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SOLVING ASSIGNMENT PROBLEM EXPLOITATION LEAST PRICE METHODOLOGY

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ABSTRACT: *In operation analysis the assignment drawback is that the necessary topic. Assignment problem has totally different strategies to unravel the matter. This article focus on latest technique for finding assignment problem. One in every of the transportation is least price methodology. It help to solve the topic simply. This paper throws lightweight o least price methodology to resolve assignment downside. This attitude deals with all range of issues. Besides this paper targeted on numerical examples and solutions.*

Keywords: *Assignment problem, Least price methodology, optimum answer*

1. Introduction

Assignment problem is giving the distribution for n jobs to n totally different machine or men to minimizing the entire price. It's necessary of transportation drawback. Particularly in call theory. During this paper needs to solve the assignment problem exploitation this methodology of least price method. I've got determine optimum answer given some numerical example. I verified the optimum answer to existing strategies. It's a simple methodology for locating assignments for problem.

2.Basic definitions:

2.1.Assignment problem

An assignment problem could be a explicit case of a transportation problem wherever the sources are assignees and therefore the destination is tasks. Moreover, every supply features a offer of one and each destination has a demand of 1. Also, the goal is to reduce the whole price or to make the most of the entire profit of allocation .

2.2.Least price methodology

The Least price methodology is another method accustomed get the initial possible answer for the transportation drawback. Here, the allocation begins with the cell that has the smallest amount price. The lower price cells unit of measurement chosen over the higher-cost cell with the target to have the tiniest quantity value of transportation. The smallest amount price methodology is taken under consideration to supply plenty of best results than the North-west Corner as a results of it considers the shipping value whereas making the allocation, whereas the North-West corner methodology exclusively considers the availability and supply demand and allocation begin with the acute left corner, irrespective of the shipping price.

3.Mathematical structure of the assignment problem:

Given n facilities and n jobs and effectiveness of every facility for every job, the matter lies in distribution every resource to 1 and just one job so the given live of effectiveness is optimized. the information matrix for this drawback is shown in tale 1.1

Table 1.1

Workers	Jobs					Supply
	A_1	A_2	A_3	………	A_n	
B_1	a_{11}	a_{12}	a_{13}	………	a_{1n}	1
B_2	a_{21}	a_{22}	a_{23}	………	a_{2n}	1
.	.	.	.	.	.	.
.	.	.	.	.	.	.
B_n	a_{n1}	a_{n2}	a_{n3}	………	a_{nn}	1
Demand	1	1	1	………..	1	N

Let xij denote the assignment of facility i to job j specified

$$x_{ij} = \begin{cases} 1 & \text{if facility i is assignmet to job j} \\ 0 & \text{otherwise} \end{cases}$$

Then, the mathematical model

Optimize z=$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{ij}$

Subjects to constraints

$$\sum_{j=1}^{n} x_{ij} = 1 \text{, for all i}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{, for all j}$$

$$x_{ij} = 0 \text{ or } 1 \text{ ,for all i,j}$$

Where a_{ij} represents the cost of assignment.

4.The steps for finding assignment problem:

This section provides the new technique to unravel assignment problem that is totally different from others. This methodology is termed lowest price method. It's simple means for all variety of assignment problem.

Stage : 1

First we have a tendency to observe that the matter is balanced or not. If the matter is balanced then attend next stage. Otherwise adding row or column to convert the matter has balanced. the value of part in dummy cells are zero.

Stage :2

We see the full downside to spot the tiniest price within the table and provides the assignment to the present cell then take away the row and column that cell contained.

Stage :3

After eliminating the row and column repeat the procedure with the subsequent lowest price cell. that's offer the assignment for lowest price cell and eliminating the row and column contain that cell.

Stage :4

Repeat the procedure till all row and column contains specifically one assignment. This methodology so as to search out initial basic assignment to the matter

Stage :5 To search out optimum assignment;

In this methodology is special case is even against that smallest price there's limit or limit to a specifically scenario then any row or column may be designated.

5.Numerical examples.

Consider a project work consists of 4 major jobs that Associate in Nursing equal range of contractors have submitted tenders. The tender quantity quoted is given within the matrix. notice the assignment that minimizes the whole value of the project once every contractor needs to be assigned a minimum of one job.

		Job			
		A	**b**	**C**	**D**
	1	10	24	30	15
contractor	**2**	16	22	28	12
	3	12	20	32	10
	4	9	26	34	16

Stage 1
The problem is 4X4 balanced problem. We see that 9 is the smallest in the problem. Assignment made in the cell (4,1), eliminating (draw line) the row 4 and column a.

	a	b	c	d
1	10	24	30	15
2	16	22	28	12
3	12	20	32	10
4	**(9)**	~~26~~	~~34~~	~~16~~

Stage 2
The new reduced matrix is given below. Again the lowest cost is 10 ,give assignment and cross out the row and column that cell appeared.

	b	c	d
1	24	30	15
2	22	28	12
3	~~20~~	~~32~~	**(10)**

Stage 3
The next new matrix contain the smallest value is22,give the assignment and remove the row and column that cell appeared. Finally the possible minimum value is 30 give the assignment

	b	c
1	24	**(30)**
2	**(22)**	~~28~~

Stage 4
Hence the complete assignment is

		Job			
		A	**b**	**c**	**d**
	1	10	24	**(30)**	15
contractor	**2**	16	**(22)**	28	12
	3	12	20	32	**(10)**
	4	**(9)**	26	34	16

Each row and each column contains exactly one assignment. Hence the initial assignment is 1 to c, 2 to b, 3 to d, 4 to a
The minimum cost is =71

Stage 5 :To find optimal solution

	A	b	c	d
1	10	24	**(30)**	15
2	16	**(22)**	28	12
3	12	20	32	**(10)**
4	**(9)**	26	34	16

Case i

In above matrix first and second row costs 30 and 22 are assigned diagonally.
That is

24	**(30)**
(22)	28

If the sum of 22 and 30 is less than or equal to 24 and 28,then go to next case. Otherwise interchange the assignments. First row assignment cannot be change.

Case ii

In second and third row 22 and 10 are form diagonally.

(22)	12
20	**(10)**

If the sum of 22 and 10 is less than or equal to sum of 20 and 12, then go to next case. Otherwise interchange the assignments. Second row assignment cannot be changed.

Case iii

In third and fourth row 9 and 10 form diagonally.

12	**(10)**
(9)	16

If the sum of 10 and 9 is less than or equal to sum of 12 and 16, then go to next case. Otherwise interchange the assignments. Hence the assignments cannot changed.
Hence the optimal assignment is

Job

		a	**b**	**c**	**d**
contractor	**1**	10	24	**(30)**	15
	2	16	**(22)**	28	12
	3	12	20	32	**(10)**
	4	**(9)**	26	34	16

The optimal allocations are
Contractor 1 → Job c
Contractor 2 → Job b
Contractor 3 → Job d
Contractor 4 → Job a
The optimal minimum cost is=30+22+10+9 =71.

6. Unbalanced problem

Consider the problem five workers are available to work with the machines and the respective costs associated with each worker machine assignment are given below. A sixth machine is available to replace one of the existing ones. Determine whether the new machine can be accepted. Also determine the optimal assignment and associated saving in cost.

Machines

		M1	**M2**	**M3**	**M4**	**M5**	**M6**
workers	**W1**	12	3	6	-	5	9
	W2	4	11	-	5	-	8
	W3	8	2	10	9	7	5
	W4	-	7	8	6	12	10
	W5	5	8	9	4	6	1

Stage 1
The given problem 5X6 is not balanced. Adding one row in last that all cost of the row are zero to convert the problem as balanced.

	M1	**M2**	**M3**	**M4**	**M5**	**M6**
W1	12	3	6	-	5	9
W2	4	11	-	5	-	8
W3	8	2	10	9	7	5
W4	-	7	8	6	12	10
W5	5	8	9	4	6	1
Wd	0	0	0	0	0	0

Where Wd is dummy worker means no worker.

Stage 2
Choose the minimum cost 0 in last row, first column give the assignment and remove the row and column.

	M1	**M2**	**M3**	**M4**	**M5**	**M6**
W1	12	3	6	-	5	9
W2	4	11	-	5	-	8
W3	8	2	10	9	7	5
W4	-	7	8	6	12	10
W5	5	8	9	4	6	1
Wd	**(0)**	0	0	0	0	0

Stage 3
Given assignment in next minimum cost 1 and remove the row and column.

	M2	**M3**	**M4**	**M5**	**M6**
W1	3	6	-	5	9
W2	11	-	5	-	8
W3	2	10	9	7	5
W4	7	8	6	12	10
W5	8	9	4	6	**(1)**

Stage 4
The lowest cost is 2 in reduced matrix, give assignment and delete the corresponding row and column.

	M2	**M3**	**M4**	**M5**
W1	3	6	-	5
W2	11	-	5	-
W3	**(2)**	10	9	7
W4	7	8	6	12

Stage 5
Next lowest cost is 5 , give assignment and remove the row and column.

	M3	M4	M5
W1	6	-	**(5)**
W2	-	5	-
W4	8	6	12

Stage 6

The next least cost is 5. Assignment is made in this cell and cross out a row and column. Finally the possible minimum value is 8, give assignment that cell.

	M3	M4
W2	-	**(5)**
W4	**(8)**	6

Hence the complete assignment is

Machines

Workers		M1	M2	M3	M4	M5	M6
	W1	12	3	6	-	**(5)**	9
	W2	4	11	-	**(5)**	-	8
Workers	W3	8	**(2)**	10	9	7	5
	W4	-	7	**(8)**	6	12	10
	W5	5	8	9	4	6	**(1)**
	Wd	**(0)**	0	0	0	0	0

The initial allocations are W1→M5, W2→M4, W3→M2, W4→M3, W5→M6, Wd→M1.
The minimum cost is =5+5+2+8+1=21

Stage 7 :To find optimal assignment

	M1	M2	M3	M4	M5	M6
W1	12	3	6	-	**(5)**	9
W2	4	11	-	**(5)**	-	8
W3	8	**(2)**	10	9	7	5
W4	-	7	**(8)**	6	12	10
W5	5	8	9	4	6	**(1)**
Wd	**(0)**	0	0	0	0	0

Case i

In above table form a 2x2 matrix from first and second row .

12	-
4	**(5)**

If the sum of 12 and 5 is less than the sum of 4 and 0. Then go to next case. Otherwise interchange the assignments. That is 12+5>4. Then interchange the assignment 5 to 4. The second row has assignment 4.

Case ii

For third and fourth row 2 and 8 assigned diagonally.

(2)	10
7	**(8)**

If 2+8≤ 10+7, then go to next case. Otherwise interchange the assignments.

Case iii

Form a another single value matrix that right side of 8.If the sum 8+9>10+6. Then change the assignment 8 to 6. Therefore the new assignment is 6.

10	9
(8)	6

Case iv
For five and sixth row the allocation 1 is minimum cost. It cannot be change. In sixth row give the assignment 0 in cell (6,3).

Hence the optimal assignments are

		Machines					
		M1	M2	M3	M4	M5	M6
Workers	W1	12	3	6	-	**(5)**	9
	W2	**(4)**	11	-	5	-	8
	W3	8	**(2)**	10	9	7	5
	W4	-	7	8	**(6)**	12	10
	W5	5	8	9	4	6	**(1)**
	Wd	0	0	**(0)**	0	0	0

Each row and every column contain specifically one assignment.
The optimal allocations are

Workers		Machines
W1	→	M5
W2	→	M1
W3	→	M2
W4	→	M4
W5	→	M 6
Wd	→	M 3

The optimal minimum cost=5+4+2+6+1=18.
Hence we see that the initial and optimal solution we conclude the new machine M6 is accepted. The sixth machine is to replace the machine M3.
The associated saving cost=21-18= Rs 3.

Conclusion
In this paper, explained the new algorithm and showed the efficiency of it by numerical example. It is easy method to solve assignment problem. This method is very used to all type of assignment problem, like maximization and minimization problem etc. the aim of this paper is to enable readers to recognize whether a problem can be formulated and analyzed as an assignment problem or as a variant of one of these problem types.

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Fuzzy Stability of A Additive Quadratic Functional Equation

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ABSTRACT: *In this paper, we established the stability of a additive quadratic functional equation in fuzzy normed space using classical Hyers direct method.*

Keywords: *Functional Equation, Fuzzy Normed Space, Fuzzy Stability*

1. Introduction

The revision of stability problems for functional equations is related to a question of Ulam [24] concerning the stability of group homomorphisms and affirmatively answered for Banach spaces by Hyers [11]. It was further generalized and excellent results obtained by number of authors [2, 11, 21, 22,23].
The solution and stability of following mixed type additive quadratic functional equations

$$f(2x+y)+f(2x-y)=2f(x+y)+2f(x-y)+2f(2x)-4f(x) \tag{1.1}$$

$$f(2x+y)+f(2x-y)=f(x+y)+f(x-y)+2f(2x)-2f(x) \tag{1.2}$$

$$g(x+y)+g(x-y)=2g(x)+g(y)+g(-y) \tag{1.3}$$

Were introduced and investigated by A. Najati, M.B.Moghimi [20], M.E. Gordji et. al., [10], M. Arunkumar, J.M. Rassias [3] .
M. Arunkumar et.al., [4] introduce and discussed the general solution and generalized Hyers-Ulam stability of a additive quadratic functional equation which is different from (1.1) and (1.2) of the form

$$f(2x+y) - f(2x-y) = 2[f(x+y) - f(x-y)] - f(y) + f(-y) \tag{1.4}$$

in Banach space.
In this paper, the authors established the stability of the functional equation (1.4) in fuzzy normed space using classical Hyers direct method.

2 DEFINITIONS ABOUT FUZZY NORMED SPACE

In this section, the authors present basic definitions about fuzzy normed space given in [18,19]. For more detailed about this sapce one can refer [5,6,7,9, 16, 17]

Definition 2.1 *Let* X *be a real linear space. A function* $N : X \times \mathrm{P} \to [0,1]$ *(3.the so-called fuzzy subset) is said to be a fuzzy norm on* X *if for all* $x, y \in X$ *and all* $s,t \in \mathrm{P}$ *,*

$(F1)$ $N(x,c)=0$ for $c \le 0;$

$(F2)$ $x=0$ if and only if $N(x,c)=1$ for all $c>0;$

$(F3)$ $N(cx,t)=N\left(x,t/|c|\right)$ if $c \ne 0;$

$(F4)$ $N(x+y,s+t) \ge min\{N(x,s),N(y,t)\};$

$(F5)$ $N(x,\cdot)$ is a non-decreasing function on P and $lim_{t\to\infty}N(x,t)=1;$

$(F6)$ for $x \neq 0, N(x,\cdot)$ is (3.upper semi) continuous on P .

The pair (X,N) is called a fuzzy normed linear space. One may regard $N(X,t)$ as the truth-value of the statement the norm of x is less than or equal to the real number t '.

$$N\left(x,t\right)=\begin{cases}\dfrac{t}{t+\|x\|}, & t>0,\, x\in X,\\ 0, & t\le 0,\, x\in X\end{cases}$$

Example 2.2 *Let* $(X,\|\cdot\|)$ *be a normed linear space. Then* is a fuzzy norm on X .

Definition 2.3 *Let* (X,N) *be a fuzzy normed linear space. Let* $\{x_n\}$ *be a sequence in* X *. Then* x_n *is said to be convergent if there exists* $x\in X$ *such that* $\lim_{n\to\infty} N(x_n-x,t)=1$ *for all* $t>0$ *. In that case,* x *is called the limit of the sequence* x_n *and we denote it by* $N-\lim_{n\to\infty} x_n = x.$

Definition 2.4 *A sequence {*x_n*} in* X *is called Cauchy if for each* $\varepsilon>0$ *and each* $t>0$ *there exists* n_0 *such that for all* $n\ge n_0$ *and all* $p>0$ *, we have* $N(x_{n+p}-x_n,t)>1-\varepsilon.$

Definition 2.5 *Every convergent sequence in a fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed space is called a fuzzy Banach space.*

To prove the stability results, we assume that (X,N) and (Y,N') are linear space and fuzzy Banach space, respectively.

3. FUZZY STABILITY RESULTS: ODD CASE

Theorem 3.1 *Let* $\beta\in\{-1,1\}$ *. Let* $\alpha: X^2\to(0,\infty]$ *be a mapping with* $0<(d/2)^{\beta}<1$

$$N'\left(\alpha\left(2^{\beta}x,2^{\beta}x\right),r\right)\ge N'\left(d^{\beta}\alpha\left(x,x\right),r\right) \tag{3.1}$$

for all $x\in X$ and all $d>0$ and

$$\lim_{n\to\infty} N'\left(\alpha\left(2^{\beta n}x,2^{\beta n}y\right),2^{\beta n}r\right)=1 \tag{3.2}$$

for all $x,y\in X$ and all $r>0$. Suppose that an odd function $f:X\to Y$ satisfies the inequality

$$N\left(f(2x+y)-f(2x-y)-2[f(x+y)-f(x-y)]+f(y)-f(-y),r\right)\ge N'\left(\alpha(x,y),r\right) \tag{3.3}$$

for all $x,y\in X$ and all $r>0$. Then there exists is a unique additive mapping $A:X\to Y$ which satisfying (1.4) and

$$N\left(f(x)-A(x),r\right)\ge N'_{AQ}\left(\alpha\left(\frac{x}{2}\right),\frac{r|2-d|}{3}\right)=\min\left\{N'\left(\alpha(x,x),\frac{r|2-d|}{18}\right),N'\left(\alpha(x,2x),\frac{r|2-d|}{9}\right)\right\}$$

(3.4)

for all $x \in X$ and all $r > 0$. The limit $A(x)$ is defined as

$$\lim_{n\to\infty} N\left(A(x) - \frac{f(2^{\beta n} x)}{2^{\beta n}}, r \right) = 1 \tag{3.4a}$$

exists for all $x \in X$

Proof. Replacing (x, y) by (x, x) and $(x, 2x)$ in (3.3) and using oddness of f, we get the following inequalities

$$N\big(f(3x) - 2f(2x) + f(x), r\big) \geq N'\big(\alpha(x,x), r\big) \tag{3.5a}$$

$$N\big(f(4x) - 2f(3x) - 2f(x) + 2f(2x), r\big) \geq N'\big(\alpha(x,2x), r\big) \tag{3.5b}$$

for all $x \in X$ and all $r > 0$. Combining (3.5a) and (3.5b) and using (F4), (F3), we achieve

$$N\left(f(4x) - 2f(2x), \frac{3r}{2} \right) \geq \min\left\{ N'\left(\alpha(x,x), \frac{r}{2} \right), N'\big(\alpha(x,2x), r\big) \right\} = N'_{AQ}\big(\alpha(x), r\big) \tag{3.6a}$$

for all $x \in X$ and all $r > 0$. Interchanging x by $x/2$ in (3.6a) and using (F3), we arrive

$$N\left(f(2x) - 2f(x), \frac{3r}{2} \right) \geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), r \right) \Rightarrow N\left(\frac{f(2x)}{2} - f(x), \frac{3r}{2\cdot 2} \right) \geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), r \right) \tag{3.6}$$

for all $x \in X$ and all $r > 0$. Replacing x by $2^n x$ in (3.6) and using (F2), we obtain

$$N\left(\frac{f(2^{n+1}x)}{2} - f(2^n x), \frac{3r}{2\cdot 2} \right) \geq N'_{AQ}\left(\alpha\left(\frac{2^n x}{2}\right), r \right) \tag{3.7}$$

for all $x \in X$ and all $r > 0$. Using (3.1), $(F3)$ in (3.7), we arrive

$$N\left(\frac{f(2^{n+1}x)}{2} - f(2^n x), \frac{3r}{2\cdot 2} \right) \geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), \frac{r}{d^n} \right) \tag{3.8}$$

for all $x \in X$ and all $r > 0$. It is simple to prove from (3.8), we get

$$N\left(\frac{f(2^{n+1}x)}{2^{(n+1)}} - \frac{f(2^n x)}{2^n}, \frac{3r}{2\cdot 2^n} \right) \geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), \frac{r}{d^n} \right) \tag{3.9}$$

holds for all $x \in X$ and all $r > 0$. Replacing r by $d^n r$ in (3.9), we obtain

$$N\left(\frac{f(2^{n+1}x)}{2^{(n+1)}} - \frac{f(2^n x)}{2^n}, \frac{3r}{2}\frac{d^n}{2^n} \right) \geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), r \right) \tag{3.10}$$

for all $x \in X$ and all $r > 0$. It is plain to see that

$$\frac{f(2^n x)}{2^n} - f(x) = \sum_{i=0}^{n-1} \frac{f(2^{i+1}x)}{2^{(i+1)}} - \frac{f(2^i x)}{2^i} \tag{3.11}$$

for all $x \in X$. From equations (3.10), (3.11) and using $(F3)$, we have

$$N\left(\frac{f(2^n x)}{2^n} - f(x), \frac{3}{2}\sum_{i=0}^{n-1}\frac{d^i}{2^i} r\right) \geq min\bigcup_{i=0}^{n-1}\left\{N\left(\frac{f(2^{i+1}x)}{2^{(i+1)}} - \frac{f(2^i x)}{2^i}, \frac{3}{2}\frac{d^i}{2^i} r\right)\right\} = N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), r\right) \tag{3.12}$$

for all $x \in X$ and all $r > 0$. Replacing x by $2^m x$ in (3.12) and using (3.1), $(F3)$, and then replacing r by $d^m r$ in the resultaing inequalty , we arrive

$$N\left(\frac{f(2^{n+m}x)}{2^{(n+m)}} - \frac{f(2^m x)}{2^m}, \frac{3}{2}\sum_{i=m}^{m+n-1}\frac{d^i}{2^i} r\right) \geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), r\right) \tag{3.13}$$

for all $x \in X$ and all $r > 0$ and all $m, n \geq 0$. Using $(F3)$ in (3.13), we obtain

$$N\left(\frac{f(2^{n+m}x)}{2^{(n+m)}} - \frac{f(2^m x)}{2^m}, r\right) \geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), r\Big/\frac{3}{2}\sum_{i=m}^{m+n-1}\frac{d^i}{2^i}\right) \tag{3.14}$$

for all $x \in X$ and all $r > 0$ and all $m, n \geq 0$. Since $0 < d < 2$ and $\sum_{i=0}^{n}\left(\frac{d}{2}\right)^i < \infty,$ the Cauchy criterion for convergence and $(F5)$ implies that $\left\{\frac{f(2^n x)}{2^n}\right\}$ is a Cauchy sequence in (Y, N). Since (Y, N) is a fuzzy Banach space, this sequence converges to some point $A(x) \in Y$. So one can define the mapping $A : X \to Y$ by $\lim_{n\to\infty} N\left(A(x) - \frac{f(2^n x)}{2^n}, r\right) = 1$ for all $x \in X.$ Letting $m = 0$ and $n \to \infty$ in (3.14) and using $(F6)$, in (3.14), we get

$$N\big(f(x) - A(x), r\big) \geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right), \frac{r(2-d)}{3}\right) \tag{3.15}$$

for all $x \in X$ and all $r > 0$. To prove A satisfies the (1.4), replacing (x, y) by $(2^n x, 2^n y)$ in (3.3), we obtain

$$N\left(\frac{1}{2^n}\Big[f(2^n(2x+y)) - f(2^n(2x-y)) - 2[f(2^n(x+y)) - f(2^n(x-y))] + f(2^n y) \ - f(-2^n y)\Big], r\right)$$
$$\geq N'\big(\alpha(2^n x, 2^n y), 2^n r\big) \tag{3.16}$$

for all $x, y \in X$ and all $r > 0$. Now,

$$N\big(A(2x+y) - A(2x-y) - 2[A(x+y) - A(x-y)] + A(y) \ - \ A(-y), r\big)$$
$$\geq min\left\{N\left(A(2x+y) - \frac{1}{2^n}f(2^n(2x+y)), \frac{r}{7}\right), N\left(-A(2x-y) + \frac{1}{2^n}f(2^n(2x-y)), \frac{r}{7}\right),\right.$$

$$N\left(-2A(x+y)+\frac{2}{2^n}f(2^n(x+y)),\frac{r}{7}\right),N\left(2A(x-y)-\frac{2}{2^n}f(2^n(x-y)),\frac{r}{7}\right),$$
$$N\left(A(y)-\frac{1}{2^n}f(2^n y),\frac{r}{7}\right),N\left(-A(-y)+\frac{1}{2^n}f(-2^n y),\frac{r}{7}\right),$$
$$N\left(\frac{1}{2^n}\left[f(2^n(2x+y))-f(2^n(2x-y))-2[f(2^n(x+y))-f(2^n(x-y))]+f(2^n y)\ -f(-2^n y)\right],\frac{r}{7}\right) \tag{3.17}$$

for all $x,y\in X$ and all $r>0$. Using (3.16) and $(F5)$ in (3.17) and letting $n\to\infty$ in (3.17), using (3.2), $(F2)$, we see that A satisfies the functional equation (1.4). In order to prove $A(x)$ is unique, we let $A'(x)$ be another additive functional equation satisfying (1.4) and (3.4). Hence,

$$N(A(x)-A'(x),r)\geq min\left\{N\left(\frac{A(2^n x)}{2^n}-\frac{f(2^n x)}{2^n},\frac{r}{2}\right),N\left(\frac{f(2^n x)}{2^n}-\frac{A'(2^n x)}{2^n},\frac{r}{2}\right)\right\}=N'_{AQ}\left(\alpha\left(\frac{x}{2}\right),\frac{r2^n\left(2-d\right)}{3d^n}\right)$$

for all $x\in X$ and all $r>0$. Since $\lim_{n\to\infty}\frac{r\,2^n(2-d)}{3d^n}=\infty,$ we obtain $\lim_{n\to\infty}N'_{AQ}\left(\alpha\left(\frac{x}{2}\right),\frac{r\,2^n(2-d)}{3d^n}\right)=1.$

Thus $N(A(x)-A'(x),r)=1$ for all $x\in X$ and all $r>0$, hence $A(x)=A'(x)$. Therefore $A(x)$ is unique. Hence the theorem is holds for $\beta=1$.

