 - k_\beta^2 - 2v^1 v_1^1}{(2k^2 - k_\beta^2)^2 - 4k2v^1 v_1^1} - \frac{2k^2 - k_\beta^2 - 2v^1 v_1^1}{(2k^2 - k_\beta^2)^2 + 4k^2 v^1 v_1^1} e^{-lkx} kdk$$

$$\int_{L\beta} = \int_{-\infty}^{0} [\frac{2\xi - k_\beta^2 + 2v^1 v_1^1}{(2\xi^2 - k_\beta^2)^2}$$

$$+4\xi^2 v^1 v_1^1 - \frac{2\xi^2 - k_\beta^2 - 2v_1 v_1^1}{(2\xi^2 - k_\beta^2)^2 - 4\xi^2 v^1 v_1^1}] e^{-lkx} \xi d\xi + \int_0^{k\alpha} [\frac{-2k^2 - k_\beta^2 + 2v^1 v_1^1}{(2k^2 - k_\beta^2)^2}$$

$$+4k^2 v^1 v_1^1 - \frac{2k^2 - k_\beta^2 - 2v_1 v_1^1}{(2k^2 - k_\beta^2)^2 - 4k^2 v^1 v_1^1}] e^{-lkx} kdk$$

$$+\int_{k\alpha}^{k\beta} [\frac{-2k^2 - k_\beta^2 + 2v^1 v_1^1}{(2k^2 - k_\beta^2)^2 - 4k^2 v v_1^1}] e^{-lkx} kdk$$

(23) can also written in the form

$$\int_{L\beta} + \int_{-\infty}^{0} [\frac{-2\xi^2 - k_\beta^2 + 2v^1 v_1^1}{(2\xi^2 - k_\beta^2)^2 - 4\xi^2 v v_1^1} - \frac{2\xi^2 - k_\beta^2 - 2v^1 v_1^1}{(2\xi^2 - k_\beta^2)^2 - 4\xi^2 v v_1^1}] e^{-l\xi x} \xi d\xi$$

$$\int + \int_0^{k\beta} [\frac{-2k^2 - k_\beta^2 + 2v^1 v_1^1}{(2k^2 - k_\beta^2)^2 - 4k^2 v v_1^1} - \frac{2\xi^2 - k_\beta^2 - 2v^1 v_1^1}{(2\xi^2 - k_\beta^2)^2 - 4\xi^2 v v_1^1}] e^{-l\xi x} \xi d\xi$$

since ξ = $-l\tau$ in first integral in (23)and we substitute $u = k_\alpha - k$ in second integral in (23), it take the forms

$$-4k_\beta^2 \int_0^{\infty} \frac{\tau(2\tau^2 - k_\beta^2 v^1 v_1^1}{(2\tau^2 - k_\beta^2)^4 - 16\tau 4 v_1^2 v_1^1 2} e^{-l\tau x} d\tau$$

$$4k_\beta^2 \int_0^{k\alpha} \sqrt{uG(u) e^{iux} du}$$

where

$$\int_{L\beta} -G(u) = u^{-1/2} \frac{[2(k_\beta - u)^2 - k_\beta^2] v_1 v_1^1 (k_\alpha - u)}{[2(k_\alpha - u)^2 - k_\beta^2]^4 - 16(k_\alpha - u)^4 v_2^1 v_1^1 2}$$

$$G(0) = \frac{(k_\alpha^2 - k_\beta^2)^{1/2}(-2)^{1/2} k_\alpha^{3/2}}{(2k_\alpha{}^2 - k_\beta^2)3}$$

Thus we obtain

$$\int_{L\alpha} = C(k_\alpha x)^{-3/2} e^{-ik_\alpha x} + O(x^{-5/2})$$

where

$$C = -2\sqrt{2\pi}\,\frac{k_\alpha^3 k_\beta^2 \sqrt{(k_\beta^2 - k_\alpha^2}}{(k_\beta^2 - 2k_\alpha^2)^3}\exp(\frac{-l\pi}{4})$$