Now, Interchanging x by $x/2$ in (3.6), we arrive

$$N\left(f(x)-2f\left(\frac{x}{2}\right),\frac{3r}{2}\right)\geq N'_{AQ}\left(\alpha\left(\frac{x}{4}\right),r\right) \tag{3.18}$$

for all $x\in X$ and all $r>0$. The rest of the proof is similar to that of $\beta=1$. Thus, for $\beta=-1$, we can prove the result. This completes the proof of the theorem.

From Theorem 3.1 we obtain the following corollary concerning the some stabilities for the functional equation (1.4).

Corollary 3.2 Let g,p be constants. Suppose that an odd function $f:X\to Y$ satisfies the inequality

$$N\big(f(2x+y)-f(2x-y)-2[f(x+y)-f(x-y)]+f(y)\ -f(-y),r\big)\geq\begin{cases}N'\big(g,r\big),\\ N'\big(g\{|x|^p+|y|^p\},r\big),\end{cases} \tag{3.20}$$

for all $x,y\in X$ and all $r>0$. Then there exists a unique additive mapping $A:X\to Y$ such that

$$N\big(f(x)-A(x),r\big)\geq\begin{cases}N'\big(12g,r\big),\\ N'\big(6g(3+2^p)|x|^p,r|2-2^p|\big), & p\neq 1\end{cases} \tag{3.21}$$

for all $x\in X$ and all $r>0$.

4. FUZZY STABILITY RESULTS: EVEN CASE

The proof of the following theorem and corollary is similar to that of pervious section by using evenness of f.

Theorem 4.1 Let $\beta \in \{-1,1\}$. Let $\alpha : X^2 \to (0,\infty]$ be a mapping with $0<(d/4)^{\beta}<1$

$$N'\left(\alpha\left(2^{\beta}x,2^{\beta}x\right),r\right)\geq N'\left(d^{\beta}\alpha\left(x,x\right),r\right) \tag{4.1}$$

for all $x \in X$ and all $d>0$ and

$$\lim_{n\to\infty}N'\left(\alpha\left(2^{\beta n}x,2^{\beta n}y\right),4^{\beta n}r\right)=1 \tag{4.2}$$

for all $x,y \in X$ and all $r>0$. Suppose that an even function $f : X \to Y$ satisfies the inequality

$$N\left(f(2x+y)-f(2x-y)-2[f(x+y)-f(x-y)]+f(y)\ -f(-y),r\right)\geq N'\left(\alpha(x,y),r\right) \tag{4.3}$$

for all $x,y \in X$ and all $r>0$. Then there exists is a unique quadratic mapping $Q: X \to Y$ which satisfying (1.4) and

$$N\left(f(x)-Q(x),r\right)\geq N'_{AQ}\left(\alpha\left(\frac{x}{2}\right),\frac{r|4-d|}{3}\right)=\min\left\{N'\left(\alpha(x,x),\frac{r|4-d|}{18}\right),N'\left(\alpha(x,2x),\frac{r|4-d|}{9}\right)\right\} \tag{4.4}$$

for all $x \in X$ and all $r>0$. The limit $Q(x)$ is defined as

$$\lim_{n\to\infty}N\left(Q(x)-\frac{f(2^{\beta n}x)}{4^{\beta n}},r\right)=1 \tag{4.4a}$$

exists for all $x \in X$.

Corollary 4.2 Let g,p be constants. Suppose that an even function $f : X \to Y$ satisfies the inequality

$$N\left(f(2x+y)-f(2x-y)-2[f(x+y)-f(x-y)]+f(y)\ -f(-y),r\right)\geq\begin{cases}N'\left(g,r\right),\\N'\left(g\{|x|^{p}+|y|^{p}\},r\right),\end{cases} \tag{4.5}$$

for all $x,y,z \in X$ and all $r>0$. Then there exists a unique quadratic mapping $Q: X \to Y$ such that

$$N\left(f(x)-Q(x),r\right)\geq\begin{cases}N'\left(4g,r\right),\\N'\left(6g(3+2^{p})|x|^{p},r|4-2^{p}|\right), & p\neq 2\end{cases} \tag{4.6}$$

for all $x \in X$ and all $r>0$.

5. FUZZY STABILITY RESULTS: MIXED CASE

Theorem 5.1 Let $\beta \in \{-1,1\}$. Let $\alpha : X^2 \to (0,\infty]$ be a mapping with $0<(d/2)^{\beta}<1$ and $0<(d/4)^{\beta}<1$, $d>0$ with conditions (3.1), (4.1), (3.2) and (4.2) for all $x,y \in X$ and all $r>0$. Suppose that a function $f: X \to Y$ satisfies the inequality

$$N\left(f(2x+y)-f(2x-y)-2[f(x+y)-f(x-y)]+f(y)\ -f(-y),r\right)\geq N'\left(\alpha(x,y),r\right) \tag{5.1}$$

for all $x, y \in X$ and all $r>0$. Then there exists is a unique additive mapping $A: X \to Y$ and a unique quadratic mapping $Q: X \to Y$ which satisfying (1.4) and

$$N\left(f(x)-A(x)-Q(x),2r\right)$$
$$\geq \min\left\{N'_{AQ}\left(\alpha\left(\frac{x}{2}\right),\frac{r|2-d|}{3}\right),N'_{AQ}\left(\alpha\left(\frac{-x}{2}\right),\frac{r|2-d|}{3}\right),N'_{AQ}\left(\alpha\left(\frac{x}{2}\right),\frac{r|4-d|}{3}\right),N'_{AQ}\left(\alpha\left(\frac{-x}{2}\right),\frac{r|4-d|}{3}\right)\right\} \tag{5.2}$$

for all $x \in X$ and all $r>0$.

Proof. Let $f_A(x)=\dfrac{f(x)-f(-x)}{2}$ for all $x \in X$. Then $f_A(0)=0; f_A(-x)=-f_A(x)$ for all $x \in X$. By Theorem 3.1, we have

$$N\left(f_A(x)-A(x),r\right)\geq \min\left\{N'_{AQ}\left(\alpha\left(\frac{x}{2}\right),\frac{r|2-d|}{3}\right),N'_{AQ}\left(\alpha\left(\frac{-x}{2}\right),\frac{r|2-d|}{3}\right)\right\} \tag{5.3}$$

for all $x \in X$ and all $r>0$. Also, let $f_Q(x)=\dfrac{f(x)+f(-x)}{2}$ for all $x \in X$. Then $f_Q(0)=0; f_Q(-x)=f_Q(x)$ for all $x \in X$. By Theorem 4.1, we have

$$N\left(f_Q(x)-Q(x),r\right)\geq \min\left\{N'_{AQ}\left(\alpha\left(\frac{x}{2}\right),\frac{r|4-d|}{3}\right),N'_{AQ}\left(\alpha\left(\frac{-x}{2}\right),\frac{r|4-d|}{3}\right)\right\} \tag{5.4}$$

for all $x \in X$ and all $r>0$. Define $f(x)=f_A(x)+f_Q(x)$, it follows from (5.3) and (5.4), we arrive our desired result.

Corollary 5.2 Let g, p be constants. Suppose that a function $f: X \to Y$ satisfies the inequality

$$N\left(f(2x+y)-f(2x-y)-2[f(x+y)-f(x-y)]+f(y)\ -f(-y),r\right)\geq \begin{cases} N'\left(g,r\right), \\ N'\left(g\{|x|^p+|y|^p\},r\right), \end{cases} \tag{5.5}$$

for all $x, y \in X$ and all $r>0$. Then there exists a unique additive mapping $A: X \to Y$ and a unique quadratic mapping $Q: X \to Y$ such that

(5.6)

for all $x \in X$ and all $r>0$.

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Stability of A Functional Equation Orginating From Sum of First ℓ - Natural Numbers in (Beta;P) Banach Spaces

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ABSTRACT: *In this paper, the authors proved the generalized Ulam-Hyers stability of a functional equation which is originating from sum of first ℓ natural numbers in (beta;p) Banach spaces. An application of this functional equation is also given.*

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1. Introduction

One of the motivating problems in the theory of functional analysis regarding the stability problem of functional equations had been first raised by S.M. Ulam [23] as follows: "When is it true that a mapping satisfying a functional equation approximately must be close to an exact solution of the given functional equation"? For very general functional equations, the concept of stability for functional equations arises when we replace the functional equation by an inequality which acts as a perturbation of the equation. Thus the stability question of functional equations is that how do the solutions of the inequality differ from those of the given functional equation?

In 1941, D. H. Hyers [12] gave an positive answer to the question of S.M. Ulam for Banach spaces. In 1950, T. Aoki [2] and in 1978, Th.M. Rassias [17] broaden Hyers' Theorem by weakening the condition for the Cauchy difference controlled by $(\| x\|^p + \| y\|^p),\ p \in [0,1)$, to be unbounded. In 1982, J.M. Rassias [16] replaced the factor $\| x\|^p + \| y\|^p$ by $\| x\|^p \| y\|^q$ for $p,q \in R$. A generalization of all the above stability results was obtained by P. Gavruta [12] in 1994 by replacing the unbounded Cauchy difference by a general control function $\varphi(x,y)$.

In 2008, a special case of Gavruta's theorem for the unbounded Cauchy difference was obtained by Ravi et.al., [19] by considering the summation of both sum and product of two $p-$ norms.

M. Arunkumar, C. Leela Sabari [5] introduced and proved the solution and Hyers - Ulam - Rassias stability of the additive functional equation

$$f(x)+f(x)=f(2x) \tag{1.1}$$

which is originating from a chemical equation.

Infact, M. Arunkumar et. al., [6] investigated the generalized Ulam-Hyers stability of a functional equation

$$f(y)=\frac{f(y+z)+f(y-z)}{2} \tag{1.2}$$

which is originating from arithmetic mean of consecutive terms of an arithmetic progression using direct and fixed point methods.

Also, M. Arunkumar et. al., [7] has established the solution and generalized Ulam - Hyers - Rassias stability of a n dimensional additive functional equation

$$f(x)=\sum_{\ell=1}^{n}\left(\frac{f(x+\ell y_\ell)+f(x-\ell y_\ell)}{2\,\ell}\right) \tag{1.3}$$

where n is a positive integer, which is originating from arithmetic mean of n consecutive terms of an arithmetic progression.

The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [3,4,8,10,13,14,15,18]) and reference cited there in.

In this paper, the authors proved the generalized Ulam-Hyers stability of a functional equation which is originating from sum of first ℓ natural numbers of the form

$$f\left[\sum_{j=1}^{\ell} j\, x_j\right]=\sum_{j=1}^{\ell}\left[j\, f[x_j]\right], \qquad \ell \ge 1 \tag{1.4}$$

in (β, p) Banach spaces. An application of this functional equation is also given.

2 DEFINITIONS AND NOTATIONS ON QUASI-BETA NORMED SPACES

In this section, we present some basic definations about quasi-β-Normed spaces.

Definition 2.1 *Let* X *be a linear space over* $R \text{ or } C$. *A quasi-*β*-norm* $\|\cdot\|$ *is a real-valued function on* X *satisfying the following:*

$(QB1)$ $\|x\| \ge 0$ for all $x \in X$ and $\|x\| = 0$ if and only if $x = 0;$

$(QB2)$ $\|\lambda x\| = |\lambda|^{\beta}\|x\|$ for all $\lambda \in R \text{ or } C$ and all $x \in X,$ with $0<\beta\le 1;$

$(QB3)$ There is a constant $K \ge 1$ such that $\|x+y\| \le K\left(\|x\|+\|y\|\right)$ for all $x, y \in X$.

The pair $\left(X,\|\cdot\|\right)$ is called quasi-β-normed space if $\|\cdot\|$ is a quasi-β-norm on X. The smallest possible K is called the modulus of concavity of $\|\cdot\|$.

Definition 2.2 *A quasi-*β*-Banach space is a complete quasi-*β*-normed space.*

Definition 2.3 *A qusi-*β*-norm* $\|\cdot\|$ *is called a* (β, p) *- norm* $(0<p\le 1)$ *if* $\|x+y\|^p \le \|x\|^p + \|y\|^p$ for all $x, y \in X$. In this case, a quasi-β-Banach space is called a (β, p)-Banach space.

For stability results and concepts for quasi-normed spaces and p-Banach space, one can refer [9,20,22] and references cited there in.

Throughout this paper, let us take X be a linear space and Y be a (β, p) Banach space with $p-$norm $\|\cdot\|_Y$. Let K be the modulus of concavity of $\|\cdot\|_Y$.

3 STABILITY RESULTS

In this section the generalized Ulam – Hyers stability of the functional equation (1.4) in (β, p) Banach space is proved.

Theorem 3.1 Let $\alpha : X^{\ell} \to [0,\infty)$ be a function satisfying the condition

$$\lim_{t\to\infty} \frac{\alpha\left[\kappa^{ti}x_1, \kappa^{ti}x_2, \cdots, \kappa^{ti}x_\ell\right]}{\kappa^{ti}} = 0 \tag{3.1}$$

for all $x_1, x_2, x_3, \cdots, x_\ell \in X$ with $i \in \{-1,1\}$. Let $f : X \to Y$ be a function satisfying the inequality

$$\left\| f\left[\sum_{j=1}^{\ell} j\, x_j\right] - \sum_{j=1}^{\ell}\left[j\, f[x_j]\right]\right\|_Y \le \alpha\left[x_1, x_2, x_3, \cdots, x_\ell\right] \tag{3.2}$$

for all $x_1, x_2, x_3, \cdots, x_\ell \in X$. Then there exists a unique additive mapping $A : X \to Y$ satisfying the functional equation (1.4) and

$$\left\| f[x] - A[x]\right\|_Y^p \le \frac{K^{(t-1)p}}{\kappa^{\beta p}} \sum_{s=\frac{1-i}{2}}^{\infty} \frac{\alpha^p\left[\underbrace{\kappa^{si}x, \kappa^{si}x, \cdots, \kappa^{si}x}_{\ell-times}\right]}{\kappa^{sip}} \tag{3.3}$$

for all $x \in X$. The mapping $A[x]$ is defined by

$$A[x] = \lim_{t\to\infty} \frac{f[\kappa^{ti}x]}{\kappa^{ti}} \tag{3.4}$$

for all $x \in X$.

Proof. Replacing $[x_1, x_2, \cdots, x_\ell]$ by $[x, x, \cdots, x]$ in (3.2), we get

$$\left\| f\left[\left[1+2+\cdots+\ell\right]x\right] - \left[1+2+\cdots+\ell\right] f[x]\right\|_Y \le \alpha\left[\underbrace{x, x, \cdots, x}_{\ell-times}\right] = \alpha_\ell\left[x\right]$$

$$\left\| f\left[\frac{\ell(\ell+1)}{2}x\right] - \left[\frac{\ell(\ell+1)}{2}\right] f[x]\right\|_Y \le \alpha_\ell\left[x\right] \tag{3.5}$$

for all $x \in X$. Define $\kappa = \dfrac{\ell(\ell+1)}{2}$, the above inequality can be written as

$$\left\| f\left[\kappa\, x\right] - \kappa f[x]\right\|_Y \le \alpha_\ell\left[x\right] \tag{3.6}$$

for all $x \in X$. Using $(QB2)$ in (3.6) , we arrive

$$\left\| \frac{f[\kappa x]}{\kappa} - f[x]\right\|_Y \le \frac{\alpha_\ell\left[x\right]}{\kappa^{\beta}} \tag{3.7}$$

for all $x \in X$. Now replacing x by κx and dividing by κ in (3.7), we obtain

$$\left\| \frac{f[\kappa x]}{\kappa} - \frac{f[\kappa^2 x]}{\kappa^2} \right\|_Y \le \frac{\alpha_\ell[\kappa x]}{\kappa^\beta \kappa} \tag{3.8}$$

for all $x \in X$. Using (QB3) it follows from (3.7) and (3.8) that

$$\left\| f[x] - \frac{f[\kappa^2 x]}{\kappa^2} \right\|_Y \le \frac{K}{\kappa^\beta} \left\{ \alpha_\ell[x] + \frac{\alpha_\ell[\kappa x]}{\kappa} \right\} \tag{3.9}$$

for all $x \in X$. Proceeding further and using induction on a positive integer t, we get

$$\left\| f[x] - \frac{f[\kappa^t x]}{\kappa^t} \right\|_Y \le \frac{K^{t-1}}{\kappa^\beta} \sum_{s=0}^{t-1} \frac{\alpha_\ell[\kappa^s x]}{\kappa^s} \tag{3.10}$$

for all $x \in X$. If we replace x by $\kappa^l x$ and divide by κ^l in (3.10), for any $t, l > 0$, it is easy to verify that the sequence $\left\{ \frac{f[\kappa^t x]}{\kappa^t} \right\}$ is a Cauchy sequence. Since Y is complete, there exists a mapping $A : X \to Y$ such that $A[x] = \lim_{t\to\infty} \frac{f[\kappa^t x]}{\kappa^t}$ for all $x \in X$. Letting $t \to \infty$ in (3.10) we see that (3.3) holds for all $x \in X$.

To show that A satisfies (1.4), replacing $[x_1, x_2, \cdots, x_\ell]$ by $[\kappa^t x_1, \kappa^t x_2, \cdots, \kappa^t x_\ell]$ and dividing by κ^t in (3.2) and using the definition of $A[x]$, and then letting $t \to \infty$, we see that A satisfies (1.4) for all $x_1, x_2, \cdots, x_\ell \in X$. To prove that A is unique, let $B[x]$ be another additive mapping satisfying (1.11) and (3.3), then

$$\|A[x] - B[x]\|_Y \le \frac{K}{\kappa^{t\beta}} \left\{ \|A[\kappa^t x] - f[\kappa^t x]\|_Y + \|f[\kappa^t x] - B[\kappa^t x]\|_Y \right\} \le \frac{2K^t}{\kappa^\beta} \sum_{s=0}^{\infty} \frac{\alpha_\ell[\kappa^{t+s} x]}{\kappa^{\beta t+s}} \to 0 \ as\ s \to \infty$$

for all $x \in X$. Hence A is unique. This completes the proof for $i = 1$.

Also, replacing x by $\frac{x}{\kappa}$ in (3.6), we get

$$\left\| f[x] - \kappa f\left[\frac{x}{\kappa}\right] \right\|_Y \le \alpha_\ell[x] \tag{3.11}$$

for all $x \in X$. The rest of the proof is similar tracing to that of the case $i = 1$. For $i = -1$ also the theorem is true. This completes the proof of the theorem.

The following corollary is an immediate consequence of Theorem 3.1 concerning the stabilities of (1.4).

Corollary 3.2 *Let* λ *and* q *be nonnegative real numbers. Let a function* $f : X \to Y$ *satisfies the inequality*

$$\left\| f\left[\sum_{j=1}^{\ell} j\, x_j\right] - \sum_{j=1}^{\ell}\left[jf[x_j]\right]\right\|_Y \le \begin{cases} \lambda, & \\ \lambda\left\{\sum_{j=1}^{\ell} \| x_j \|_X^q\right\}, & q \ne 1; \\ \lambda \prod_{j=1}^{\ell} \| x_j \|_X^q, & \ell q \ne 1; \\ \lambda\left\{\prod_{j=1}^{\ell} \| x_j \|_X^q + \left\{\sum_{j=1}^{\ell} \| x_j \|_X^{\ell q}\right\}\right\}, & \ell q \ne 1; \end{cases} \tag{3.12}$$

for all $x_1, x_2, \cdots, x_\ell \in X$. Then there exists a unique additive function $A: X \to Y$ such that

$$\| f[x] - A[x] \|_Y^p \le \begin{cases} \frac{K^{(t-1)p}}{\kappa^{\beta p}} \frac{\kappa^p\, \lambda^p}{|\kappa - 1|^p}, \\ \frac{K^{(t-1)p}}{\kappa^{\beta p}} \frac{\ell^p \kappa^{\beta p q}\, \lambda^p \| x \|_X^{pq}}{|\kappa - \kappa^{\beta q}|^p}, \\ \frac{K^{(t-1)p}}{\kappa^{\beta p}} \frac{\kappa^{\beta p q}\, \lambda^p \| x \|_X^{\ell p q}}{|\kappa - \kappa^{\beta \ell q}|^p}, \\ \frac{K^{(t-1)p}}{\kappa^{\beta p}} \frac{(\ell+1)^p \kappa^{\beta p q}\, \lambda^p \| x \|_X^{\ell p q}}{|\kappa - \kappa^{\beta \ell q}|^p} \end{cases} \tag{3.13}$$

for all $x \in X$.

4. Application of The Functional Equation (1.4)

The functional equation (1.4)

$$f\left[\sum_{j=1}^{\ell} j\, x_j\right] = \sum_{j=1}^{\ell}\left[jf[x_j]\right], \qquad \ell \ge 1.$$

is used to find the sum of first ℓ natural numbers. Since $f(x) = x$ is the solution of the above functional equation, then it can be written as follows

$$\sum_{j=1}^{\ell} j\, x_j = \sum_{j=1}^{\ell} j x_j, \qquad \ell \ge 1.$$

Now, let us take the variables as consecutive terms, the above identity transforms as sum of first ℓ natural numbers. Mathematically, if we replace $(x_1, x_2, x_3, \cdots, x_\ell)$ by $(x, x, x, \cdots, x)$, we get

$$1x + 2x + 3x + 4x + \cdots + \ell x = \frac{\ell(\ell+1)}{2} x.$$

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An Heuristic approach for solving OSSP with the objective of minimizing the Total Completion Time

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ABSTRACT: *The Open Shop Scheduling Problem(OSSP) may be broadly defined as allocation of resources over a period of time in which the jobs can be processed in any order. OSSP as the origin in 1976 by Gonzles and Sahni(1976). In his paper he developed an algorithm for solving the 2 machine case. For more than 2 machine cases this problem wasn't solved till 2000. Atlast Jayakumar (2001) solved the problem for more than 2 job 2 machine cases and generalized for 'n' job 'm' machine cases with the objective of minimizing the Makespan. In our present work we have solved the general case that of 'n' job 'm' machine cases with the objective of minimizing the total completion time(TCT) using the heuristic approach.*

Keywords: *Open shop Scheduling problem, Heuristic, Total Completion Time*

1. Introduction

Scheduling as its origin in Manufacturing Industries in the early 1950's. It has been classified into three main categories namely, Flow Shop Scheduling Problem (FSSP), Job Shop Scheduling Problem (JSSP) and Open Shop Scheduling Problem(OSSP). In Flow Shop Scheduling Problem jobs can be processed through sequencing. In Job Shop Scheduling Problem Jobs can not only been sequencing but also routing is applicable for different jobs on different machines. In Open Shop Scheduling Jobs can be processed in any conceivable order i.e. there is no routing and sequencing. Whenever the machine is free job can be allocated if it is free and similarly whenever a job is free it can be allocated on any machine which is free. In this paper we focus our attention on OSSP alone with the objective of minimizing the total completion time.

Statement of the Problem

The founder of Open Shop Scheduling Problem (OSSP) is Gonzalez and Sahni during the year 1976. They had defined it in such a way that, if the jobs can be processed in any order it can be called as OSSP. In that paper they have solved the 2 job 2 machine problem with the objective of minimizing the Makespan. For more than 2 job 2 machine case there is no available literature is found till 2000 with the objective of minimizing the Makespan or Minimize the total idle time or minimizing the total completion time. Jayakumar(2001) has solved the problem only for the 2 job 2 machine case but also for the general case that of 'n' job 'm' machine cases. In this paper we are going to minimize the total completion time for more than 2 job 2 machine cases which is the extension of Jayakumar's previous work. We proposed three Heuristics algorithms and compared and found which algorithm is superior to the other two, when pre-emptions are not allowed.

Literature Review

For the single machine cases

The machine problem with the processing time of a job is defined by an increasing function of total normal processing time of job in front of it is the sequence by Ji-Bo Wang , Lin-Hui Sun , Lin-Yan Sun(2011). Later in the same year single machine scheduling problem with a learning effect with the objective of minimizing the total completion time by chinyao Low & Wen-Yi Lin(2011).

For the two machine cases

Gonzalez & Sahni(1976) started the open schedule shop solved the 2 job 2 machine case with the objective of minimize finish times.

For the general cases

For the general case that of 'n' job 'm' machine case Jayakumar (2001) solved the problem with the objective of minimizing makespan. Later on less attention is given by the researchers on OSSP with the objective of minimizing the Total Completion Time(TCT) which is the most important one for manufacturing Industries.

Objective

Choosing the appropriate objective for solving this type of problem is based on the demand and according to the needs of the society. Suppose let us consider the large automotive garage with specialized shops. A car comes for service may require the following works: replace exhaust pipeline and muffler, align wheels and tune up. According to the situation, theses three tasks may be carried out in any order. However, since the exhaust system, alignment and tune up shops are in different buildings, so it is not possible to perform two tasks simultaneously and the Objective is to minimize the Total Completion time. Instead of finding the Makespan, if we find the total completion time we will come to an conclusion that how many hours the machines becomes busy, can be found and how far the machines has been utilized in total can be calculated and its performance can be measured easily.

For the 2 job 2 machine case

Gonzalez and Sahni(1976) solved the Open Shop Scheduling Problem for the 2 job 2 machine case. For more than 2 job 2 machine case the problem is combinatorial in nature and complete enumeration is tedious. Hence for the general cases even though it is NP-hard in nature it can be solved by Heuristic Method which is the alternate way of solving it to get a reasonably good and better schedule and if possible we can go for benchmarking.

For the General case

Akturk, Ghosh and Gunes(2004) focused on the performance of the SPT list scheduling heuristic and provided theoretical worst-case bounds on it. L. Tang and D. Bai (2010) considered the OSSP to minimize TCT. They developed a Shortest Processing Time Block (SPTB) heuristic for the special case of the problem i.e., the job number is the multiple of the machine number and extend the heuristic to solve the general problems. They proved that the heuristic is asymptotically optimal when the job number goes to infinity. Scheduling Open Shops with parallel machines to minimize TCT was considered by Naderi et al(2011) They developed an effective MILP model and apply memetic algorithm to solve the problem.
Jayakumar(2001) solved the OSSP with the objective of minimizing the Makespan for general cases i.e. 'n' job 'm' machine cases. The Literature Review reveals that the number of scholars who works in this area is the need of the hour and it is a mixer of both discrete maths and Operations Research and called sometimes combinatorial optimization problems. so we focused our attention with the objective of Minimizing the Total Completion Time for the 'n' job 'm' machine cases, since it is needed for the Industrial as well as combinatorial problems. We have made one such attempt in this paper and identified that the proposed algorithm 'A' performs better than the other two algorithms found in this Paper.

Model Building

We have developed three algorithms with priority rules using MWP, Longest Processing Time (LPT), Shortest Processing Time (SPT) and Randomly chosen among the available times. In order to build the model we use the following Assumptions:

1. Jobs and machine are continuously available for processing.
2. Pre-emption not allowed.
3. All jobs are ready at time Zero.
4. Processing times are arbitrary.
5. If there is a tie, choose arbitrarily.
6. There is no routing.
7. There is no pre defined sequencing.

Proposed Algorithm 'A'

MWP(Maximum Work Pending): select the operation associated with the job having the most work pending to be processed.
LPT(Longest Processing Time): select the operation with the maximum processing time.

RAN(Random): If there is a tie, Select the operation at random. At each stage, it is necessary to identify the operation in 'S_K' and to keep track of the times at which the machines are available for processing. MWP_{ij} is the work pending of the job associated with operation (i,j).

Heuristic Schedule Generation:

Step 1: let t=0 and assume P_k= (Empty), S_k= (All operation).

Step 2: Examine P^*=min(P_{ij}),(i,j) in S_K as explained and the corresponding operation for which P^* could be released. If 'P^*' occurs only for operation in S_K . Then add that operation to 'P_k'and create the next partial schedule P_{k+1} otherwise go to step 3.

Step 3: Among the operation in S_K for which 'P_{ij}' is equal to P^*, identify an operation according to the order of priorities as given in the earlier section and add this operation to 'P_k'as early as possible , thus creating only on partial schedule 'P_{k+1}' for the next stage .

Step 4: For each new partial schedule 'P_{k+1}' created in step 3, update the data set as follows: From 'S_{k+1}' by deleting operation (i,j) from S_K . Increment 't' by one.

Step 5: Repeat from step 2 to step 4 for each P_{k+1} created in step 3 and continue in this manner until all the operations are added in to 'S_K' .

Proposed Algorithm 'B'

Priority Rules:

MWP (Maximum Work Pending): select the operation associated with the job having the most work pending to be processed.

SPT (Shortest processing time): select the operation with the shortest processing time.

RAN(Random): If there is a tie, select the operation at random . At each stage, it is necessary to identify the operation in 'S_K' and to keep track of the times at which the machines are available for processing. MWP_{ij} is the work pending of the job associated with operation (i,j).

Heuristic Schedule Generation:

Step 1: let t=0 and assume P_k= (empty), S_k= {All operation}.

Step 2: Determine P^*=min(P_{ij}),(i,j) in S_K as explained and the corresponding operation for which P^* could be released. If 'P^*' occurs only for operation in S_K . Then add that operation to 'P_k'and create the next partial schedule P_{k+1} otherwise go to step 3.

Step 3: Among the operation in S_K for which 'P_{ij}' is equal to P^*, identify an operation according to the order of priorities as given in the earlier and add this operation to 'P_k'as early as possible , thus creating only on partial schedule 'P_{k+1}' for the next stage .

Step 4: For each new partial schedule 'P_{k+1}' created in step 3, update the data set as follows: From 'S_{k+1}' by deleting operation (i,j) from S_K . Increment 't' by one.

Step 5: P_{ij}=0 from step 2 to step 4 for each P_{k+1} created in step 3 and finding in this manner until all the operations are added in to 'S_K' .

Proposed Algorithm 'C'

Priority Rules

MWP (Maximum Work Pending): select the operation associated with the job having the most work pending to be processed.

RAN(Random): Select the operation at random. At each stage, it is necessary to identify the operation in 'S_K' and to keep track of the times at which the selections are available for processing. MWP_{ij} is the work pending of the job associated with operation (i,j).

Heuristic Schedule Generation:

Step 1: let t=0 and assume P_k= (empty), S_k= {All operation}.

Step 2: Determine P^*=min(P_{ij}),(i,j) in S_K as explained and the corresponding operation for which P^* could be released. If 'P^*' occurs only for operation in S_K . Then add that operation to 'P_k'and create the next partial schedule P_{k+1} otherwise go to step 3.

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Step 3: Among the operation in S_K for which 'P_{ij}' is equal to P^*, identify an operation according to the order of priorities as given in the earlier and add this operation to 'P_k'as early as possible , thus creating only on partial schedule 'P_{k+1}' for the next stage .

Step 4: For each new partial schedule 'P_{k+1}' created in step 3, update the data set as follows: From 'S_{k+1}' by deleting operation (i,j) from S_K . Increment 't' by one.

Step 5: Repeat from step 2 to step 4 for each P_{k+1} created in step 3 and continue in this manner until all the operations are added in to 'S_K' .

Three algorithm have been developed to achieve the objective that of minimizing the total completion time. For more than two machines with arbitrary processing time, the investigation is made between the three algorithm presented above and it as been found that the algorithm 'A' performs better than the other two in finding total completion time. The computational result indicates that for the square matrix instances(number of jobs equal to the number of machines) as well as the rectangular matrix instances(the number of jobs greater than the number of machines) the proposed Algorithm 'A' performs better than the other two. Whereas the number of jobs less than the number of machine instances the proposed algorithm 'B' performs better than the other two.

Illustration

Consider the three jobs, three machine case as illustrated in the table. The numbers given in the table are processing time (in hours) requirements for each job on each machine.

	Machine 1	Machine 2	Machine 3
Job 1	2	4	5
Job 2	3	6	4
Job 3	5	2	5

Using the proposed Algorithm 'A' The Total Completion Time is 39 hours, whereas using the proposed Algorithm 'B' The Total Completion Time is 41 hours and using the proposed Algorithm 'C' The Total Completion Time is 45 hours. The Gantt chart related to the Total Completion Time (TCT) is given in the Appendix.

Conclusion

As far as the Open Shop Scheduling Problem with the objective of minimizing the Total Completion Time is concern the performance of the Proposed Algorithm 'A' is superior to the other two for not only the 3 job 3 machine case but also for the 'n' job 'm' machine cases in general.

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Cohomological aspects of Product of Several n - Spheres

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ABSTRACT: *In this article I shall review the product of several n-spheres in transformation groups to study the Cohomological aspects of several n-spheres and its roll of play in various transformation groups and shall also obtain some new results in this article.*

Keywords: *Transformation groups, Product of spheres, product of two spheres and its roll.*

1. Introduction

Throughout our discussion of transformation groups via cohomology theory, we adopt the following notations and assumptions. p stands for either prime or zero. G is the cyclic group of order p where $p \neq 0$. For $p = 0$, G is the circle group. K is the prime field whose characteristic is p. Some times K is the coefficient domain for cohomology. X is a G - space that is to say X is a space on which G acts. F stands for fixed point of G on X. We use Sheaf theoretic cohomology in the cohomological theory.

We say that a space is locally paracompact if it is Hausdorff and point has a closed paracompact neighbourhood. Such a space has a paracompactifying family ϕ with $E(\phi) = X$. That is to say the set of all closed K such that K has a closed paracompact neighbourhood in X is such a family.

Since any type of fixed point set can occur if X is a Hilbert space therefore our space X assumes that X is paracompact and $\dim_K X < \infty$ or X is compact. It is easy to observe that the space X is paracompact with $\dim_K X$ is finite is much easier to deal with than X is compact and it is enough for most applications. If X is infinite dimensional, certain results if X is compact have proven to be of value in the theory of compact semigruop.

We also assume that if X has certain type of cohomology then so does F. We use the following notations. $X \sim Y$ (X is homeomorphic to Y) to mean $H^*(X,K) \approx H^*(Y,K)$ (isomorphic as a ring). The simplest type of space is an acyclic space that is a point. $X \sim$ point $\Rightarrow F \sim$ point. The next simplest space is a sphere and $X \sim S^n \Rightarrow F \sim S^r$ for some $r \leq n$. Note that the coefficients used and the restrictions on the group acting are vital for these results.

Product of spheres

The next simplest space, as far as the cohomology ring structure is concerned is a space whose ring is generated by one element and hence is a truncated polynonimal ring. These include the real, complex and quaternionic projective spaces $(P^n, \mathbb{C}P^n and\ \mathbb{H}P^n)$ and the Cayley projective plane as well as other more exotic spaces. The first theorem regarding these is also due to Smith [35] who proved that $X = P^n \Rightarrow F \sim P^n + P^{n-r-1}$ or $F = \phi$ for the case $p = 2$, '+' denotes the disjoint union. The method used was to lift the action to a $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ action on S^n and study the fixed points of elements of this group. J.C.Su used the same method to obtain the result for more general $X \sim P^n (p = 2)$ and also for $p = 0$ and $X \sim \mathbb{C}P^n$.

We consider the case $X \sim S^n \times S^m$. This is the case in which fixed point sets can occur. This is the first case is which X need not be totally non homologous to zero in X_G when $F \neq \phi$. There are two possibilities one is G may act nontrivially on $H^n(X)$ if $n = m$, the another one is G may act trivially but the spectral sequence may be non trivial. In this we assume X to be totally non homologous to zero in X_G. The following cases are the only possibilities for F :

1. $F \sim S^q \times S^r$,
2. $H^*(F)$ generated by u, v in degree q with $u^2 \neq 0$, $v^2 \neq 0$, and $uv = 0$,
3. $F \sim p^3(q)$,
4. $F \sim pt + p^2(q)$,
5. $F \sim S^q + S^r$ (q and / or r can be zero).

In this section our main result is the following.

Theorem: 1.1
Let $p = 2$ and $X \sim S^n \times S^m$ with $n \leq m$. Suppose $F \sim S^r + S^q$ with $n < r < q$. Then $q \leq m$. Moreover if $2^k < n$ or if $2^k = n$ and $S_q^{2k} : H^m(X) \longrightarrow H^{m+n}(X)$ is trivial.

Theorem: 1.2
Let T be the involution on $X \sim S^n \times S^n$ such that $T^* \neq I$ on $H^n(X, \mathbb{Z}_2)$. Then $F \sim S^n$ and the restriction $H^n(X) \longrightarrow H^n(F)$ is onto. Moreover if $X = S^n \times S^n$, $n \neq 1, 3, 7$ and if $\pi : X \longrightarrow S^n$ is the projection on either factor then $(\pi / F)^* : H^n(S^n) \longrightarrow H^n(F)$ is an isomorphism

Products of several n-spheres
In this section we study involutions on spaces $X \sim S^n \times S^n \times \ldots \times S^n$ (k factors). (Thus $p = 2$ and coefficients are in $\mathbb{Z}_2$). If $H^i(F) = 0$ for $0 < i < n$ we shall obtain quite complete information. The case in which $X = S^1 \times \ldots \times S^1$ was studied by Conner in by the use of covering space methods. Conner's method generalizes to the case in which X is a finite dimensional $K(\pi, 1)$ -space. Cohomological methods are not applicable to this generalization to $K(\pi, 1)$ -spaces but, on the other hand, the covering space method is not available when $n > 1$. Our first result shows that $\dim H^*(F)$ depends only on the action of T^* on $H^n(X)$ (where T is the given involution).

Theorem: 1.3
Let T be an involution on $X \sim S^n \times \ldots \times S^n$ (k -times). Let r be the dimension of the subspace of $H^n(X; \mathbb{Z}_2)$ consisting of the elements fixed by T^*. Then if $F \neq \phi$ we have $\dim H^n(X; \mathbb{Z}_2) = 2^r$ and the spectral sequence of $\pi : X_G \longrightarrow B_G$ degenerates (i.e., all differentials are zero).
Before giving the proof we need some elementary algebraic lemmas.

Lemma: 1.4

Let V be a $\mathbb{Z}_2$-vector space and let $T:V\longrightarrow V$ be an automorphism of period two. Put $\tau=1-T=1+T$. Let $x_1,\dots,x_p,z_1,\dots,z_q$ be a basis of $\ker\tau$ such that $z_1,\dots,z_q$ is a basis of $\operatorname{Im}\tau$. Write $z_i=y_i+Ty_i$ in any way. Then $x_1,\dots,x_p,y_1,Ty_1,\dots,y_q,Ty_q$ is a basis of V.

The proof is accomplished by showing inductively that $x_1,\dots,x_p,y_1,Ty_1,\dots,y_r,Ty_r$ are linearly independent and then noting that $\dim V=\dim\ker\tau+\dim\operatorname{Im}\tau$.

Corollary: 1.5

With the notation of 1.4 we can write V as a direct sum $V=U\oplus TU\oplus W$ where $W\subset\ker\tau$.

Corollary: 1.6

With the notation of 1.4 suppose we are given a set of vectors which span V, is invariant (as a set) under T, and contains vectors of $\ker\tau$ which map to a spanning set in $\ker\tau/\operatorname{Im}\tau$. Then from this set we may extract vectors $x_1,\dots,x_p,y_1,\dots,y_q$ such that $Tx_i=x_i$ and $x_1,\dots,x_p,y_1,\dots,y_q,Ty_1,\dots,Ty_q$ from a basis of V.

We shall now prove theorem 1.3 By 1.5 we can write $H^j(X)=U_j\oplus T^*U_j\oplus W_j$ for any given j. Then $H^*(B_G;\mathrm{H}^j(X))$ is the cohomology of $G=\mathbb{Z}_2$ with coefficient in the G-module $H^j(X)$. Thus in the spectral sequence of $X_G\longrightarrow B_G$ we have $E_2^{i,j}=H^i\left(\mathbb{Z}_2;(U_j\oplus T^*U_j)\oplus W_j\right)=H^i(\mathbb{Z}_2;U_j\oplus T^*U_j)\oplus H^i(\mathbb{Z}_2;W_j)\longrightarrow(1)$. For $i>0$ the term $H^i(\mathbb{Z}_2;U_j\oplus T^*U_j)$ vanishes while the term $H^i(\mathbb{Z}_2;W_j)$ is isomorphic to W_j. For $i=0$ the first term in (1) is the diagonal $\{(u,T^*u):u\in U_j\}$ and will have no importance in out discussion. Note that operation by $t\in H^1(B_G)=H^1(\mathbb{Z}_2;\mathbb{Z}_2)$ annihilates the first summand of (1) and is an isomorphism $H^i(\mathbb{Z}_2;W_j)\longrightarrow H^{i+1}(\mathbb{Z}_2;W_j)$ on the second for $i\geq 0$.

Now we can find elements $a_1,\dots,a_\mu,b_1,\dots,b_\nu$ of $H^n(X)$ such that $T^*a_i=a_i$ and the a_i,b_i and T^*b_i from a basis of $H^n(X)$. Note that $r=\mu+\nu$ in 1.3

Let $x\in F$, assuming $F\neq\phi$. Then $1\otimes a_i\in E_2^{0,n}$ is transgressive and must transgress to zero. Thus there are elements $\alpha_i\in H^n(X_G)$ representing a_i and with $\eta_x(\alpha_i)=0$. We also put $\beta_i=\mathbb{N}(b_i)\in H^{2n}(X_G)$ which represents $b_i\cup T^*b_i$. Note that both $1\otimes a_i$ and $1\otimes(b_i\cup T^*b_i)$ are permanent cocycles in the spectral sequence. Now for any integer j we see from 4.4.4 that the subspace W_j of (1) may, and shall, be assumed to have, as a basis, the elements which are cup products of several a_i with several $(b_i\cup T^*b_i)$. It follows that the summand $H^0(Z_2;W_j)$ of $E_2^{0,j}$ consists of permanent cocycles. The other summand of $E_2^{0,j}$ also consists of permanent cocycles by an easy inductive argument on j recalling that t annihilates this summand.

Thus we have shown that the spectral sequence degenerates. It follows that $\dim H^*(F)=\dim H^N(X_G)=\sum_j\dim E_2^{N-j,j}$ for N large, which is just $\sum_j\dim W_j$. For $j=\ell n$ the dimension of W_j is $\sum_i \binom{\nu}{i}\binom{\mu}{\ell-2i}$ which is the coefficient of x^{ℓ} in $(1+x^2)^{\nu}(1+x)^{\mu}$. Summing over ℓ we obtain $\dim H^*(F) = 2^{\nu}2^{\mu} = 2^r$ as claimed.

If we assume that $H^i(F;\mathbb{Z}_2)=0$ for $0<i<n$ we obtain the following complete information about the cohomology structure over $\mathbb{Z}_2$ of F :

Theorem:1.7

With the hypotheses of 4.4.1 assume further that $H^i(F,\mathbb{Z}_2)=0$ for $0<i<n$. Then F consists of 2^{ℓ} components each of which has the $\bmod 2$ cohomology ring of $S^n \times \ldots \times S^n\left((r-\ell)-times\right)$ for some $0 \le \ell \le \mu$. Here $\mu = \dim(\ker\tau / \operatorname{Im}\tau)$, where $\tau = 1-T^*$ on $H^n(X;\mathbb{Z}_2)$.

If n is odd, then $r-\ell$ is the dimension of the image in $H^n(X;\mathbb{Z}_2)$ of the subgroup of $H^n(X;\mathbb{Z}_4)$ consisting of the elements invariant under T^* on $H^n(X;\mathbb{Z}_4)$.

Proof

We shall use the notation and facts established during the proof of 1.3 Let W $=\oplus$ W$^j \subset H^*(X)$ and let W $=\oplus$ W$^j \subset H^*(X_G)$ be the cohomology extension of W which has as a $\mathbb{Z}_2$-basis the monomials in the α_i and $\mathbb{Z}(b_i)$ (including $1\in H^0(X_G)$). It is clear that the map

$$H^*(B_G)\otimes \mathrm{W} \longrightarrow H^*(X_G) \longrightarrow (2)$$

is monomorphism and is an isomorphism in high degrees. Moreover, j^* must be a monomorphism on the image of (2) to $H^*(F_G)$. Thus the situation is similar to the case in which X is totally non-homologous to zero with the exception that now W is an extension of only part of $H^*(X)$ and that (2) is not onto in low degrees. Note that the homogeneous elements of W of positive degree (i.e., except for $1\in$ W) are all in $\ker\eta_x$.