The integral (23)reduces to

$$\int_{L\beta} = D(k_\beta x)^{-3/2} e^{-ik\beta x} + O(x)^{-5/2}$$

where

$$D = 2\pi\sqrt{2\pi}\sqrt{1 - \frac{k_\alpha^2}{k_\beta^2}}\exp(\frac{-i\pi}{4})$$

Including branch integral in (21),(22),the displacements becomes

$$u_0 = \frac{-QH}{\mu}\exp[l(wt - kx)] + \frac{iQ}{2\pi\mu}C(k_\alpha x)^{-3/2}\exp[l(wt - kx)]$$

$$+D(k_\beta x)^{-3/2}\exp[i(wt - k_\beta x)] + \ldots$$

$$w_0 = -i\frac{-Qk}{\mu}\exp[l(wt - kx)] + \frac{Q}{2\pi\mu}C_1(k_\alpha x)^{-3/2}\exp[l(wt - k_\alpha x)]$$

$$+D_1(k_\beta x)^{-3/2}\exp[l(wt - k_\beta x)] + \ldots$$

where

$$C_1 = -l\sqrt{2\pi}\,\frac{k_\alpha^2 k_\beta^2}{(k_\beta^2 - 2k_\alpha^2)^2}\exp(\frac{-i\pi}{4})$$

$$D_1 = -4l\sqrt{2\pi}(1 - \frac{k_\alpha^2}{k_\beta^2})\exp(\frac{-i\pi}{4})$$

Romberg integration Romberg integration solve improper integral if it is improper due to one or more of the following problems

(i)its integrand goes to finite limiting value at finite upper and lower limits but cannot be evaluated right on one of those limits.

(ii)its upper limit is 1 ,or lower limit is −1

(iii)it has an integrable singularity at either limit (iv)it has an integrable singularity at known place between upper and lower limit

The algorithm for Romberg integration is

1.Inputa,b,M

where a=lower limit of integration

b=upper limit of integration

M=

2.h=b-a

3.r(0,0)=h *[f(a)+f(b)]/2

4.for n=1 to M ,do

h=h/2

r(n,0)=r(n-1,0)/ $2+h\sum_{i=1}^{2^{n-1}} f(a+2i-h)$

5.for m=1 to n ,do

r(n,m)=r(n,m-1)+[r(n,m-1)-r(n-1,m-1)]/4m -1

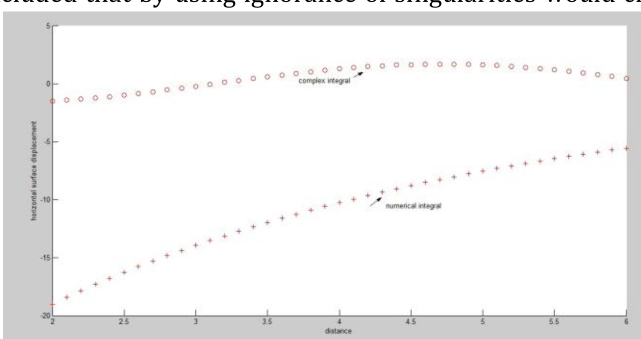
6.end do

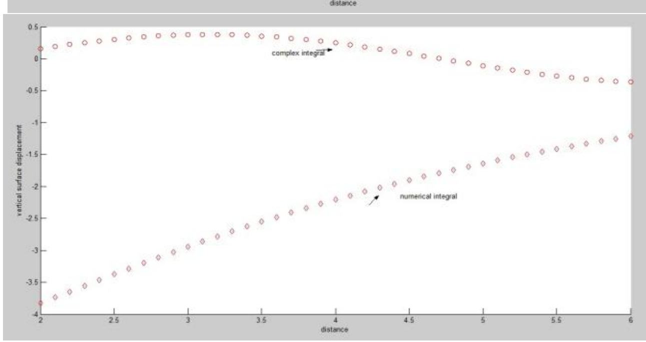
7.end do

8.output r(n,m)

Conclusion:

These computational programs, equations and methods with their graphical representation show that hows the results are differ from each other, when computed from different methods and their interpretations. This discussion has concluded that by using ignorance of singularities would create difference in values of integrals.





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