Now suppose that $\alpha \in$ W$^n \subset H^*(X_G)$ (i.e., α is a linear combination of the α_i). Consider α^2 and note that since $\eta_x(\alpha)=0$ and $i^*(\alpha)^2=0$ there are only two possibilities: $\alpha^2=0$ or $\alpha^2=t^n\gamma$ for some $\gamma\in H^*(X_G)$. If $\alpha^2=t^n\gamma$ we must have that $\eta_x(\gamma)=0$ and, moreover, we can alter γ by an element annihilated by t and hence we can assume that $\gamma\in$ W^n. Now suppose, in the case $\alpha^2=t^n\gamma$, that

$$\left\{\frac{j^*(\alpha)=1\otimes a+t^n\otimes c}{j^*(\gamma)=1\otimes d+t^n\otimes c'}\right\}$$

where $a,d\in H^n(F)$ and $c,c'\in H^0(F)$. Then we have $j^*(\alpha^2)=t^{2n}\otimes c^2=t^{2n}\otimes c$ (since $a^2=i^*(\alpha)^2 \mid F=0$). Thus we have $d=0$ and $c=c'$. Then

$$\left\{\frac{j^*(\alpha-\gamma)=1\otimes a}{j^*(\gamma)=t^n\otimes c}\right\}$$

Since $a^2 = 0$ we have $j^*\left(Sq^n(\alpha-\gamma)\right) = 1\otimes a^2 = 0$ so that $\alpha-\gamma \in \ker Sq^n$ and clearly $\gamma \notin \ker Sq^n$. It follows that we may alter the given basis $\{a_i\}$ of W_n so that for some integer where and $\ell \le \mu$,

$$j^*(\alpha_i) = \left\{\frac{t^n\otimes c_i \quad for \quad 1\le i\le \ell}{1\otimes a_i' \quad for \quad \ell\le i\le\mu}\right\} \longrightarrow (3)$$

where $a_i' = a_i \mid F \in H^n(F)$ and $c_i \in H^0(F)$. Also note that

$$\alpha_i^2 = \left\{\frac{t^n\alpha_i \quad for \quad 1\le i\le \ell}{0 \quad for \quad \ell\le i\le\mu}\right\} \longrightarrow (4)$$

Of course we also have,

$$j^*(\beta_i) = j^*\left(\square\,(b_i)\right)t^n \otimes b_i' \longrightarrow (5)$$

where $b_i' = b_i \mid F \in H^n(F)$

It follows from (3), (5) and the fact that W generates $H^*(X_G)$ over $H^*(B_G)$, that $1, c_1, \ldots, c_\ell$ generate $H^0(F)$ as an algebra.(Presently we shall see that they are an algebra basis). It also follows that for any component F_0 of F the $a_i \mid F_0 (i > \ell)$ and $b_i \mid F_0$ and 1 generate $H^*(F_0)$ as an algebra. Moreover, $a_i^2 = 0 = b_i^2$ so that all squares of elements of $H^*(F_0)$ of positive degree vanish.

Now let $c \in H^0(F)$ be the element which distinguishes F_0 (i.e., $c = 1$ on F_0 and $c = 0$ on other components). Note that F_0 is an arbitrary component and need not contain the base point x. We can write $c = P(c_1, \ldots, c_\ell)$ a square-free polynomial in the c_i. Define γ to be that element of $H^*(X_G)$ obtained by replacing c_i by α_i in P and multiplying each monomial by an appropriate power of t to bring the total degree up to ℓn. Then clearly $j^*(\gamma) = t^{\ell n}\otimes c$.

Also let $\alpha = \alpha_{\ell+1}\cup\ldots\cup\alpha_\mu$ so that $j^*(\alpha) = 1\otimes a'$ where $a' = a_{\ell+1}\cup\ldots\cup a_\mu \mid F$. Finally put $\beta = \beta_1\cup\ldots\cup\beta_\nu$ so that $j^*(\beta) = t^{n\nu}\otimes b'$ where $b' = (b_1\cup\ldots\cup b_\nu \mid F$. Now it is clear that $\gamma\alpha\beta \ne 0$ since it is some power of t times an element of $H^*(X_G)$ which represents in $H^*(X)$ an element which is a monomial in the a_i and $(b_i \cup T^* b_i)$ with no repetitions. Moreover $\gamma\alpha\beta$ is in the image of $H^*(B_G)\otimes$ W $\longrightarrow H^*(X_G)$ so that $j^*(\gamma\alpha\beta) \ne 0$. But $j^*(\gamma\alpha\beta) = t^{n(\ell+\nu)}\otimes ca'b'$ and $ca'b'$ is just $a'b' \mid F_0$, that is $0 \ne ca'b' = a_{\ell+1}\cup\ldots\cup a_\mu \cup b_1\cup\ldots\cup b_\nu \mid F_0$ Thus, for any component F_0 of F, $H^*(F_0)$ is the exterior algebra on the restrictions to F_0 of $a_{\ell+1}, \ldots, a_\mu, b_1, \ldots, b_\nu$ so that $F_0 \square\, S^n\times\ldots\times S^n (r-\ell\ factors)$.Since $\dim H^*(F) = 2^r$, F must have 2^ℓ components. (Note that is follows that 1 and the square-free monomials in the c_i are a basis for $H^0(F)$).

It remains to prove the last statement of the theorem. To do this, one notes that the cohomology sequence associated with $0 \to \mathbb{Z}_2 \to \mathbb{Z}_4 \to \mathbb{Z}_2 \to 0$ shows that $H^i(X;\mathbb{Z}_4) = 0$ for $0 < i < n$. Thus the spectral sequences $\bmod 2$ and $\bmod 4$ give rise to the following diagram.

The columns are split exact and the second row is exact. A diagram chase shows that i^* maps $\ker \eta_x \cap \ker Sq^1$ isomorphically onto $\operatorname{Im}\rho$. Now to find $\operatorname{Im}\rho$ we note first that $b_i + T^* b_i$ is in $\operatorname{Im}\rho$ since $H^n(X;Z_4) \longrightarrow H^n(X;Z_2)$ is onto (Sq^1 being zero). Let $a = \sum \lambda_i a_i \in W_n$ and put $\alpha = \sum \lambda_i \alpha_i$. Then $a \in \operatorname{Im}\rho$ iff $Sq^1(\alpha) = 0$ which holds iff $Sq^1\left(j^*(\alpha)\right) = 0$. But $Sq^1\left(j^*(\alpha)\right) = t^{n+1} \otimes \sum_{i=1}^{\ell} \lambda_i c_i$ since n is odd, and this is zero iff a is in the span of $a_{\ell+1}, \ldots, a_\mu$ since the c_i are independent. Thus $a_{\ell+1}, \ldots, a_\mu, b_1 + T^* b_1, \ldots, b_\nu + T^* b_\nu$ form a basis of $\operatorname{Im}\rho$ whence $\dim \operatorname{Im}\rho = \mu + \nu - \ell = r - \ell$ as claimed.

$$\begin{tikzcd}
H^n(B_G;Z_4) \arrow[r, "o"] \arrow[d, shift right, "\pi^!"'] & H^n(B_G;Z_2) \arrow[d, shift right, "\pi^!"'] & \\
H^n(X_G;Z_4) \arrow[r] \arrow[u, shift right, "\eta_!"'] \arrow[d] & H^n(X_G;Z_2) \arrow[r, "Sq"] \arrow[u, shift right, "\eta_x"'] \arrow[d, "i^*"] & H^{n+1}(X_G;Z_2) \\
H^n(X;Z_4)^G \arrow[r, "\rho"] & H^n(X;Z_2)^G &
\end{tikzcd}$$

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Intuitionistic Fuzzy Stability of Mixed Undecic- Dodecic Functional Equation: A Fixed Point Approach

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ABSTRACT: *In this study, we determine the stability of a generalized Hyers-Ulam type theorem concerning the single variable mixed undecic- dodecic functional equation in the framework of intuitionistic fuzzy normed spaces through the fixed-point alternative.*

Keywords: *Intuitionistic Fuzzy Banach Space, Intuitionistic Fuzzy Normed Space*

1. Introduction

The study of stability problems for functional equations is connected to a question of Ulam [25] concerning the stability of group homomorphisms and positively answered for Banach spaces by Hyers [8]. It was further generalized and outstanding results was obtained by number of authors see ([9, 12, 17, 19, 20]). During the last seven decades, the above problem was tackled by numerous authors and its solutions via various forms of functional equations were discussed one can refer [7, 10, 14, 18, 26, 27, 29] and references cited there in.

Recently M. Arunkumar and P. Agilan introduced and discussed the generalized Ulam - Hyers stability simple mixed quintic - sextic, septic - octic, nonic - decic functional equations of the form

$$\vartheta(2p) = 48\vartheta(p) + 16\vartheta(-p) \tag{1}$$

$$\eta(2q) = 192\eta(q) + 64\eta(-q) \tag{2}$$

$$\psi(2r) = 768\psi(r) + 256\psi(-r) \tag{3}$$

in Banach space.

In this paper, the authors introduce and establish the stability of single variable mixed undecic - dodecic functional equation

$$g(5w) = 146,484,375g(w) + 97,656,250g(-w) \tag{4}$$

in intuitionistic fuzzy Banach spaces using fixed point method. The above equation having solution

$$g(w) = c_1\, x^{11} + c_2\, x^{12} \tag{5}$$

Lemma 1.1 *An odd function* $g : X \to Y$ *satisfies the functional equation (4) then*

$$g(5w) = 48,828,125g(w) = 5^{11}\, g(w)$$

for all $x \in X$

Lemma 1.2 *An even function* $g : X \to Y$ *satisfies the functional equation (4) then*

$$g(5w) = 244,140,625g(w) = 5^{12}\, g(w)$$

for all $x \in X$

2 Definitions and Notations of Intuitionistic Fuzzy Banach Space

Now, we recall the basic definitions and notations in the setting of intuitionistic fuzzy normed space are cited in [16, 22, 23, 24].

Definition 2.1 *A binary operation* $*:[0,1]\times[0,1]\to[0,1]$ *is said to be continuous* t *-norm if* $*$ *satisfies the following conditions:*

1. $*$ is commutative and associative;
2. $*$ is continuous;
3. $a*1=a$ for all $a\in[0,1]$;
4. $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Definition 2.2 *A binary operation* $\Diamond:[0,1]\times[0,1]\to[0,1]$ *is said to be continuous* t *-conorm if* $\Diamond$ *satisfies the following conditions:*

1. $\Diamond$ is commutative and associative;
2. $\Diamond$ is continuous;
3. $a\Diamond 0=a$ for all $a\in[0,1]$;
4. $a\Diamond b\le c\Diamond d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Using the notions of continuous t -norm and t -conorm, Saadati and Park [22] introduced the concept of intuitionistic fuzzy normed space as follows:

Definition 2.3 *The five-tuple* $(X,\mu,\nu,*,\Diamond)$ *is said to be an intuitionistic fuzzy normed space (for short, IFNS) if* X *is a vector space,* $*$ *is a continuous* t *-norm,* $\Diamond$ *is a continuous* $t-$ *conorm, and* μ,ν *are fuzzy sets on* $X\times(0,\infty)$ *satisfying the following conditions. For every* $x,y\in X$ *and* $s,t>0$

- $\mu(x,t)+\nu(x,t)\le 1$,
- $\mu(x,t)>0$,
- $\mu(x,t)=1$, if and only if $x=0$.
- $\mu(\alpha x,t)=\mu\left(x,\dfrac{t}{\alpha}\right)$ for each $\alpha\ne 0$,
- $\mu(x,t)*\mu(y,s)\le\mu(x+y,t+s)$,
- $\mu(x,\cdot):(0,\infty)\to[0,1]$ is continuous,
- $\lim_{t\to\infty}\mu(x,t)=1$ and $\lim_{t\to 0}\mu(x,t)=0$,
- $\nu(x,t)<1$,
- $\nu(x,t)=0$, if and only if $x=0$.

- $\nu(\alpha x,t)=\nu\left(x,\frac{t}{\alpha}\right)$ for each $\alpha\neq 0$,
- $\nu(x,t)\Diamond\nu(y,s)\geq\nu(x+y,t+s)$,
- $\nu(x,\cdot):(0,\infty)\to[0,1]$ is continuous,
- $\lim\limits_{t\to\infty}\nu(x,t)=0$ and $\lim\limits_{t\to 0}\nu(x,t)=1$.

In this case, (μ,ν) is called an intuitionistic fuzzy norm.

Example 2.4 *Let* $(X,\|\cdot\|)$ *be a normed space. Let* $a*b=ab$ *and* $a\Diamond b=\min\{a+b,1\}$ *for all* $a,b\in[0,1]$*. For all* $x\in X$ *and every* $t>0$*, consider*

$$\mu(x,t)=\begin{cases}\dfrac{t}{t+\|x\|} & if \quad t>0;\\ 0 & if \quad t\leq 0;\end{cases}$$

and

$$\nu(x,t)=\begin{cases}\dfrac{\|x\|}{t+\|x\|} & if \quad t>0;\\ 0 & if \quad t\leq 0.\end{cases}$$

Then $(X,\mu,\nu,*,\Diamond)$ is an IFN-space.

The concepts of convergence and Cauchy sequences in an intuitionistic fuzzy normed space are investigated in [22].

Definition 2.5 *Let* $(X,\mu,\nu,*,\Diamond)$ *be an IFNS. Then, a sequence* $x=\{x_k\}$ *is said to be intuitionistic fuzzy convergent to a point* $L\in X$ *if*

$$\lim \quad \mu(x_k-L,t)=1 \quad and \quad \lim \quad \nu(x_k-L,t)=0$$

for all $t>0$. In this case, we write

$$x_k\overset{IF}{\to}L \qquad as \qquad k\to\infty$$

Definition 2.6 *Let* $(X,\mu,\nu,*,\Diamond)$ *be an IFN-space. Then,* $x=\{x_k\}$ *is said to be intuitionistic fuzzy Cauchy sequence if*

$$\mu\left(x_{k+p}-x_k,t\right)=1 \qquad and \qquad \nu\left(x_{k+p}-x_k,t\right)=0$$

for all $t>0$, and $p=1,2\cdots$.

Definition 2.7 *Let* $(X,\mu,\nu,*,\Diamond)$ *be an IFN-space. Then* $(X,\mu,\nu,*,\Diamond)$ *is said to be complete if every intuitionistic fuzzy Cauchy sequence in* $(X,\mu,\nu,*,\Diamond)$ *is intuitionistic fuzzy convergent* $(X,\mu,\nu,*,\Diamond)$.

3. Fundamental results in fixed point theory

Theorem 3.1 *(Banach's contraction principle) Let* (X,d) *be a complete metric space and consider a mapping* $T:X\to X$ *which is strictly contractive mapping, that is*

1. $d(Tx,Ty)\le Ld(x,y)$ for some (Lipschitz constant) $L<1$. Then,

(i) The mapping T has one and only fixed point $x^*=T(x^*);$

(ii)The fixed point for each given element x^* is globally attractive, that is

2. $lim_{n\to\infty}T^n x=x^*,$ for any starting point $x\in X$;

(iii) One has the following estimation inequalities:

3. $d(T^n x,x^*)\le\dfrac{1}{1-L}\ d(T^n x,T^{n+1}x),\forall\ n\ge 0,\forall\ x\in X;$

4. $d(x,x^*)\le\dfrac{1}{1-L}\ d(x,x^*),\forall\ x\in X.$

Theorem 3.2 *Suppose that for a complete generalized metric space* (Ω,δ) *and a strictly contractive mapping* $T:\Omega\to\Omega$ *with Lipschitz constant* L. *Then, for each given* $x\in\Omega$ *, either*

$$d(T^n x,T^{n+1}x)=\infty \qquad \forall \qquad n\ge 0,$$

or there exists a natural number n_0 such that

(FP1) $d(T^n x,T^{n+1}x)<\infty$ for all $n\ge n_0$;

(FP2) The sequence $(T^n x)$ is convergent to a fixed to a fixed point y^* of T ;

(FP3) y^* is the unique fixed point of T in the set $\Delta=\{y\in\Omega: d(T^{n_0}x,y)<\infty\};$

(FP4) $d(y^*,y)\le\dfrac{1}{1-L}d(y,Ty)$ for all $y\in\Delta.$

by using above theorem we have to prove the stability results.

4. Stability Results in Intuitionistic Fuzzy Normed Space

Hereafter throughout this paper, assume that X is a linear space, (Z,μ',ν') is an intuitionistic fuzzy normed space and (Y,μ,ν) an intuitionistic fuzzy Banach space.

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Theorem 4.1 *Let* $g_u : X \to Y$ *be an odd mapping for which there exists a function* $\Psi_{UD} : X \to Z$ *with the double condition*

$$\left.\begin{aligned} &\lim_{n\to\infty}\mu'\left(\Psi_{UD}\left(T_i^n w\right), T_i^{11n} t\right) = 1 \\ &\lim_{n\to\infty}\nu'\left(\Psi_{UD}\left(T_i^n w\right), T_i^{11n} t\right) = 0 \end{aligned}\right\} \tag{1}$$

for all $w \in X$ and all $t > 0$ where T_i is defined as

$$T_i = \begin{cases} 5 & if \quad i = 0 \\ \dfrac{1}{5} & if \quad i = 1 \end{cases} \tag{2}$$

and satisfying the double functional inequality

$$\left.\begin{aligned} &\mu\left(g_u(5w) - 146,484,375 g_u(w) - 97,656,250 g_u(-w), t\right) \geq \mu'\left(\Psi_{UD}\left(x\right), t\right) \\ &\nu\left(g_u(5w) - 146,484,375 g_u(w) - 97,656,250 g_u(-w), t\right) \leq \nu'\left(\Psi_{UD}\left(x\right), t\right) \end{aligned}\right\} \tag{3}$$

for all $x \in X$ and all $t > 0$. If there exists $L = L(i)$ such that the function

$$\Psi_{UD}(w) = \Psi_{UD}\left(\frac{x}{5}\right), \tag{4}$$

has the property

$$\left.\begin{aligned} &\mu'\left(T_i \Psi_{UD}(T_i w), t\right) = \mu'\left(\Psi_{UD}(w), Lt\right) \\ &\nu'\left(T_i \Psi_{UD}(T_i w), t\right) = \nu'\left(\Psi_{UD}(w), Lt\right) \end{aligned}\right\} \tag{5}$$

for all $w \in X$ and all $t > 0$, then there exists a unique undecic function $\mathrm{U} : X \to Y$ satisfying the functional equation (4) and

$$\left.\begin{aligned} &\mu\left(g_u(w) - \mathrm{U}(w), t\right) \geq \mu'\left(\Psi_{UD}(w), \frac{L^{1-i}}{1-L} t\right) \\ &\nu\left(g_u(w) - \mathrm{U}(w), t\right) \leq \nu'\left(\Psi_{UD}(w), \frac{L^{1-i}}{1-L} t\right) \end{aligned}\right\} \tag{6}$$

for all $w \in X$ and all $t > 0$.

Proof. Consider the set

$$\mathrm{L}=\{h\,|h:X\to Y,h(0)=0\}$$

and introduce the generalized metric on L, as

$$d(h,f)=\inf\left\{L\in(0,\infty):\left\{\begin{matrix}\mu(h(w)-g_u(w),t)\geq\mu'(\Psi_{UD}(w),Lt),\\ \\ \nu(h(w)-g_u(w),t)\leq\nu'(\Psi_{UD}(w),Lt),\end{matrix}\right\}\right\} \quad (7)$$

for all $w\in X$ and all $t>0$. It is easy to see that (7) is complete with respect to the defined metric. Define $\Gamma:\mathrm{L}\to\mathrm{L}$ by

$$\Gamma h(w)=\frac{1}{T_i^{11}}h(T_iw),$$

for all $w\in\mathrm{X}$. Now, from (7) and $h,f\in\mathrm{L}$

$$\begin{cases}\mu(h(w)-g_u(w),t)\geq\mu'(\Psi_{UD}(w),t),x\in X,t>0\\ \mu(\frac{1}{T_i^{11}}h(T_iw)-\frac{1}{T_i^{11}}f(T_iw),t)\geq\mu'(\Psi_{UD}(T_iw),T_it),x\in X,t>0\\ \mu(\frac{1}{T_i^{11}}h(T_iw)-\frac{1}{T_i^{11}}f(T_iw),t)\geq\mu'(\Psi_{UD}(w),Lt),w\in X,t>0\\ \mu(\Gamma h(w)-\Gamma g_u(w),t)\geq\mu'(\Psi_{UD}(w),Lt),w\in X,t>0\end{cases}$$

$$\begin{cases}\nu(h(w)-g_u(w),t)\leq\nu'(\Psi_{UD}(w),t),w\in X,t>0\\ \nu(\frac{1}{T_i^{11}}h(T_iw)-\frac{1}{T_i^{11}}f(T_ix),t)\leq\nu'(\Psi_{UD}(T_iw),T_it),w\in X,t>0\\ \nu(\frac{1}{T_i^{11}}h(T_iw)-\frac{1}{T_i}f(T_i^{11}x),t)\leq\nu'(\Psi_{UD}(w),Lt),w\in X,t>0\\ \nu(\Gamma h(w)-\Gamma g_u(w),t)\leq\nu'(\Psi_{UD}(w),Lt),w\in X,t>0\end{cases}$$

This implies $d(\Gamma h,\Gamma g)\leq Ld(h,g)$. i.e., Γ is a strictly contractive mapping on L with Lipschitz constant L.

Using oddness of f in (3), we reach

$$\left.\begin{matrix}\mu(g_u(5w)-5^{11}g_u(w),t)\geq\mu'\left(\Psi_{UD}(w),t\right)\\ \nu(g_u(5w)-5^{11}g_u(w),t)\leq\nu'\left(\Psi_{UD}(w),t\right)\end{matrix}\right\} \quad (8)$$

for all $w\in X$ and all $t>0$. Now, from (8) and (5) for the case $i=0$, we reach

$$\begin{cases} \mu(g_u(5w)-5^{11}g_u(w),t)\ge \mu'\left(\Psi_{UD}(w),t\right) \\ \mu(\frac{g_u(5w)}{5^{11}}-g_u(w),t)\ge \mu'\left(\Psi_{UD}(w),5^{11}t\right) \\ \mu(\Gamma g_u(w)-g_u(w),t)\ge \mu'\left(\Psi_{UD}(w),Lt\right) \\ \mu(\Gamma g_u(w)-g_u(w),t)\ge \mu'\left(\Psi_{UD}(w),Lt\right) \\ \mu(\Gamma g_u(w)-g_u(w),t)\ge \mu'\left(\Psi_{UD}(w),Lt\right) \end{cases}$$

$$\begin{cases} \nu(g_u(5w)-5^{11}g_u(w),t)\le \nu'\left(\Psi_{UD}(w),t\right) \\ \nu(\frac{g_u(5w)}{5^{11}}-g_u(w),t)\le \nu'\left(\Psi_{UD}(w),5^{11}t\right) \\ \nu(\Gamma g_u(w)-g_u(w),t)\le \nu'\left(\Psi_{UD}(w),Lt\right) \\ \nu(\Gamma g_u(w)-g_u(w),t)\le \nu'\left(\Psi_{UD}(w),Lt\right) \\ \nu(\Gamma g_u(w)-g_u(w),t)\le \nu'\left(\Psi_{UD}(w),Lt\right) \end{cases} \tag{9}$$

for all $w\in X$ and all $t>0$. Again by interchanging w into $\frac{x}{5^{11}}$ in (8) and (5) for the case $i=1$, we get

$$\begin{cases} \mu(g_u(5w)-5^{11}g_u(w),t)\ge \mu'\left(\Psi_{UD}(\frac{x}{5^{11}}),t\right) \\ \mu(g_u(w)-\Gamma g_u(w),t)\ge \mu'\left(\Psi_{UD}(w),t\right) \\ \mu(g_u(w)-\Gamma g_u(w),t)\ge \mu'\left(\Psi_{UD}(w),t\right) \\ \mu(g_u(w)-\Gamma g_u(w),t)\ge \mu'\left(\Psi_{UD}(w),t\right) \end{cases}$$

$$\begin{cases} \nu(g_u(5w)-5^{11}g_u(w),t)\le \nu'\left(\Psi_{UD}(\frac{w}{5}),t\right) \\ \nu(g_u(w)-\Gamma g_u(w),t)\le \nu'\left(\Psi_{UD}(w),t\right) \\ \nu(g_u(w)-\Gamma g_u(w),t)\le \nu'\left(\Psi_{UD}(w),t\right) \\ \nu(g_u(w)-\Gamma g_u(w),t)\le \nu'\left(\Psi_{UD}(w),t\right) \end{cases} \tag{10}$$

for all $w\in X$ and all $t>0$. Thus, from (8) and (10), we arrive

$$\left.\begin{aligned} \mu(\Gamma g_u(w)-g_u(w),t)\ge \mu'(\Psi_{UD}(w),L^{1-i}t), w\in X \\ \nu(\Gamma g_u(w)-g_u(w),t)\le \nu'(\Psi_{UD}(w),L^{1-i}t), w\in X \end{aligned}\right\} \tag{11}$$

Hence property (FP1) holds.

By (FP2), it follows that there exists a fixed point U of T in L such that

$$\lim_{n\to\infty}\mu\left(\frac{f(T_i^n x)}{T_i^{11n}}-\text{U}(w),t\right)=1, \lim_{n\to\infty}\nu\left(\frac{f(T_i^n x)}{T_i^{11n}}-\text{U}(w),t\right)=0$$

for all $w\in X$ and all $t>0$. In order to prove $U: X\to Y$ is udecic, it is trivial.

By (FP3), U is the unique fixed point of Γ in the set $\Delta=\{\mathrm{U}\in\mathrm{L}:d(f,U)<\infty\},\mathrm{U}$ is the unique function such that

$$\left.\begin{aligned}\mu(g_u(w)-\mathrm{U}(w),t)\geq\mu'(\Psi_{UD}(w),L^{1-i}t),w\in X\\ \nu(g_u(w)-\mathrm{U}(w),t)\leq\nu'(\Psi_{UD}(w),L^{1-i}t),w\in X\end{aligned}\right\}$$

for all $w\in X$ and and all $t>0$. Finally by (FP4), we obtain

$$\left.\begin{aligned}\mu\left(g_u(w)-\mathrm{U}(w),t\right)\geq\mu'\left(\Psi_{UD}(w),\frac{L^{1-i}}{1-L}t\right)\\ \nu\left(g_u(w)-\mathrm{U}(w),t\right)\leq\nu'\left(\Psi_{UD}(w),\frac{L^{1-i}}{1-L}t\right)\end{aligned}\right\}$$

for all $w\in X$ and all $t>0$. So, the proof is complete.

Corollary 4.2 *Suppose that an odd function* $g_u:X\to Y$ *satisfies the double inequality*

$$\left.\begin{aligned}\mu\left(g_u(5w)-146,484,375g_u(w)-97,656,250g_u(-w),t\right)\geq\begin{cases}\mu'\left(\mathrm{L},t\right),\\ \mu'\left(\mathrm{L}\,\|w\|^H),t\right),\end{cases}\\ \nu\left(g_u(5w)-146,484,375g_u(w)-97,656,250g_u(-w),t\right)\leq\begin{cases}\nu'\left(\mathrm{L},t\right),\\ \nu'\left(\mathrm{L}\,\|w\|^H,t\right),\end{cases}\end{aligned}\right\} \tag{12}$$

for all $w\in X$ and all $t>0$, where $\mathrm{L},H\neq 11$ are constants with $\mathrm{L}>0$. Then there exists a unique undecic mapping $\mathrm{U}:X\to Y$ such that the double inequality

$$\left.\begin{aligned}\mu\left(g_u(w)-\mathrm{U}(w),t\right)\geq\begin{cases}\mu'\left(\mathrm{L},|5^{11}-1|t\right),\\ \mu'\left(\mathrm{L}\,\|w\|^H,\dfrac{5^{11H}}{|5^{11}-5^H|}\right),\end{cases}\\ \nu\left(g_u(w)-\mathrm{U}(w),t\right)\leq\begin{cases}\nu'\left(\mathrm{L},|5^{11}-1|t\right),\\ \nu'\left(\mathrm{L}\,\|w\|^H,\dfrac{5^{11H}}{|5^{11}-5^H|}\right),\end{cases}\end{aligned}\right\} \tag{13}$$

holds for all $w\in X$ and all $t>0$.

Proof. Now,

$$\mu'\left(\Psi_{UD}(T_i^n w),T_i^k t\right)=\begin{cases}\mu'\left(\mathrm{L},T_i^k t\right),\\ \mu'\left(\mathrm{L}\,\|w\|^H,T_i^{k-1}t\right),\end{cases}$$
$$=\begin{cases}\to 1\,as\;\;k\to\infty\\ \to 1\,as\;\;k\to\infty\end{cases}$$

$$\nu'\left(\Psi_{UD}(T_i^n w), T_i^k t\right) = \begin{cases} \nu'\left(\mathrm{L}, T_i^k t\right), \\ \nu'\left(\mathrm{L}\,\|w\|^H, T_i^{k-1} t\right), \end{cases}$$

$$= \begin{cases} \to 0 \; as \;\; k \to \infty \\ \to 0 \; as \;\; k \to \infty \end{cases}$$

for all $w \in X$ and all $t > 0$. Thus, the relation (1) holds. It follows from (4), (5) and (12), we arrive

$$\mu'\left(\Psi_{UD}, t\right) = \mu'\left(\Psi_{UD}\left(\frac{w}{5}\right), t\right) = \begin{cases} \mu'\left(\mathrm{L}, t\right) \\ \mu'\left(\dfrac{\mathrm{L}\,\|w\|^H}{5^H}, t\right) \end{cases}$$

$$\nu'\left(\Psi_{UD}, t\right) = \nu'\left(\Psi_{UD}\left(\frac{w}{5}\right), t\right) = \begin{cases} \nu'\left(\mathrm{L}, t\right) \\ \nu'\left(\dfrac{\mathrm{L}\,\|w\|^H}{5^H}, t\right) \end{cases}$$

for all $w \in X$ and all $t > 0$. Also from (5), we have

$$\mu'\left(T_i \Psi_{UD}(T_i w), t\right) = \begin{cases} \mu'\left(\mathrm{L}, T_i^{-1} t\right) \\ \mu'\left(\mathrm{L}\,\|w\|^H, T_i^{H-1} t\right) \end{cases}$$

$$\nu'\left(T_i \Psi_{UD}(T_i w), t\right) = \begin{cases} \nu'\left(\mathrm{L}, T_i^{-1} t\right) \\ \nu'\left(\mathrm{L}\,\|w\|^H, T_i^{H-1} t\right) \end{cases}$$

for all $w \in X$ and all $t > 0$.

For the case $L = T_i^{-11} = \left(5^{-11}\right)$ for $i = 0$ and $L = T_i^{-11} = \left(\dfrac{1}{5^{-11}}\right) = 5^{11}$ for $i = 1$ from the inequality (6), we arrive

$$\left.\begin{aligned} \mu\left(g_u(w) - \mathrm{U}(w), t\right) &\geq \mu'\left(\Psi_{UD}(w), \frac{(5^{-11})^{1-0}}{1-5^{-11}} t\right) = \mu'\left(\mathrm{L}, \left(1-5^{11}\right)t\right) \\ \nu\left(g_u(w) - \mathrm{U}(w), t\right) &\leq \nu'\left(\Psi_{UD}(w), \frac{(5^{-11})^{1-0}}{1-5^{-11}} t\right) = \nu'\left(\mathrm{L}, \left(1-5^{11}\right)t\right) \end{aligned}\right\}$$

$$\left.\begin{aligned} \mu\left(g_u(w) - \mathrm{U}(w), t\right) &\geq \mu'\left(\Psi_{UD}(w), \frac{(5^{11})^{1-1}}{1-5^{11}} t\right) = \mu'\left(\mathrm{L}, \left(5^{11}-1\right)t\right) \\ \nu\left(g_u(w) - \mathrm{U}(w), t\right) &\leq \nu'\left(\Psi_{UD}(w), \frac{(5^{11})^{1-1}}{1-5^{11}} t\right) = \nu'\left(\mathrm{L}, \left(5^{11}-1\right)t\right) \end{aligned}\right\}$$

for all $w \in X$ and all $t > 0$.

For the case $L = T_i^{H-11} = 5^{H-11}$ for $i = 0$ and $L = T_i^{H-1} = \left(\frac{1}{5}\right)^{H-11} = 5^{11-H}$ for $i = 1$ from the inequality (6), we arrive

$$\left.\begin{aligned} \mu\left(g_u(w) - \mathrm{U}(w), t\right) &\geq \mu'\left(\Psi_{UD}(w), \frac{(5^{H-11})^{1-0}}{1-5^{H-11}} t\right) = \mu'\left(\mathrm{L}\,\|w\|^H, \frac{5^{11r}}{5^{11} - 5^H} t\right) \\ \nu\left(g_u(w) - \mathrm{U}x, t\right) &\leq \nu'\left(\Psi_{UD}, x\frac{(5^{11r-11})^{1-0}}{1-5^{H-11}} t\right) = \nu'\left(\mathrm{L}\,\|w\|^H, \frac{5^{11r}}{5^{11} - 5^H} t\right) \end{aligned}\right\}$$

$$\left.\begin{aligned} \mu\left(g_u(w) - \mathrm{U}(w), t\right) &\geq \mu'\left(\Psi_{UD}(w), \frac{(5^{11-r})^{1-1}}{1-5^{11-r}} t\right) = \mu'\left(\mathrm{L}\,\|w\|^H, \frac{5^{11H}}{5^H - 5^{11}} t\right) \\ \nu\left(g_u(w) - \mathrm{U}(w), t\right) &\leq \nu'\left(\Psi_{UD}(w), \frac{(5^{11-11r})^{1-1}}{1-5^{r-11}} t\right) = \nu'\left(\mathrm{L}\,\|w\|^H, \frac{5^{11H}}{5^H - 5^{11}} t\right) \end{aligned}\right\}$$

for all $w \in X$ and all $t > 0$.

Theorem 4.3 *Let* $g_d : X \to Y$ *be an even mapping for which there exists a function* $\Psi_{UD} : X \to Z$ *with the double condition*

$$\left.\begin{aligned} \lim_{n\to\infty} \mu'\left(\Psi_{UD}\left(T_i^n w\right), T_i^{12n} t\right) = 1 \\ \lim_{n\to\infty} \nu'\left(\Psi_{UD}\left(T_i^n w\right), T_i^{12n} t\right) = 0 \end{aligned}\right\} \tag{14}$$

for all $w \in X$ and all $t > 0$ where T_i is defined as

$$T_i = \begin{cases} 5 & if \quad i = 0 \\ \frac{1}{5} & if \quad i = 1 \end{cases} \tag{15}$$

and satisfying the double functional inequality

$$\left.\begin{aligned} \mu\left(g_d(5w) - 146,484,375 g_d(w) - 97,656,250 g_d(-w), t\right) \geq \mu'\left(\Psi_{UD}\left(x\right), t\right) \\ \nu\left(g_d(5w) - 146,484,375 g_d(w) - 97,656,250 g_d(-w), t\right) \leq \nu'\left(\Psi_{UD}\left(x\right), t\right) \end{aligned}\right\} \tag{16}$$

for all $w \in X$ and all $t > 0$. If there exists $L = L(i)$ such that the function

$$\Psi_{UD}(w) = \Psi_{UD}\left(\frac{w}{5}\right), \tag{17}$$

has the property

$$\left.\begin{aligned} \mu'\left(T_i \Psi_{UD}(T_i w), t\right) = \mu'\left(\Psi_{UD}(w), Lt\right) \\ \nu'\left(T_i \Psi_{UD}(T_i w), t\right) = \nu'\left(\Psi_{UD}(w), Lt\right) \end{aligned}\right\} \tag{18}$$

for all $w \in X$ and all $t > 0$, then there exists a unique dodecic function $\mathrm{D} : X \to Y$ satisfying the functional equation (4) and

$$\left.\begin{array}{l}\mu\left(g_d(w)-\mathrm{D}(w),t\right) \geq \mu'\left(\Psi_{UD}(w), \dfrac{L^{1-i}}{1-L}t\right)\\ \nu\left(g_d(w)-\mathrm{D}(w),t\right) \leq \nu'\left(\Psi_{UD}(w), \dfrac{L^{1-i}}{1-L}t\right)\end{array}\right\} \tag{19}$$

for all $w \in X$ and all $t > 0$.

Proof. Consider the set

$$\mathrm{L} = \{h \,|\, h : X \to Y, h(0) = 0\}$$

and introduce the generalized metric on L,

$$d(h,f) = \inf\left\{L \in (0,\infty) : \begin{Bmatrix}\mu(h(w)-g_d(w),t) \geq \mu'(\Psi_{UD}(w), Lt), x \in X, t > 0\\ \nu(h(w)-g_d(w),t) \leq \nu'(\Psi_{UD}(w), Lt), x \in X, t > 0\end{Bmatrix}\right\} \tag{20}$$

It is easy to see that (20) is complete with respect to the defined metric. Define $\Gamma : \mathrm{L} \to \mathrm{L}$ by

$$\Gamma\, h(w) = \frac{1}{T_i^{12}} h(T_i x),$$

for all $w \in \mathrm{X}$. The rest of the proof is similar to that of Theorem 4.1.

Corollary 4.4 *Suppose that an even function* $g_d : X \to Y$ *satisfies the double inequality*

$$\left.\begin{array}{l}\mu\left(g_d(5w)-146{,}484{,}375g_d(w)-97{,}656{,}250g_d(-w),t\right) \geq \begin{cases}\mu'\left(\mathrm{L},t\right),\\ \mu'\left(\mathrm{L}\,\|w\|^H),t\right),\end{cases}\\ \nu\left(g_d(5w)-146{,}484{,}375g_d(w)-97{,}656{,}250g_d(-w),t\right) \leq \begin{cases}\nu'\left(\mathrm{L},t\right),\\ \nu'\left(\mathrm{L}\,\|w\|^H,t\right),\end{cases}\end{array}\right\} \tag{21}$$

for all $w \in X$ and all $t > 0$, where $\mathrm{L}, H \neq 12$ are constants with $\mathrm{L} > 0$. Then there exists a unique dodecic mapping $\mathrm{D} : X \to Y$ such that the double inequality

$$\left.\begin{array}{l}\mu\left(g_d(w)-\mathrm{D}(w),t\right) \geq \begin{cases}\mu'\left(\mathrm{L}, |5^{12}-1|\,t\right),\\ \mu'\left(\mathrm{L}\,\|w\|^H, \dfrac{5^{12H}}{|5^{12}-5^H|}\right),\end{cases}\\ \nu\left(g_d(w)-\mathrm{D}(w),t\right) \leq \begin{cases}\nu'\left(\mathrm{L}, |5^{12}-1|\,t\right),\\ \nu'\left(\mathrm{L}\,\|w\|^H, \dfrac{5^{12H}}{|5^{12}-5^H|}\right),\end{cases}\end{array}\right\} \tag{22}$$

holds for all $w \in X$ and all $t > 0$.

Theorem 4.5 *Let* $f : X \to Y$ *be a mapping for which there exists a function* $\Psi_{UD} : X \to Z$ *with the double conditions (1), (14) for all* $w \in X$ *and all* $t > 0$ *and satisfying the double functional inequality*

$$\left.\begin{aligned} \mu\left(g(5w)-146,484,375g(w)-97,656,250g(-w),t\right) &\geq \mu'\left(\Psi_{UD}(w),t\right) \\ \nu\left(g(5w)-146,484,375g(w)-97,656,250g(-w),t\right) &\leq \nu'\left(\Psi_{UD}(w),t\right) \end{aligned}\right\} \quad (23)$$

for all $w \in X$ and all $t > 0$. If there exists $L = L(i)$ such that the functions (4) and (17) has the properties (5) and (18) for all $w \in X$ and all $t > 0$, then there exists a unique undecic function $\mathrm{U} : X \to Y$ and a unique dodecic function $\mathrm{D} : X \to Y$ satisfying the functional equation (4) and

$$\left.\begin{aligned} \mu\left(g(w)-\mathrm{U}(w)-\mathrm{D}(w),t\right) &\geq \mu'\left(\Psi_{UD}(w),\frac{L^{1-i}}{1-L}t\right) * \mu'\left(\Psi_{UD}(-w),\frac{L^{1-i}}{1-L}t\right) \\ &* \mu'\left(\Psi_{UD}(w),\frac{L^{1-i}}{1-L}t\right) * \mu'\left(\Psi_{UD}(-w),\frac{L^{1-i}}{1-L}t\right) \\ \nu\left(g(w)-\mathrm{U}(w)-\mathrm{D}(w),t\right) &\leq \nu'\left(\Psi_{UD}(w),\frac{L^{1-i}}{1-L}t\right) \Diamond \nu'\left(\Psi_{UD}(-w),\frac{L^{1-i}}{1-L}t\right) \\ &\Diamond \nu'\left(\Psi_{UD}(w),\frac{L^{1-i}}{1-L}t\right) \Diamond \nu'\left(\Psi_{UD}(-w),\frac{L^{1-i}}{1-L}t\right) \end{aligned}\right\} \quad (24)$$

for all $w \in X$ and all $t > 0$.

Corollary 4.6 *Suppose that a function* $f : X \to Y$ *satisfies the double inequality*

$$\left.\begin{aligned} \mu\left(g(5w)-146,484,375g(w)-97,656,250g(-w),t\right) &\geq \begin{cases} \mu'\left(\mathrm{L},t\right), \\ \mu'\left(\mathrm{L}\,\|w\|^{H}),t\right), \end{cases} \\ \nu\left(g(5w)-146,484,375g(w)-97,656,250g(-w),t\right) &\leq \begin{cases} \nu'\left(\mathrm{L},t\right), \\ \nu'\left(\mathrm{L}\,\|w\|^{H},t\right), \end{cases} \end{aligned}\right\} \quad (25)$$

for all $x, y \in X$ and all $t > 0$, where $\mathrm{L}, H \neq 11, 12$ are constants with $\mathrm{L} > 0$. Then there exists a unique undecic mapping $\mathrm{U} : X \to Y$ and a unique dodecic function $\mathrm{D} : X \to Y$ such that the double inequality

$$\left.\begin{aligned} \mu\left(g(w)-\mathrm{U}(w)-\mathrm{D}(w),t\right) &\geq \begin{cases} \mu'\left(\mathrm{L},|5^{11}-1|\,t\right) * \mu'\left(\mathrm{L},|5^{12}-1|\,t\right), \\ \mu'\left(\mathrm{L}\,\|w\|^{H},\frac{5^{11r}}{|5^{11}-5^{r}|}\right) * \mu'\left(\mathrm{L}\,\|w\|^{H},\frac{5^{12r}}{|5^{12}-5^{r}|}\right), \end{cases} \\ \nu\left(g(w)-\mathrm{U}(w)-\mathrm{D}(w)),t\right) &\leq \begin{cases} \nu'\left(\mathrm{L},|5^{11}-1|\,t\right) \Diamond \nu'\left(\mathrm{L},|5^{12}-1|\,t\right), \\ \nu'\left(\mathrm{L}\,\|w\|^{H},\frac{5^{11r}}{|5^{11}-5^{r}|}\right) \Diamond \nu'\left(\mathrm{L}\,\|w\|^{H},\frac{5^{12r}}{|5^{12}-5^{r}|}\right) \end{cases} \end{aligned}\right\} \quad (26)$$

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holds for all $x \in X$ and all $t > 0$.

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Additive Quadratic Functional Equation and Inequality are Stable in Banach Space

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ABSTRACT: *In this paper, the authors proved the generalized Ulam - Hyers stability of a mixed type general additive quadratic functional equation and inequality*

$$h_a^q\left(p^{-1}x+q^{-1}y+r^{-1}z\right)-h_a^q\left(p^{-1}x-q^{-1}y+r^{-1}z\right)+h_a^q\left(p^{-1}x+q^{-1}y-r^{-1}z\right)$$
$$-h_a^q\left(p^{-1}x-q^{-1}y-r^{-1}z\right)=4h_a^q\left(p^{-1}x\right)+2\left[h_a^q\left(q^{-1}y\right)+h_a^q\left(-q^{-1}y\right)\right]-4h_a^q\left(p^{-1}x-q^{-1}y\right)$$
$$\left\|Ia^q\left(p^{-1}x+q^{-1}y+r^{-1}z\right)-Ia^q\left(p^{-1}x-q^{-1}y+r^{-1}z\right)+Ia^q\left(p^{-1}x+q^{-1}y-r^{-1}z\right)\right.$$
$$\left.-Ia^q\left(p^{-1}x-q^{-1}y-r^{-1}z\right)-4Ia^q\left(p^{-1}x\right)-2\left[Ia^q\left(q^{-1}y\right)+Ia^q\left(-q^{-1}y\right)\right]\right\|\le\left\|4Ia^q\left(p^{-1}x-q^{-1}y\right)\right\|$$

where $p,q,r\in\mathrm{R}$ *with* $p=q=r\neq 0$ *in Banach spaces.*

Keywords: *Banach Space, Stability of Functional Equation*

1. Introduction

The stability problem of functional equations originated from a question of S.M. Ulam [21] concerning the stability of group homomorphisms. D.H. Hyers [10] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' theorem was generalized by T. Aoki [2] for additive mappings and by Th.M. Rassias [20] for linear mappings by considering an unbounded Cauchy difference.

The paper of Th.M. Rassias [20] has provided a lot of influence in the development of what we call generalized Hyers-Ulam stability of functional equations. A generalization of the Th.M. Rassias theorem was obtained by P. Gavruta [7] by replacing the unbounded Cauchy difference by a general control function in the spirit of Rassias' approach.

In 1982, J.M. Rassias [14] followed the innovative approach of the Th.M. Rassias theorem [20] in which he replaced the factor $\| x \|^p + \| y \|^p$ by $\| x \|^p \| y \|^q$ for $p,q\in R$ with $p+q=1.$

In 2008, a special case of Gavruta's theorem for the unbounded Cauchy difference was obtained by Ravi etal., [19] by considering the summation of both the sum and the product of two $p-$ norms in the sprit of Rassias approach. The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [1, 3, 4, 5, 6, 8, 9, 11, 12, 16, 17]).

In this paper, the authors proved the generalized Ulam - Hyers stability of a mixed type general additive quadratic functional equation and inequality

$$h_a^q\left(p^{-1}x+q^{-1}y+r^{-1}z\right)-h_a^q\left(p^{-1}x-q^{-1}y+r^{-1}z\right)+h_a^q\left(p^{-1}x+q^{-1}y-r^{-1}z\right)$$
$$-h_a^q\left(p^{-1}x-q^{-1}y-r^{-1}z\right)=4h_a^q\left(p^{-1}x\right)+2\left[h_a^q\left(q^{-1}y\right)+h_a^q\left(-q^{-1}y\right)\right]-4h_a^q\left(p^{-1}x-q^{-1}y\right) \tag{1.1}$$

$$\left\|Ia^q\left(p^{-1}x+q^{-1}y+r^{-1}z\right)-Ia^q\left(p^{-1}x-q^{-1}y+r^{-1}z\right)+Ia^q\left(p^{-1}x+q^{-1}y-r^{-1}z\right)\right.$$
$$\left.-Ia^q\left(p^{-1}x-q^{-1}y-r^{-1}z\right)-4Ia^q\left(p^{-1}x\right)-2\left[Ia^q\left(q^{-1}y\right)+Ia^q\left(-q^{-1}y\right)\right]\right\|\le\left\|4Ia^q\left(p^{-1}x-q^{-1}y\right)\right\| \tag{1.2}$$

where $p,q,r\in\mathrm{R}$ are positive numbers with $p=q=r\neq 0$ in Banach spaces.

In Section 2, the generalized Ulam - Hyers stability of the functional equation (1.1) is proved.

In Section 3, the generalized Ulam - Hyers stability of the functional inequality (1.2) given.

Hereafter through out this paper, let us consider X and Y to be a normed space and a Banach space, respectively. Define a mapping $Dh_a^q : X \to Y$ by

$$Dh_a^q(x,y,z) = h_a^q\left(p^{-1}x + q^{-1}y + r^{-1}z\right) - h_a^q\left(p^{-1}x - q^{-1}y + r^{-1}z\right) + h_a^q\left(p^{-1}x + q^{-1}y - r^{-1}z\right)$$
$$-h_a^q\left(p^{-1}x - q^{-1}y - r^{-1}z\right) - 4h_a^q\left(p^{-1}x\right) - 2\left[h_a^q\left(q^{-1}y\right) + h_a^q\left(-q^{-1}y\right)\right] + 4h_a^q\left(p^{-1}x - q^{-1}y\right)$$

for all $x, y, z \in X$.

2. STABILITY RESULTS

In this section, the generalized Ulam - Hyers stability of the functional equation (1) is provided.

Theorem 2.1 *Let* $j \in \{-1, 1\}$ *and* $\Lambda : X^3 \to [0, \infty)$ *be a function such that*

$$\sum_{n=0}^{\infty} \frac{\Lambda\left(3^{nj}x, 3^{nj}y, 3^{nj}z\right)}{3^{nj}} \text{ converges in } \mathbb{R} \text{ and } \lim_{n\to\infty} \frac{\Lambda\left(3^{nj}x, 3^{nj}y, 3^{nj}z\right)}{3^{nj}} = 0 \tag{1}$$

for all $x, y, z \in X$. Let $h_a : X \to Y$ be an odd function satisfying the inequality

$$\left\| Dh_a(x,y,z) \right\| \le \Lambda\left(x, y, z\right) \tag{2}$$

for all $x, y, z \in X$. Then there exists a unique additive mapping $A : X \to Y$ such that

$$\left\| h_a(x) - A(x) \right\| \le \frac{1}{3} \sum_{k=\frac{1-j}{2}}^{\infty} \frac{\Lambda(3^{kj}x, 3^{kj}x, 3^{kj}x)}{3^{kj}} \tag{3}$$

for all $x \in X$. The mapping $A(x)$ is defined by

$$A(x) = \lim_{n\to\infty} \frac{h_a(3^{nj}x)}{3^{nj}} \tag{4}$$

for all $x \in X$.

Proof. Assume $j = 1$. Replacing (x, y, z) by (px, qx, rx) in (2) and using oddness of h_a, we get

$$\left\| h_a(x) - \frac{h_a(3x)}{3} \right\| \le \frac{1}{3} \Lambda\left(px, qx, rx\right) \tag{5}$$

for all $x \in X$. Now replacing x by $3x$ and dividing by 3 in (15), we obtain

$$\left\| \frac{h_a(3x)}{3} - \frac{h_a(3^2x)}{3^2} \right\| \le \frac{\Lambda\left(3px, 3qx, 3rx\right)}{9} \tag{6}$$

for all $x \in X$. It follows from (15) and (6) that

$$\left\| h_a(x) - \frac{h_a(3^2x)}{3^2} \right\| \le \left\| h_a(x) - \frac{h_a(3x)}{3} \right\| + \left\| \frac{h_a(3x)}{3} - \frac{h_a(3^2x)}{3^2} \right\|$$

$$\leq \frac{1}{3}\left[\Lambda\left(px,qx,rx\right)+\frac{\Lambda\left(3px,3qx,3rx\right)}{3}\right] \tag{7}$$

for all $x \in X$. In general for any positive integer n, we get

$$\left\|h_a(x)-\frac{h_a(3^n x)}{3^n}\right\| \leq \frac{1}{3}\sum_{k=0}^{n-1}\frac{\Lambda(3^k px,3^k qx,3^k rx)}{3^k} \leq \frac{1}{3}\sum_{k=0}^{\infty}\frac{\Lambda(3^k px,3^k qx,3^k rx)}{3^k} \tag{8}$$

for all $x \in X$. In order to prove the convergence of the sequence

$$\left\{\frac{h_a(3^n x)}{3^n}\right\},$$

replace x by $3^m x$ and divide by 3^m in (8), for any $m,n > 0$, to deduce

$$\left\|\frac{h_a(3^m x)}{3^m}-\frac{h_a(3^{n+m} x)}{3^{(n+m)}}\right\| = \frac{1}{3^m}\left\|h_a(3^m x)-\frac{h_a(3^n \cdot 3^m x)}{3^n}\right\|$$

$$\leq \frac{1}{6}\sum_{k=0}^{n-1}\frac{\Lambda(3^{k+m} px,3^{k+m} qx,3^{k+m} rx)}{3^{k+m}}$$

$$\leq \frac{1}{6}\sum_{k=0}^{\infty}\frac{\Lambda(3^{k+m} px,3^{k+m} qx,3^{k+m} rx)}{3^{k+m}}$$

$$\to 0 \; as \; m \to \infty$$

for all $x \in X$. Hence the sequence $\left\{\frac{h_a(3^n x)}{3^n}\right\}$ is a Cauchy sequence. Since Y is complete, there exists a mapping $A : X \to Y$ such that

$$A(x) = \lim_{n\to\infty}\frac{h_a(3^n x)}{3^n} \; \forall x \in X.$$

Letting $n \to \infty$ in (8) we see that (3) holds for all $x \in X$. To prove A satisfies (1.1), replacing (x,y,z) by $(3^n x, 3^n y, 3^n z)$ and dividing by 3^n in (2), we obtain

$$\frac{1}{3^n}\| Dh_a(3^n x,3^n y,3^n z)\| \leq \frac{1}{3^n}\Lambda(3^n x,3^n y,3^n z)$$

for all $x,y,z \in X$. Letting $n \to \infty$ in the above inequality and using the definition of $A(x)$, we see that

$$DA(x,y,z) = 0.$$

Hence A satisfies (1.1) for all $x,y,z \in X$. To show A is unique, let $B(x)$ be another additive mapping satisfying (1) and (3), then

$$\|A(x)-B(x)\| = \frac{1}{3^n}\|A(3^n x)-B(3^n x)\|$$

$$\leq \frac{1}{3^n}\left\{\left\|A(3^n x)-h_a(3^n x)\right\|+\left\|h_a(3^n x)-B(3^n x)\right\|\right\}$$
$$\leq \frac{1}{3}\sum_{k=0}^{\infty}\frac{\Lambda(3^{k+n}px,3^{k+n}qx,3^{k+n}rx)}{3^{(k+n)}}$$
$$\rightarrow 0 \; as\, n \rightarrow \infty$$

for all $x \in X$. Hence A is unique.

For $j=-1$, we can prove a similar stability result. This completes the proof of the theorem.

The following Corollary is an immediate consequence of Theorem 2.1 concerning the stability of (1.1).

Corollary 2.2 *Let* λ *and* s *be non negative real numbers. Let an odd function* $h_a : X \rightarrow Y$ *satisfy the inequality*

$$\left\|Dh_a(x,y,z)\right\| \leq \begin{cases} \lambda, & \\ \lambda\left\{\|x\|^s+\|y\|^s+\|z\|^s\right\}, & s\neq 1; \\ \lambda\|x\|^s\|y\|^s\|z\|^s & s\neq \frac{1}{3}; \\ \lambda\left\{\|x\|^s\|y\|^s\|z\|^s+\left\{\|x\|^{3s}+\|y\|^{3s}+\|z\|^{3s}\right\}\right\}, & s\neq \frac{1}{3}; \end{cases} \tag{9}$$

for all $x,y,z \in X$. Then there exists a unique additive function $A : X \rightarrow Y$ such that

$$\left\|h_o(x)-A(x)\right\| \leq \begin{cases} \frac{\lambda}{2}, \\ \frac{\lambda[p^s+q^s+r^s]^s\|x\|^s}{|3-3^s|}, \\ \frac{\lambda[(pqr)^s]^p\|x\|^{3s}}{|3-3^{3s}|}, \\ \frac{\lambda[(pqr)^s+p^{3s}+q^{3s}+r^{3s}]\|x\|^{3s}}{|3-3^{3s}|}, \end{cases} \tag{10}$$

for all $x \in X$.

Theorem 2.3 *Let* $j \in \{-1,1\}$ *and* $\Lambda : X^3 \rightarrow [0,\infty)$ *be a function such that*

$$\sum_{n=0}^{\infty}\frac{\Lambda\left(3^{nj}x,3^{nj}y,3^{nj}z\right)}{3^{nj}} \; converges \;\; in \mathrm{R} \;\; and \; \lim_{n\rightarrow\infty}\frac{\Lambda\left(3^{nj}x,3^{nj}y,3^{nj}z\right)}{9^{nj}}=0 \tag{11}$$

for all $x,y,z \in X$. Let $h_Q : X \rightarrow Y$ be an even function satisfying the inequality

$$\left\|Dh_q(x,y,z)\right\| \le \Lambda\left(x,y,z\right) \tag{12}$$

for all $x,y,z \in X$. Then there exists a unique quadratic mapping $Q: X \to Y$ such that

$$\left\|h_q(x) - Q(x)\right\| \le \frac{1}{9}\sum_{k=\frac{1-j}{2}}^{\infty} \frac{\Lambda(3^{kj}x, 3^{kj}x, 3^{kj}x)}{3^{kj}} \tag{13}$$

for all $x \in X$. The mapping $Q(x)$ is defined by

$$Q(x) = \lim_{n\to\infty} \frac{h_q(3^{nj}x)}{9^{nj}} \tag{14}$$

for all $x \in X$.

Proof. Assume $j=1$. Replacing (x,y,z) by (px,qx,rx) in (12) and using evenness of h_q, we get

$$\left\|h_q(x) - \frac{h_q(3x)}{9}\right\| \le \frac{1}{9}\Lambda\left(px,qx,rx\right) \tag{15}$$

for all $x \in X$.
Hence the proof is similar to that of Theorem 2.1.

Corollary 2.4 *Let* λ *and* s *be non negative real numbers. Let an even function* $h_q: X \to Y$ *satisfy the inequality*

$$\left\|Dh_q(x,y,z)\right\| \le \begin{cases} \lambda, & \\ \lambda\left\{\|x\|^s + \|y\|^s + \|z\|^s\right\}, & s \ne 2; \\ \lambda\|x\|^s\|y\|^s\|z\|^s & s \ne \dfrac{2}{3}; \\ \lambda\left\{\|x\|^s\|y\|^s\|z\|^s + \left\{\|x\|^{3s} + \|y\|^{3s} + \|z\|^{3s}\right\}\right\}, & s \ne \dfrac{2}{3}; \end{cases} \tag{16}$$

for all $x,y,z \in X$. Then there exists a unique quadratic function $Q: X \to Y$ such that

$$\left\|h_q(x) - Q(x)\right\| \le \begin{cases} \dfrac{\lambda}{8}, \\ \dfrac{\lambda[p^s + q^s + r^s]^s\|x\|^s}{|9-3^s|}, \\ \dfrac{\lambda[(pqr)^s]^p\|x\|^{3s}}{|9-3^{3s}|}, \\ \dfrac{\lambda[(pqr)^s + p^{3s} + q^{3s} + r^{3s}]\|x\|^{3s}}{|9-3^{3s}|}, \end{cases} \tag{17}$$

for all $x \in X$.

Theorem 2.5 *Let* $j \in \{-1,1\}$ *and* $\Lambda : X^3 \to [0,\infty)$ *be a function satisfying (1) and (11) for all* $x, y, z \in X$. *Let* $f : X \to Y$ *be a function satisfying the inequality*

$$\|Dh(x,y,z)\| \leq \Lambda(x,y,z) \tag{18}$$

for all $x, y, z \in X$. Then there exists a unique additive mapping $A : X \to Y$ and a unique quadratic mapping $Q : X \to Y$ such that

$$\|h(x)-A(x)-Q(x)\| \leq \frac{1}{2}\left[\frac{1}{3}\sum_{k=\frac{1-j}{2}}^{\infty}\left(\frac{\Lambda(3^{kj}x,3^{kj}x,3^{kj}x)}{3^{kj}}+\frac{\Lambda(-3^{kj}x,-3^{kj}x,-3^{kj}x)}{3^{kj}}\right)\right.$$
$$\left.+\frac{1}{9}\sum_{k=\frac{1-j}{2}}^{\infty}\left(\frac{\Lambda(3^{kj}x,3^{kj}x,3^{kj}x)}{9^{kj}}+\frac{\Lambda(-3^{kj}x,-3^{kj}x,-3^{kj}x)}{9^{kj}}\right)\right] \tag{19}$$

for all $x \in X$. The mapping $A(x)$ and $Q(x)$ are defined in (4) and (14) respectively for all $x \in X$.

Proof. Let $h_o(x) = \dfrac{h_a(x)-h_a(-x)}{2}$ for all $x \in X$. Then $h_o(0)=0$ and $h_o(-x) = -h_o(x)$ for all $x \in X$. Hence

$$\|Dh_o(x,y,z)\| \leq \frac{\Lambda(x,y,z)}{2}+\frac{\Lambda(-x,-y,-z)}{2} \tag{20}$$

for all $x, y, z \in X$. By Theorem 2.1, we have

$$\|h_o(x)-A(x)\| \leq \frac{1}{6}\sum_{k=\frac{1-j}{2}}^{\infty}\left(\frac{\Lambda(3^{kj}x,3^{kj}x,3^{kj}x)}{3^{kj}}+\frac{\Lambda(-3^{kj}x,-3^{kj}x,-3^{kj}x)}{3^{kj}}\right) \tag{21}$$

for all $x \in X$. Also, let $h_e(x) = \dfrac{h_q(x)+h_q(-x)}{2}$ for all $x \in X$. Then $h_e(0)=0$ and $h_e(-x) = h_e(x)$ for all $x \in X$. Hence

$$\|Dh_e(x,y,z)\| \leq \frac{\Lambda(x,y,z)}{2}+\frac{\Lambda(-x,-y,-z)}{2} \tag{22}$$

for all $x, y, z \in X$. By Theorem 2.3, we have

$$\|h_e(x)-Q(x)\| \leq \frac{1}{18}\sum_{k=\frac{1-j}{2}}^{\infty}\left(\frac{\Lambda(3^{kj}x,3^{kj}x,3^{kj}x)}{9^{kj}}+\frac{\Lambda(-3^{kj}x,-3^{kj}x,-3^{kj}x)}{9^{kj}}\right) \tag{23}$$

for all $x \in X$. Define

$$h(x) = h_e(x) + h_o(x) \tag{24}$$

for all $x \in X$. From (21),(23) and (24), we arrive

$$\|h(x)-A(x)-Q(x)\| = \|h_e(x)+h_o(x)-A(x)-Q(x)\|$$
$$\leq \|h_o(x)-A(x)\| + \|h_e(x)-Q(x)\|$$
$$\leq \frac{1}{6}\sum_{k=\frac{1-j}{2}}^{\infty}\left(\frac{\Lambda(3^{kj}x,3^{kj}x,3^{kj}x)}{3^{kj}}+\frac{\Lambda(-3^{kj}x,-3^{kj}x,-3^{kj}x)}{3^{kj}}\right)$$
$$+\frac{1}{18}\sum_{k=\frac{1-j}{2}}^{\infty}\left(\frac{\Lambda(3^{kj}x,3^{kj}x,-3^{kj}x)}{9^{kj}}+\frac{\Lambda(-3^{kj}x,-3^{kj}x,-3^{kj}x)}{9^{kj}}\right)$$

for all $x \in X$. Hence the theorem is proved.

Using Corollaries 2.2 and 2.4, we have the following Corollary concerning the stability of (1).

Corollary 2.6 *Let* λ *and* s *be non negative real numbers. Let a function* $f : X \to Y$ *satisfy the inequality*

$$\|Dh_a(x,y,z)\| \leq \begin{cases} \lambda, & \\ \lambda\{\|x\|^s+\|y\|^s+\|z\|^s\}, & s\neq 1,2; \\ \lambda\|x\|^s\|y\|^s\|z\|^s & s\neq \frac{1}{3},\frac{2}{3}; \\ \lambda\{\|x\|^s\|y\|^s\|z\|^s+\{\|x\|^{3s}+\|y\|^{3s}+\|z\|^{3s}\}\}, & s\neq \frac{1}{3},\frac{2}{3}; \end{cases} \tag{25}$$

for all $x, y, z \in X$. Then there exists a unique additive function $A : X \to Y$ and a unique quadratic function $Q : X \to Y$ such that

$$\|A(x)-Q(x)-h(x)\|$$
$$\leq \begin{cases} \left[\left(\frac{\lambda}{2}\right)+\left(\frac{\lambda}{8}\right)\right], \\ \left(\left[\frac{1}{|3-3^s|}+\frac{3}{|9-3^s|}\right]\left(p^s+q^s+r^s\right)\|x\|^s\right), \\ \left(\left[\frac{1}{|3-3^s|}+\frac{3}{|9-3^{3s}|}\right]\left(pqr\right)^s\|x\|^{3s}\right), \\ \left(\left[\frac{1}{|3-3^{3s}|}+\frac{3}{|9-3^{3s}|}\right]\left[\left(pqr\right)^s+\left(p^{3s}+q^{3s}+r^{3s}\right)\right]\|x\|^{3s}\right) \end{cases} \tag{26}$$

for all $x \in X$.

3 . STABILITY RESULTS IN FUNCTIONAL INEQUALITY

In this section, the generalized Ulam - Hyers stability of the functional inequality (1.2) is provided.

Theorem 3.1 *Let* $j \in \{-1,1\}$ *and* $\Lambda : X^3 \to [0,\infty)$ *be a function such that*

$$\sum_{n=0}^{\infty}\frac{\Lambda\left(3^{nj}x,3^{nj}y,3^{nj}z\right)}{3^{nj}}\ converges\ in\,\mathbb{R}\ \ and\ \lim_{n\to\infty}\frac{\Lambda\left(3^{nj}x,3^{nj}y,3^{nj}z\right)}{3^{nj}}=0 \tag{1}$$

for all $x,y,z\in X$. Let $I_a:X\to Y$ be an odd function satisfying the inequality

$$\begin{aligned}&\left\|Ia^q\left(p^{-1}x+q^{-1}y+r^{-1}z\right)-Ia^q\left(p^{-1}x-q^{-1}y+r^{-1}z\right)+Ia^q\left(p^{-1}x+q^{-1}y-r^{-1}z\right)\right.\\&\left.-Ia^q\left(p^{-1}x-q^{-1}y-r^{-1}z\right)-4Ia^q\left(p^{-1}x\right)-2\left[Ia^q\left(q^{-1}y\right)+Ia^q\left(-q^{-1}y\right)\right]\right\|\\&\qquad\le\left\|4Ia^q\left(p^{-1}x-q^{-1}y\right)\right\|+\Lambda\left(x,y,z\right)\end{aligned} \tag{2}$$

with $\left\|I_a^q(0)\right\|=0$ for all $x,y,z\in X$. Then there exists a unique additive mapping $A:X\to Y$ such that

$$\left\|h_a(x)-A(x)\right\|\le\frac{1}{3}\sum_{k=\frac{1-j}{2}}^{\infty}\frac{\Lambda(3^{kj}x,3^{kj}x,3^{kj}x)}{3^{kj}} \tag{3}$$

for all $x\in X$. The mapping $A(x)$ is defined by

$$A(x)=\lim_{n\to\infty}\frac{I_a(3^{nj}x)}{3^{nj}} \tag{4}$$

for all $x\in X$.

Proof. Hence the proof of all theorems and corollary in functinal inequality (1.2) with the condition $\left\|I_a^q(0)\right\|=0$ is similar to the stability results in functional equation (1. 1)

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Solving Transshipment Problem Using Zero Point and ICMM Method

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ABSTRACT: *Transshipment problem is a special case of transportation problem where the shipment takes place through transient nodes before reaching the final destination. In this paper, the transshipment problem is converted to a transportation problem and the converted transportation problem is solved using the proposed procedure namely Zero point method. The result obtain through zero point method is compared with the ICMM and VAM methods to point out the conclusion.*

Keywords: *ICMM method, VAM method, Initial basic feasible solution, Zero point method, Optimal solution.*

1. Introduction

The transportation problem is one of the subclasses of Linear programming problems. In a transportation problem shipments are allowed only between source-sink pairs. There is a possibility of existing points via which units of goods may be transshiped from a source to a sink. It is a strong assumption that shipments may be allowed between sources and between sinks and also inter-linking source-sink. Transportation models which have these additional features are called as transshipment problem.

The paper is structured as follows: Basic definitions and the Mathematical formulations of transportation problem and transshipment problem are reviewed in section 2. In section 3, the procedure is proposed for the converted transportation problem using zero point method. The section also includes the procedure for the transshipment problem. In section 4, suitable numerical examples are presented for the two types of transshipment models. Comparative study is made under results and discussions in section 5. Section 6 concludes the paper. It is followed by the list of references.

2. Preliminaries

2.1 Mathematical Formulation of Transportation Problem

Minimize $Z = \sum\sum c_{ij} x_{ij}$, i = 1,2,3,…..,m and j = 1,2,3,…..,n

subject to $\sum_{j=1}^{n} x_{ij} = a_i$, i = 1,2,3,…..,m

$\sum_{i=1}^{m} x_{ij} = b_j$, j = 1,2,3,…..,n

$x_{ij} \geq 0$ for all i,j.

2.2 Mathematical Formulation of Transshipment Problem

Min $Z = \sum\sum c_{ij} x_{ij}$, i=1,2,….m+n and j=1,2,…..m+n, j≠i.

subject to $\sum_{j=1, j\neq i}^{m+n} x_{ij} - \sum_{j=1, j\neq i}^{m+n} x_{ji} = a_i$, for all i = 1,2,…..m

$\sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} = b_j$, for all j = m+1,m+2,…..m+n

$x_{ij} \geq 0$, i,j = 1,2,……….m+n, j ≠ i

Transient nodes: It is the node through which the shipment takes place when the goods are transshipped on their journey from the sources to the destinations.

Buffer Amount: It is the Maximum (sum of the supplies, sum of the demands)

3. Procedure for Transportation problem and Transshipment problem

Procedure for ICMM method for transportation problem is developed by Priya et al.[3]. Pandian and Natarajan [2] proposed a new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. Elizabeth and Sujatha [1] proposed the procedure for transportation problem using Zero point method under fuzzy environment in k stages. In this paper, we have applied the Zero point method for crisp case.

3.1. Procedure for Transportation model using Zero point method

Step 3.1.1: Construct the transportation model. Check whether the given problem is a balanced one. If not convert the problem into the balanced one by introducing a dummy column or dummy row with cost entries '0'.
Step 3.1.2: Choose the least element in each row and subtract it from all the elements of that row.
Step 3.1.3: Choose the least element in each column and subtract it from all the elements of that column.
Step 3.1.4: Check whether each row and each column has atleast one '0' entry. If so, choose the row or column with only one '0' entry and allocate the minimum of supply and demand to that corresponding cell. If the supplies are fully used and the demands are fully received, the optimal solution is reached. If not, go to Step 3.1.5.
Step 3.1.5: Draw minimum number of lines horizontally and vertically to cover all the zeros. Choose the least uncovered element and subtract it from all the uncovered elements and add it at the intersection of lines. Now check whether each row and each column has atleast one '0' entry. If so go to Step 3.1.4.
Step 3.1.6: Repeat the procedure until the optimal solution is reached.

3.2. Procedure for Transshipment problem

Step 3.2.1: Construct the transshipment model where (i) Sources and destinations acting as the transient nodes or (ii) Intermediate nodes acting as the transient nodes.
Step 3.2.2: Convert the transshipment problem to the transportation problem by adding the buffer amount to the transient nodes.
Step 3.2.3: Obtain the optimal solution for the converted transportation problem by applying a) Zero point method or b) ICMM method. The optimal solution obtained will be the optimal solution to the given transshipment problem.

4. Numerical Examples

Example 4.1 (Sources and destinations acting as the transient nodes) The following is the transshipment problem with 4 sources and 2 destinations. The supply values of the sources S_1, S_2, S_3 and S_4 are 100, 200,150 and 350 units respectively. The demand values of destinations D_1 and D_2 are 350 and 450 units respectively. Transportation cost per unit between various defined sources and destinations are given in the following table 4.1. Solve the transshipment problem.

Table 4.1: Transshipment problem

Sources	Destinations						Supply
	S_1	S_2	S_3	S_4	D_1	D_2	
S_1	0	4	20	5	25	12	100
S_2	10	0	6	10	5	20	200
S_3	15	20	0	8	45	7	150
S_4	20	25	10	0	30	6	350
D_1	20	18	60	15	0	10	
D_2	10	25	30	23	4	0	
Demand					350	450	

Buffer amount = max (800,800) = 800

Table 4.2: Converted Transportation problem

Sources	Destinations						Supply
	S_1	S_2	S_3	S_4	D_1	D_2	
S_1	0	4	20	5	25	12	900
S_2	10	0	6	10	5	20	1000
S_3	15	20	0	8	45	7	950
S_4	20	25	10	0	30	6	1150
D_1	20	18	60	15	0	10	800
D_2	10	25	30	23	4	0	800
Demand	800	800	800	800	1150	1250	5600

a) Zero point method

By applying the procedure given in the subsection 3.1, we obtain the table 4.3.

Table 4.3: The final optimal table obtained using Zero point method

Sources	Destinations						Supply
	S_1	S_2	S_3	S_4	D_1	D_2	
S_1	800 *	100 *					900
S_2		700 *			300 *		1000
S_3			800 *			150 *	950
S_4				800 *		350 *	1150
D_1					800 *		800
D_2					50 *	750 *	800
Demand	800	800	800	800	1150	1250	5600

* denote the places of 0's

The optimal objective value for transshipment problem = Rs.5250.

b) ICMM method

Table 4.4: Initial basic feasible solution obtained using ICMM method

Sources	Destination						Supply
	S_1	S_2	S_3	S_4	D_1	D_2	
S_1	**800** 0	**100** 4	20	5	25	12	900
S_2	10	**700** 0	6	10	**300** 5	20	1000
S_3	15	20	**800** 0	8	45	**150** 7	950
S_4	20	25	10	**800** 0	30	**350** 6	1150
D_1	20	18	60	15	**800** 0	10	800
D_2	10	25	30	23	**50** 4	**750** 0	800
Demand	800	800	800	800	1150	1250	5600

Initial basic feasible solution = Rs.5250.

Table 4.5: The final optimal table obtained using MODI method

Sources	Destinations						Supply	u_i
	S_1	S_2	S_3	S_4	D_1	D_2		
S_1	**800** 0	**100** 4	20	5	25	12	900	5
S_2	10	**700** 0	6	10	**300** 5	20	1000	1
S_3	15	20	**800** 0	8	45	**150** 7	950	7

S_4	20	25	10	**800** 0	30	**350** 6	1150	6
D_1	20	18	60	15	**800** 0	10	800	-4
D_2	10	25	30	23	**50** 4	**750** 0	800	0
Demand	800	800	800	800	1150	1250	5600	
v_j	-5	-1	-7	-6	4	0		

All $\Delta_{ij} = c_{ij} - (u_i + v_j) \geq 0$. The optimal objective value obtained using MODI method =Rs.5250.

Example 4.2 (**Intermediate nodes acting as the transient nodes**) A company owns two factories (Factory 1 and Factory 2). Factory 1 and Factory 2 can produce up to 150 and 200 products per day, respectively. Products are shipped to Customer 1 and Customer 2. Each customer requires at least 130 products per day. It may be cheaper to first ship to Transfer center 1 or Transfer center 2 and then to the final customers. Minimize the total shipping cost.

Table 4.6: Transshipment problem

	C_1	C_2	T_1	T_2	Supply
F_1	25	28	8	13	150
F_2	26	25	15	12	200
T_1	16	17	0	6	
T_2	14	16	6	0	
Demand	130	130			

Buffer amount = max (350, 260) = 350

Table 4.7: Converted transportation problem

	C_1	C_2	T_1	T_2	Dummy column	Supply
F_1	25	28	8	13	0	150
F_2	26	25	15	12	0	200
T_1	16	17	0	6	0	350
T_2	14	16	6	0	0	350
Demand	130	130	350	350	90	1050

a) Zero point method

By applying the procedure given in the subsection 3.1, we obtain the table 4.8.

Table 4.8: The final optimal table obtained using Zero point method

	C_1	C_2	T_1	T_2	Dummy column	Supply
F_1			130 *		20 *	150
F_2		130 *			70 *	200
T_1	130 *	*	220 *			350
T_2	*			350 *		350
Demand	130	130	350	350	90	1050

* denote the places of 0's

The optimal objective value for transshipment problem = Rs.6370.

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b) ICMM method

Table 4.9: Initial basic feasible solution using ICMM method

	C_1	C_2	T_1	T_2	Dummy column	Supply
F_1	25	28	**60** 8	13	**90** 0	150
F_2	26	**130** 25	**70** 15	12	0	200
T_1	**130** 16	17	**220** 0	6	0	350
T_2	14	16	6	**350** 0	0	350
Demand	130	130	350	350	90	1050

Initial basic feasible solution = Rs. 6860.

Table 4.10: The final optimal table obtained using MODI method

	C_1	C_2	T_1	T_2	Dummy column	Supply	u_i
F_1	25	28	**130** 8	13	**20** 0	150	0
F_2	26	**130** 25	15	12	**70** 0	200	0
T_1	**130** 16	17	**220** 0	6	0	350	-8
T_2	14	16	6	**350** 0	0	350	-10
Demand	130	130	350	350	90	1050	
v_j	24	25	8	10	0		

All $\Delta_{ij} = c_{ij} - (u_i + v_j) \geq 0$. The optimal objective value obtained using MODI method = Rs.6370.

5. Results and Discussions

Table 5.1.

Methods	Example 4.1		Example 4.2	
	Initial basic feasible solution	Optimal solution	Initial basic feasible solution	Optimal solution
Zero point	-	5250	-	6370
ICMM	5250	5250	6860	6370
VAM	5250	5250	6370	6370

From the table 5.1, it is clear that the optimal solution obtained using Zero point method coincides with the optimal solution obtained using ICMM method. Also the optimal solution obtained using both the methods coincide with the optimal solution obtained using the traditional VAM method.

6. Conclusion

The ICMM method and VAM method gives only the Initial basic feasible solution and MODI method is applied to obtain the optimal solution, whereas the method proposed in this paper namely Zero point method gives the optimal solution directly. Hence, we conclude that the Zero point method is an easiest way to obtain the optimal solution for the transshipment problem.

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More on Pairwise Fuzzy σ-Baire Spaces

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ABSTRACT: *In this paper the several characterizations of pairwise fuzzy σ-Baire spaces are studied and inter relations between pairwise fuzzy σ-Baire spaces and pairwise fuzzy open hereditarily irresolvable spaces are studied.*

Keywords: *Pairwise fuzzy open set, pairwise fuzzy F_σ-set, pairwise fuzzy G_δ-set, pairwise fuzzy σ-nowhere dense set, pairwise fuzzy σ-first category sets, pairwise fuzzy Baire space.*

1. Introduction

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by L.A.Zadeh [14] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. In 1968, C.L.Chang [4] defined fuzzy topological space by using fuzzy sets introduced by Zadeh. The concept of σ-nowhere dense sets in classical topology was introduced and studied by Jiling Cao and Sina Greenwood in [6].

In 1989, A.Kandil [7] introduced and studied fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. The concept of Baire bitopological spaces have been studied extensively in classical topology in [1] ,[2] and [5]. The concept of Pairwise fuzzy σ-nowhere dense set is introduced and studied in [9]. By using pairwise fuzzy σ-nowhere dense sets, the concept of pairwise fuzzy σ-Baire space is studied by authors in [11], [12] and [13]. In this paper several characterizations of pairwise fuzzy σ-Baire spaces are studied and the inter relations between pairwise fuzzy σ-Baire spaces and pairwise fuzzy open hereditarily irresolvable spaces are investigated.

2. PRELIMINARIES

In order to make the exposition self-contained, some basic notations and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X. Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I.

Definition 2.4. [9] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ (i = 1, 2). The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set.

Definition 2.5. [9] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy open sets in (X, T_1, T_2).

Definition 2.6. [9] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy closed sets in (X, T_1, T_2).

Lemma 2.7. [3] For a family of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X, T), $\vee cl(\lambda_\alpha) \leq cl(\vee \lambda_\alpha)$. In case *A* is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee \lambda_\alpha)$. Also $\vee\, int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$ in (X, T).

Definition 2.8. [8] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T1}\, cl_{T2}(\lambda) = cl_{T2}\, cl_{T1}(\lambda) = 1$, in (X, T_1, T_2).

Definition 2.9 [10]. A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T1}\, cl_{T2}(\lambda) = int_{T2}\, cl_{T1}(\lambda) = 0$, in (X, T_1, T_2).

Definition 2.10. [9] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $int_{T1}int_{T2}(\lambda) = int_{T2}\, int_{T1}(\lambda) = 0$.

Definition 2.11. [11] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy σ-first category set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy σ-second category set in (X, T_1, T_2).

Definition 2.12. [11] If λ is a pairwise fuzzy σ-first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy σ-residual set in (X, T_1, T_2).

Definition 2.13. [11] A fuzzy bitopological space (X, T_1, T_2) is called pairwise fuzzy σ-first category space if the fuzzy set 1_X is a pairwise fuzzy σ-first category set in (X, T_1, T_2). That is $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Otherwise (X, T_1, T_2) will be called a pairwise fuzzy σ-second category space.

Definition 2.14. [10] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $int_{Ti}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(i=1,2)$ where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

3. PAIRWISE FUZZY σ -BAIRE SPACES

Definition 3.1 [9] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-Baire space if $int_{Ti}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(i = 1, 2)$ where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2).

Theorem 3.1 [11] Let (X,T_1,T_2) be a fuzzy bitopological space. Then the following are equivalent:
(1) (X,T_1,T_2) is a pairwise fuzzy σ-Baire space.
(2) $int_{Ti}(\lambda) = 0$, $(i = 1,2)$ for every pairwise fuzzy σ-first category set λ in (X,T_1,T_2).
(3) $cl_{Ti}(\mu) = 1$, $(i = 1,2)$ for every pairwise fuzzy σ-residual set μ in (X,T_1,T_2).

Proposition 3.1. If (X,T_1,T_2) is a pairewise fuzzy σ-Baire space. Then,
(i) $int_{Ti}int_{Tj}(\lambda)=0$, $(i=1,2)$ for every pairwise fuzzy σ-first category set λ in (X,T_1,T_2).
(ii) $cl_{Ti}cl_{Tj}(\mu)=1$, $(i=1,2)$ for every pairwise fuzzy σ-residual set μ in (X,T_1,T_2).

Proof. Let (X,T_1,T_2) be a pairwise fuzzy σ-Baire space.
(i) Let λ be a pairwise fuzzy σ-first category set in (X,T_1,T_2). Then by theorem 3.1, $int_{Ti}(\lambda)=0$. Now, $int_{Ti}int_{Tj}(\lambda)= int_{Ti}(int_{Tj}(\lambda)) = int_{Ti}(0) = 0$. Hence, $int_{Ti}int_{Tj}(\lambda)= 0$ $(i,j=1,2$ and $i \neq j)$ in (X,T_1,T_2).
(ii) Let μ be a pairwise fuzzy σ-residual set in (X,T_1,T_2). Then by theorem 3.1, $cl_{Ti}(\mu)=1$. Now, $cl_{Ti}cl_{Tj}(\mu)= cl_{Ti}(cl_{Tj}(\mu)) = cl_{Ti}(1) = 1$. Hence, $cl_{Ti}cl_{Tj}(\mu)= 1$ $(i,j=1,2$ and $i \neq j)$ in (X,T_1,T_2).

Proposition 3.2. If $cl_{Tj}(\lambda)$ $(j=1,2)$ is a pairwise fuzzy σ-first category set in a pairwise fuzzy σ-Baire space (X,T_1,T_2), then λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proof. Let $cl_{Tj}(\lambda)$ be a pairwise fuzzy σ-first category set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy σ-Baire space, by theorem 3.1, $int_{Ti}[cl_{Tj}(\lambda)] = 0$ $(i,j=1,2$ and $i \neq j)$ in (X,T_1,T_2). Therefore λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proposition 3.3. If $int_{Ti}(\mu)$ $(i=1,2)$ is a pairwise fuzzy σ-residual set in a pairwise fuzzy σ-Baire space (X,T_1,T_2), then $1-\mu$ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proof. Let $int_{Ti}(\mu)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy σ-Baire space, by theorem 3.1, $cl_{Tj}[int_{Ti}(\mu)] = 1$ $(i,j=1,2$ and $i \neq j)$ in (X,T_1,T_2). This implies that, $1- cl_{Tj}int_{Ti}(\mu) = 0$ and hence $int_{Ti}cl_{Tj}(1-\mu) = 0$ in (X,T_1,T_2). Thus $1-\mu$ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proposition 3.4. If $[int_{T_i}(\mu_k)]$'s (k=1 to ∞) are pairwise fuzzy σ-residual sets in a pairwise fuzzy σ-Baire space, then $cl_{T_i}[\wedge_{k=1}^{\infty}(\mu_k)] = 1$ in (X,T_1,T_2).

Proof. Let $[int_{T_i}(\mu_k)]$'s (k=1 to ∞) be a pairwise fuzzy σ-residual sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy σ-Baire space. By proposition 3.3, $(1-\mu_k)$'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Hence, $int_{T_i}[\vee_{k=1}^{\infty}(1-\mu_k)] = 0$ in (X,T_1,T_2). This implies that, $int_{T_i}[1-\wedge_{k=1}^{\infty}(\mu_k)] = 0$. Thus, $1-cl_{T_i}[\wedge_{k=1}^{\infty}(\mu_k)] = 0$ in (X,T_1,T_2), then $cl_{T_i}[\wedge_{k=1}^{\infty}(\mu_k)] = 1$ in (X,T_1,T_2).

Proposition 3.5. If λ is a pairwise fuzzy σ -nowhere dense set in a pairwise fuzzy topological space (X,T_1,T_2) , then λ is a pairwise fuzzy first category set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy σ-nowhere dense set in a pairwise fuzzy topological space (X,T_1,T_2) .Then λ is a pairwise fuzzy F_σ-set such that $int_{T_1}int_{T_2}(\lambda) = 0= int_{T_2}int_{T_1}(\lambda)$. Since λ is a pairwise fuzzy F_σ-set, $\lambda =\vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy closed sets in (X,T_1,T_2). Now $int_{T_1}int_{T_2}(\lambda) = 0= int_{T_2}int_{T_1}(\lambda)$ implies that $int_{T_1}int_{T_2}(\vee_{k=1}^{\infty}(\lambda_k)) = 0= int_{T_2}int_{T_1}(\vee_{k=1}^{\infty}(\lambda_k))$. But, from lemma 2.7., we have $\vee_{k=1}^{\infty}int_{T_i}(\lambda_k)\leq int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k))$ (i=1,2). This implies that $\vee_{k=1}^{\infty}int_{T_i}(\lambda_k)\leq 0$. That is $\vee_{k=1}^{\infty}int_{T_i}(\lambda_k)=0$. Hence we have $int_{T_1}int_{T_2}(\lambda_k) = 0= int_{T_2}int_{T_1}(\lambda_k)$ for each k. Since (λ_k)'s are pairwise fuzzy closed, $cl_{T_i}(\lambda_k) = \lambda_k$. Then $int_{T_i}cl_{T_i}(\lambda_k) = int_{T_i}(\lambda_k) =0$. That is, $int_{T_i}cl_{T_i}(\lambda_k) =0$ for each k. Hence (λ_k) 's are pairwise fuzzy nowhere dense sets in (X,T_1,T_2) . Therefore $\lambda =\vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k) 's are pairwise fuzzy nowhere dense sets, implies that λ is a pairwise fuzzy first category set in (X,T_1,T_2).

Proposition 3.6. If each pairwise fuzzy nowhere dense set λ is a pairwise fuzzy F_σ-set in a pairwise fuzzy topological space (X,T_1,T_2), then λ is a pairwise fuzzy first category set in (X,T_1,T_2).

Proof. Let each pairwise fuzzy nowhere dense set λ be a pairwise fuzzy F_σ-set in (X,T_1,T_2) .
Since λ is a pairwise fuzzy nowhere dense set in (X,T_1,T_2) , we have $int_{T_i}cl_{T_i}(\lambda) = 0$. Now
$int_{T_i}(\lambda) \leq int_{T_i}cl_{T_i}(\lambda)\leq 0$, implies that $int_{T_i}(\lambda) \leq 0$. That is, $int_{T_i}(\lambda) = 0$. Hence λ is a pairwise fuzzy F_σ -set such that $int_{T_i}(\lambda) = 0$. Then λ is a pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2) . Then, by proposition 3.5., λ is a pairwise fuzzy first category set in (X,T_1,T_2).

4. PAIRWISE FUZZY σ -BAIRE SPACE AND PAIRWISE FUZZY OPEN HEREDITARILY IRRESOLVABLE SPACES

Definition 4.1 A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy open hereditarily irresolvable space if $int_{T_1}cl_{T_2}(\lambda)\neq 0\neq int_{T_2}cl_{T_1}(\lambda)$, then $int_{T_1}int_{T_2}(\lambda)\neq 0\neq int_{T_2}int_{T_1}(\lambda)$ for any non-zero fuzzy set in (X,T_1,T_2).

Proposition 4.1 If (X,T_1,T_2) is a pairwise fuzzy open hereditarily irresolvable space, any pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2) is a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy σ-nowhere dense set in a pairwise fuzzy open hereditarily irresolvable space (X,T_1,T_2). Then λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2) such that $int_{T_1}int_{T_2}(\lambda)=0= int_{T_2}int_{T_1}(\lambda)$. Since (X,T_1,T_2) is a pairwise fuzzy open hereditarily irresolvable space, $int_{T_1}int_{T_2}(\lambda)=0=int_{T_2}int_{T_1}(\lambda)$ implies that $int_{T_1}cl_{T_2}(\lambda)=0= int_{T_2}cl_{T_1}(\lambda)$. Hence λ is a a pairwise fuzzy nowhere dense set in (X,T_1,T_2).

Proposition 4.2 If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy σ-Baire space and pairwise fuzzy open hereditarily irresolvable space, Then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proof. Let (X,T_1,T_2) be a pairwise fuzzy σ-Baire space and pairwise fuzzy open hereditarily irresolvable space. Then $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k))=0$ (i=1,2), Where (λ_k)'s are pairwise fuzzy σ-nowhere dense set in (X,T_1,T_2). By proposition 4.1, (λ_k)'s are pairwise fuzzy nowhere dense set in (X,T_1,T_2). Hence $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k))=0$, where (λ_k)'s are pairwise fuzzy nowhere dense set in (X,T_1,T_2). Therefore (X,T_1,T_2) is a pairwise fuzzy Baire space.

5. Conclusion

In this paper, conditions under which pairwise fuzzy σ -nowhere dense sets become pairwise fuzzy first category sets are established, the conditions under which pairwise fuzzy σ- first category sets become pairwise fuzzy first category sets are established, the conditions under which pairwise fuzzy σ- first category sets become pairwise fuzzy σ-Baire space are established. The conditions under which pairwise fuzzy σ-Baire space become pairwise fuzzy Baire spaces are also established. The inter relations between pairwise fuzzy open hereditarily irresolvable spaces are also studied.

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Algorithmic Structure of Smarandache-Lattice

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ABSTRACT: *In this paper, we introduced Smarandache-2-algebraic structure of Lattice namely Smarandache-. A samarandache-2-algebraic structure on a set N means a weak algebraic structure S_1 on N such that there exist a proper subset M of N, which is embedded with a stronger algebraic structure S_2, stronger algebraic structure means satisfying more axioms, that is S_1<<S_2, by proper subset one can understand a subset different from the empty set, from the unit element if any, from the whole set. We define Smarandache-Lattice and construct its algorithms through orthomodular lattice ,residuated lattice,pseudocomplment lattice, arbitrary lattice and congruence and ideal lattice . For basic concept of near-ring we refer to Padilla Raul [21] and for smarandache algebraic structure we refer to Florentin Smarandache [8]*

1. Introduction

In order that, new notions are introduced in algebra to better study the congruence in number theory by Florentin smarandache [8]. By<proper subset> of a set A we consider a set P included in A, and different from A, different from empty set and from the unit element in A-if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures S_1<<S_2 if : both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1laws or S_2 has more laws than S_1.

For example: Semi group<< Monoid <<group<<ring<<field, or semi group<< commutative semi group, ring<<unitary ring etc. they define a general special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is a structure, where SM<<SN.In additionwe have published [13],[14],[15],[16].

Thecharacterization of Smarandache-lattice by the substructures of Lattice namely orthomodular lattice ,ideal lattice, pseudo complement lattice, Arbitrary lattice, Residuated lattice was studied. From that it is observed that orthomodular lattice of Boolean algebra are {0} and itself, ideals of Boolean algebra are {0} and itself, pseudo complement lattice of Boolean algebra are {0} and itself, arbitrary lattice of Boolean algebra are {0} and itself, residuated lattice of Boolean algebra are {0} and itself, The converse of the above are also true if non-zero substructures are considered .Then the Boolean algebra itself is orthomodular lattice ,ideals, pseudo complement lattice, Arbitrary lattice, Residuated lattice by this hypothesis, in this paper algorithms to construct the Smarandache lattice from its characterization obtained in paper[13],[14],[15],[16] are obtained.

2. Preliminaries

Definition: 2.1

A partially ordered set $(L,<)$ is said to form a **Lattice** if for every $a,b\in L$, Sup {a, b} and Inf {a, b} exist in L. In that case, we write Sup {a, b} = $a\vee b$, Inf{a, b} = $a\wedge b$ Other notations like a + b and a. b or $a\cup b$ and $a\cap b$ are also used for Sup {a, b} and Inf {a, b}.

Definition: 2.2 (Lattice as an Algebraic Structure)

A **lattice** as an algebraic structure is a set on which two binary operations are defined, called join and meet, denoted by ∨ and ∧, satisfying the following axioms (i) Commutative law (ii) Associative law (iii) Absorption law (iv) Idempotent law.

Definition: 2.3
A **Boolean algebra** consists of a set B, two binary operations $\wedge$ and $\vee$ (called meet and join respectively), a unary operation $'$ and two constants 0 and 1.These obey the following laws: (i) Commutative Laws (ii) Associative Laws (iii) Distributive Laws (iv) Identity Laws (v) Complement Laws (vi) Idempotent Laws (vii) Null Laws(viii) Absorption Laws (ix) DeMorgan's Laws(x) Involution Law.

Definition 2.4
The **Smarandache lattice** is defined to be a lattice S, such that a proper subset of S is a Boolean algebra with respect to with same induced operations. By proper subset we understand a set included in S, different from the empty set, from the unit element if any, and from S.

Definition 2.5 (Alternate definition for Smarandache lattice)
If there exists superset of a Boolean algebra is a Lattice with respect to the same induced operations, then that Boolean algebra is said to be Smarandache lattice.

Definition 2.6
A **Boolean algebra** is **a lattice** that contains a least element and a greatest element and that is both complemented and distributive

Definition 2.7
An ortho poset P is called **orthomodular** if for every pair $a,b \in P$ with $a < b$ there is $a,c \in P$ such that $c \perp a$ and $b = a \vee c$.We will write shortly P instead of $< P,\leq,\wedge,\vee,' >$.For every $a,b \in P$ with $a \leq b$ let us denote $b - a = (b \wedge a') = (b' \vee a)' \in P$ According to the orthomodular law, $b = a \wedge (b-a) \in P$ $a,b \in P$ with $a \leq b$ and, moreover $a \perp (b-a)$.

Definition 2.8
The **orthomodular lattice** L is called **Boolean algebr**a if and if for every $a,b \in L$ the condition $a \wedge b = a \wedge b' = 0$ implies $a = 0$,

Definition 2.9
A **residuated lattice** is an algebra $< L \wedge,\vee \otimes,' \rightarrow,0,1 >$ such that
(i) $< L \leq,\wedge,\vee,' 0,1 >$ is Lattice (the corresponding order will be denoted by $\leq$) with the least element 0 and the greatest element 1 (ii) $< L \wedge,\vee \otimes,' \rightarrow,0,1 >$ is a commutative monoid (i.e. $\otimes$is commutative, associative, and $x \otimes x = 1$ holds) (iii) $x \otimes y \leq z$ if and only if $x \leq y \rightarrow z$ holds (adjointness condition).

Definition 2.10
A Boolean algebra is a **residuated lattice** which is both a Heyting algebra and an MV-algebra (relation to the usual axiomatization is $a \rightarrow b = \neg a \vee b$)

Definition 2.11
Let L be **a lattice** and $U \subseteq L$. U is said to be an **ideal** of L iff U is nonempty, $b \leq a \in U$ implies $B \in U$, and $a,b \in U$ implies $a \vee b \in U$.

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Definition 2.12

The **Lattice ideal** L is called a **Boolean algebr**a if for all $0 \in I$, $a \in I \Rightarrow b \leq a$ then $b \in I$, $a \vee b \in I$

Definition 2.13

The arbitrary **Lattice ideal** L is called a **Boolean algebr**a if $L \cong \square (L)$

Definition 2.14

The **Pseudo complement Lattice** L is called a **Boolean algebr**a if $a \subseteq L$ and $b \subseteq L$ are such that $a \bigcap a' = a$

3. Algorithms

In Gunder pliz[19] in section 1.60(d).The theorem by Gratzer and Fain is given the following conditions for a near ring $N \neq \{0\}$ are equivalent

1. $\bigcap I \neq \{0\}$, $\{0\} \neq 1 \subseteq N$ 2. N contains a unique minimal ideal, contained in all other non-zero ideals.

Cosequently the following conditions for a lattice $N \neq \{0\}$ are equivalent

1. $\bigcap I \neq \{0\}$, $\{0\} \neq 1 \subseteq N$ 2. N contains a unique minimal larttice ideal, contained in all other non-zero lattice ideals

Algorithms 3.1 (Orthomodular Lattice)

Step 1: Consider a non-empty set B
Step 2:
Verify that B is a Boolean algebra under meet and join
For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$

(ii) For all $a \in B$, $a \wedge a = a \in B$

2 (i) For all $a,b \in B$, $a \vee b = b \vee a \in B$,

(ii) For all $a,b \in B$, $a \wedge b = b \wedge a \in B$

3 (i) For all $a,b,c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$

(ii) For all $a,b,c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4 (i) For all $a,b \in B$, $a \vee (a \wedge b) = a \in B$

(ii) For all $a,b \in B$, $a \wedge (a \vee b) = a \in B$

5(i). For all $a,b,c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a,b,c \in B$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6 (i) For all $a'' = a$

7 (i) For all $a \in B$, $a \vee a' = 1 \in B$,

(ii) For all $a \in B$, $a \wedge a' = 0 \in B$

8(i) For all $a \in B$, $a \vee 0 = a \in B$,

(ii) For all $a \in B$, $a \wedge 1 = a \in B$

9(i) For all $a \in B$, $a \vee 1 = 1 \in B$,

(ii) For all $a \in B$, $a \wedge 0 = 0 \in B$

10(i) $a,b \in B$, $\overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) $a,b \in B$, $\overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied,
$(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let B_i , $i = 0,1,2 \ldots .n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j's from B_i's subject to for all $a,b \in B$ such that

$$a \wedge b = a \wedge b' \Rightarrow a = 0$$,

Step 7: $\bigcap B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice.

Algorithms 3.2 (Pseudo complemented lattice)

Step 1: Consider a non-empty set M
Step 2: Verify that B is a Boolean algebra under meet and join
For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$

(ii) For all $a \in B$, $a \wedge a = a \in B$

2 (i) For all $a,b \in B$, $a \vee b = b \vee a \in B$,

(ii) For all $a,b \in B$, $a \wedge b = b \wedge a \in B$

3. (i) For all $a,b,c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$

(ii) For all $a,b,c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4.(i) For all $a,b \in B$, $a \vee (a \wedge b) = a \in B$

(ii) For all $a,b \in B$, $a \wedge (a \vee b) = a \in B$

5(i). For all $a,b,c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a,b,c \in B$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6(i) For all $a'' = a$

7(i) For all $a \in B$, $a \vee a' = 1 \in B$,

(ii) For all $a \in B$, $a \wedge a' = 0 \in B$

8(i) For all $a \in B$, $a \vee 0 = a \in B$,

(ii) For all $a \in B$, $a \wedge 1 = a \in B$

9(i) For al $a \in B$, $a \vee 1 = 1 \in B$,

(ii) For all $a \in B$, $a \wedge 0 = 0 \in B$

10(i) $a, b \in B$, $\overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) $a, b \in B$, $\overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied,
$(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let B_i , $i = 0, 1, 2 \ldots n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j's from B_i's subject to for all $a \subseteq L$ and $b \subseteq L$ are such that $a \bigcap a' = a$

Step 7: $\bigcap B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice

Algorithmsn 3.3 (Residuated lattice**)**

Step 1: Consider a non-empty set M
Step 2: Verify that B is a Boolean algebra under meet and join
For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$

(ii) For all $a \in B$, $a \wedge a = a \in B$

2.(i) For all $a, b \in B$, $a \vee b = b \vee a \in B$,

(ii) For all $a, b \in B$, $a \wedge b = b \wedge a \in B$

3. (i) For all $a, b, c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$

(ii) For all $a, b, c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4.(i) For all $a, b \in B$, $a \vee (a \wedge b) = a \in B$

(ii) For all $a, b \in B$, $a \wedge (a \vee b) = a \in B$

5 (i). For all $a, b, c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a, b, c \in B$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6 (i) For all $a''=a$

7 (i) For all $a\in B$, $a\vee a'=1\in B$,

(ii) For all $a\in B$, $a\wedge a'=0\in B$

8 (i) For all $a\in B$, $a\vee 0=a\in B$,

(ii) For all $a\in B$, $a\wedge 1=a\in B$

9 (i) For al $a\in B$, $a\vee 1=1\in B$,

(ii) For all $a\in B$, $a\wedge 0=0\in B$

10 (i) $a,b\in B$, $\overline{(a\vee b)}=\bar{a}\wedge\bar{b}\in B$

(ii) $a,b\in B$, $\overline{(a\wedge b)}=\bar{a}\vee\bar{b}\in B$

If the above conditions are satisfied,
$(B,\wedge,\vee,',0,1)$ is a Boolean algebra.

Step 3: Let $B=B_0$

Step 4: Let B_i , $i=0,1,2....n$ be supersets of B_0

Step 5: Let $S=\bigcup_{i=1}^{n}B_i$

Step 6: Choose sets B_j's from B_i's subject to for all $a,b\in B$ such that $a\rightarrow b=\neg a\vee b$,

Step 7: $\bigcap B_j=B_0\neq\{0\}\subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice

Algorithms 3.4 (Arbitrary lattice and Congruence's)

Step 1: Consider a non-empty set M

Step 2: Verify that B is a Boolean algebra under meet and join

For, check the following conditions

1. (i) For all $a\in B$, $a\vee a=a\in B$

(ii) For all $a\in B$, $a\wedge a=a\in B$

2.(i) For all $a,b\in B$, $a\vee b=b\vee a\in B$,

(ii) For all $a,b\in B$, $a\wedge b=b\wedge a\in B$

3. (i) For all $a,b,c\in B$, $a\vee(b\vee c)=(a\vee b)\vee c\in B$

(ii) For all $a,b,c\in B$, $a\wedge(b\wedge c)=(a\wedge b)\wedge c\in B$

4.(i) For all $a,b\in B$, $a\vee(a\wedge b)=a\in B$

(ii) For all $a,b\in B$, $a\wedge(a\vee b)=a\in B$

5(i). For all $a,b,c \in B$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a,b,c \in B$, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6(i) For all $a'' = a$

7(i) For all $a \in B$, $a \vee a' = 1 \in B$,

(ii) For all $a \in B$, $a \wedge a' = 0 \in B$

8(i) For all $a \in B$, $a \vee 0 = a \in B$,

(ii) For all $a \in B$, $a \wedge 1 = a \in B$

9(i) For al $a \in B$, $a \vee 1 = 1 \in B$,

(ii) For all $a \in B$, $a \wedge 0 = 0 \in B$

10(i) $a,b \in B$, $\overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) $a,b \in B$, $\overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied,
$(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let B_i , $i = 0,1,2 \ldots n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j's from B_i's subject to for all $L \cong \square\,(L)$ (Isomorphic to congruence of L)

Step 7: $\bigcap B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice

Algorithms 3.5 (Lattice Ideal)

Step 1: Consider a non-empty set M
Step 2: Verify that B is a Boolean algebra under meet and join
For, check the following conditions

1. (i) For all $a \in B$, $a \vee a = a \in B$

(ii) For all $a \in B$, $a \wedge a = a \in B$

2.(i) For all $a,b \in B$, $a \vee b = b \vee a \in B$,

(ii) For all $a,b \in B$, $a \wedge b = b \wedge a \in B$

3. (i) For all $a,b,c \in B$, $a \vee (b \vee c) = (a \vee b) \vee c \in B$

(ii) For all $a,b,c \in B$, $a \wedge (b \wedge c) = (a \wedge b) \wedge c \in B$

4.(i) For all $a,b \in B$, $a \vee (a \wedge b) = a \in B$

(ii) For all $a,b \in B,\ a \wedge (a \vee b) = a \in B$

5(i). For all $a,b,c \in B,\ a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in B$

(ii) For all $a,b,c \in B,\ a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \in B$

6 (i) For all $a'' = a$

7 (i) For all $a \in B,\ a \vee a' = 1 \in B$,

(ii) For all $a \in B,\ a \wedge a' = 0 \in B$

8 (i) For all $a \in B,\ a \vee 0 = a \in B$,

(ii) For all $a \in B,\ a \wedge 1 = a \in B$

9 (i) For all $a \in B,\ a \vee 1 = 1 \in B$,

(ii) For all $a \in B,\ a \wedge 0 = 0 \in B$

10 (i) For all $a,b \in B,\ \overline{(a \vee b)} = \bar{a} \wedge \bar{b} \in B$

(ii) For all $a,b \in B,\ \overline{(a \wedge b)} = \bar{a} \vee \bar{b} \in B$

If the above conditions are satisfied,
$(B, \wedge, \vee, ', 0, 1)$ is a Boolean algebra.

Step 3: Let $B = B_0$

Step 4: Let $B_i\ ,\ i = 0,1,2\ldots n$ be supersets of B_0

Step 5: Let $S = \bigcup_{i=1}^{n} B_i$

Step 6: Choose sets B_j's from B_i's subject to for all $0 \in I$ if $a \in I \Rightarrow b \leq a$ then $b \in I$
$a \vee b \in I$

Step 7: $\bigcap B_j = B_0 \neq \{0\} \subset S$

Step 8: If step (7) is a true, then we write S is a Smarandache-Lattice

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A Study on Domination of Distinct Non-Zero Zero Divisor Graphs and Its Complement Graph

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ABSTRACT: *In this paper we established the bounds for domination of distinct non-zero zero divisor graphs and its complement graph.*

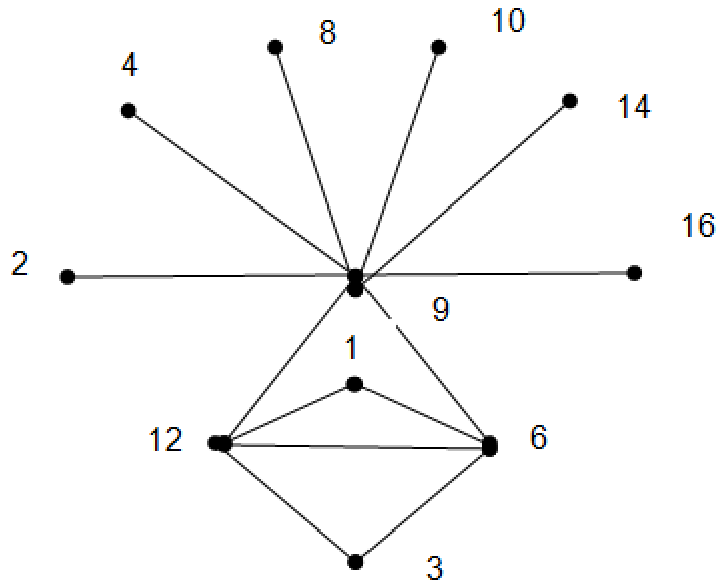
Keywords: *Non-zero zero divisor grap,7p zero divisor graph,pq zero divisor graph,p^2q zero divisor graph,Complement of zero divisor graph,Domination number.*

1. Introduction

All the graphs considered here are simple ,finite,connected and undirected .In this paper we have taken some of the non-zero zero divisor graphs such as 7p,pq,p^2q,where p&q are prime number.we determined the domination number of distinct non-zero divisor graphs and its complement graph.The zero divisor graph (Beck) of a commutative ring R with unit element 1 is a simple graph whose set of vertices consists of all elements of the ring R with an edge (x,y) iff xy=0.It is denoted by Γ (Z)

Definition: 1.1[2] The **Zero-divisor graph** [Beck] of a commutative ring R with unit element 1 is a simple graph whose set of vertices consists of all elements of the ring with an edge defined between a and b if and only if ab=0.Its denoted by Γ (Z)

Example:1.2
i)Let, Z_{18}=The set of all congruent modulo 18.
Γ (Z_{18})= {(9,2),(9,4),(9,6),(9,8),(9,10),(9,12),(9,14),(9,16),(12,3),(12,6),(12,15), (6.3),(6,15)}



ii) Z_{22}= The set of all congruence modulo 22, $\Gamma(Z_{22})$= {(11,2), (11,4), (11,6), (11,8), (11,10), (11,12), (11,14),11,16), (11,18),(11,20)} .we get a star graph.

Definition:1.3[10]
The **complement** or inverse of a graph of a graph G is denoted by $\bar{G}$ on the same vertices such that two distinct vertices of G are adjacent if and only if they are not adjacent in $\bar{G}$

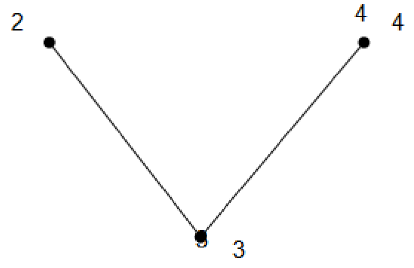
Definition:1.4[7]
Let G = (V,E) be a graph. The sub set D is said to be a **dominating set** of G if for each v∈ V eitherv∈ V or v is adjacent to some vertex in D.

Definition:1.5[8]
The minimum number of a dominating set in G is called **domination number** of G. And it is denoted by γ(G).

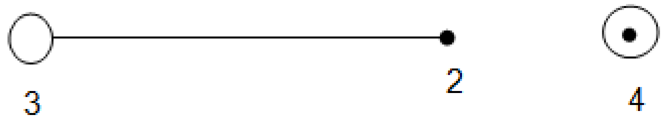
2. INTRODUCTION:
In this section, we discussed some basic definitions, examples, priliminaries and domination number of pq,7p, p^2q non-zero zero-divisor graphs and its complement graph.

Definition2.1:[2]pq zero divisor graph and its complement graph
If p and q are distinct then the pq non-zero zero divisor graph will be a complete bipartite graph. On one side of the graph we can list all the factors of the first prime number and on the other side we can list all the factors of the second prime. Then we construct a complete bipartite graph. It isdenoted by $\Gamma(Z_{pq})$.

Example :2.2
Consider,Z_6= {0,1,2,3,4,5},$Z_{3.2}$ = Z_6, the set of congruence modulo 6 and
$\Gamma(Z_{3.2}) = \Gamma(Z_6)$= {(2,3),(4,3)} be the set of non-zero zero divisor graph.



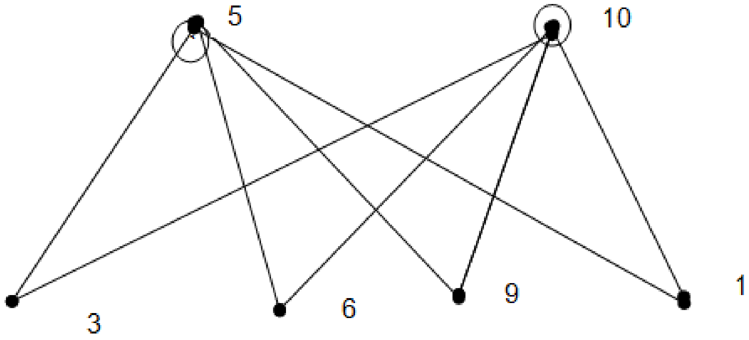
The minimum dominating set D={3},Domination number $\gamma(\Gamma(Z_6))$=1`
The complement of $\Gamma(Z_6)$ graphis a disconnected graph with two components and it is denoted by $\Gamma(\bar{Z}_6)$, which is shown below.i.e) $\Gamma(\bar{Z}_6)$ ={(4,2)} and {3}



The dominating sets are D_1 ={3} and D_2 = {4},The domination number of each component is 1.The domination number of $\Gamma(\bar{Z}_6)$is 2.i.e) $\gamma(\Gamma(\bar{Z}_6))$= 2

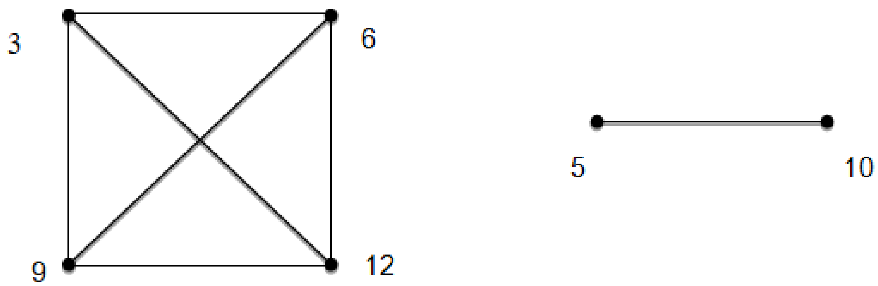
Example:2.3

Consider Z_{15} = {0,1,2,3,4,5,6,7,8,9,10,11,12,13,14},Let $Z_{5.3}=Z_{15}$ be the set of congruence modulo 15 and Γ (Z_{15})={(5,3),`(5,6), (5,9), (5,12), (10,3), (10,6), (10,9), (10,12)} be the set of non-zero zero divisor graph.



The minimum dominating set is D={5,10} andThe domination number $\gamma(\Gamma(Z_{15}))= 2$

The complement of Γ (Z_{15}) graph is a disconnected graph with two components and it is denoted by Γ $(\bar{Z}_{15})$, which is shown in belowi.e) Γ $(\bar{Z}_{15})$={(3,6), (3,9),(3,12),(6,9),(6,12),(9,12)} and {(5,10)}



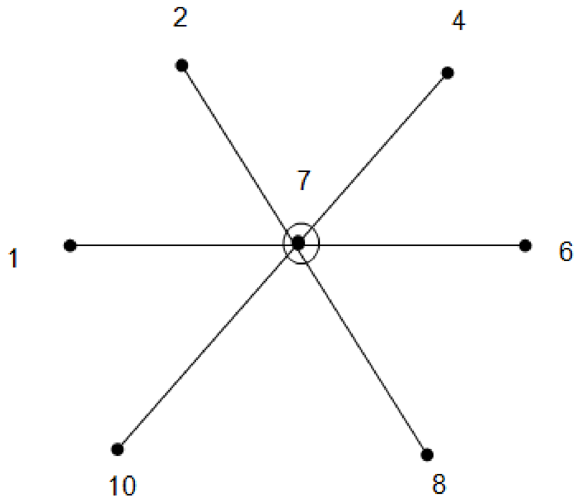
The dominating sets are D_1= {3} and D_2= {5}The domination number of each component is 1.The domination number of Γ $(\bar{Z}_{15})$ is 2.i.e) $\gamma(\Gamma(\bar{Z}_{15})) = 2$

Definition:2.4[2] 7p Zero divisor graphs and its complement graphs

Let Γ (Z_{7p}) the complete bipartite graph. On the one side we can list all the factors of 7 and on the other side we can list all the factors of prime p, $\forall$ prime 'p'.It is denoted by Γ (Z_{7p}).

Example: 2.5

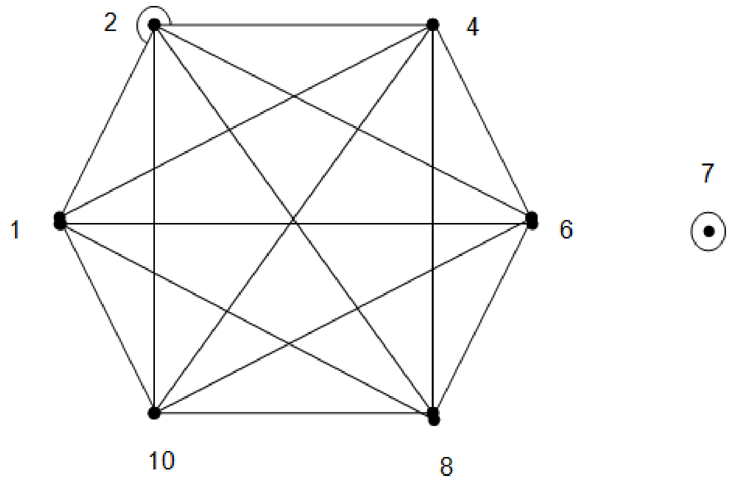
Consider Z_{14}= {0,1,2,3,4,5,6,7,8,9,10,11,12,13},$Z_{7.2}=Z_{14}$ be the set of congruence modulo 14.
Γ $(Z_{7.2})$ = Γ (Z_{14}) = {(7,2),(7,4),(7,6),(7,8),(7,10),(7,12)} be the set of non-zero zero divisor graph.



The minimum dominating set D= {7}.The domination number $\gamma(\Gamma(Z_{14})) = 1$

The complement of $\Gamma(Z_{14})$ is a disconnected graph with two components and it is denoted by $\Gamma(\bar{Z}_{14})$,Which is shown in below.

$\Gamma(\bar{Z}_{14})$= {(2,4),(2,4),(2,6),(2,8),(2,10),(2,12),(4,6),(4,8),(4,10),(4,12),(6,8),(6,10),(6,12),
(8,10),(8,12),(12,15)} and {7}

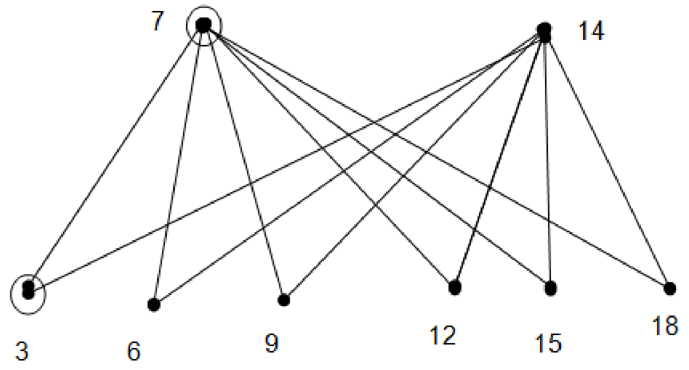


The dominating sets are D_1= {2} and D_2= {7},The domination number of each component is 1.The domination number of $\Gamma(\bar{Z}_{14})$ is 2.i.e) $\gamma(\Gamma(\bar{Z}_{14})) = 2$

Example: 2.6

Consider Z_{21} = {0,1,2,3,4,5,6,...,20},$Z_{7.3}$= Z_{21} be the set of congruence modulo 21.

$\Gamma(Z_{21})$= {(7,3),(7,6),(7,9),(7,12),(7,15),(7,18),(14,3),(14,3),(14,6),(14,9),(14,12),
(14,15),(14,18)} be the set of non- zero zero divisor graph.



The minimum dominating set D= {7,3},Domination number $\gamma(\Gamma(Z_{21})) = 2$

The complement of $\Gamma(Z_{21})$ is a disconnected graph with two components and it is denoted by $\Gamma(\bar{Z}_{21})$, which is shown below.

$\Gamma(\bar{Z}_{21})$ = {(7,14)}and{(3,6),(3,9),(3,12),(3,15),(3,18),(6,9),(6,12),(6,15),(6,18), (9,12),(9,15),(9,18),(12,15),(12,18),(15,18),}

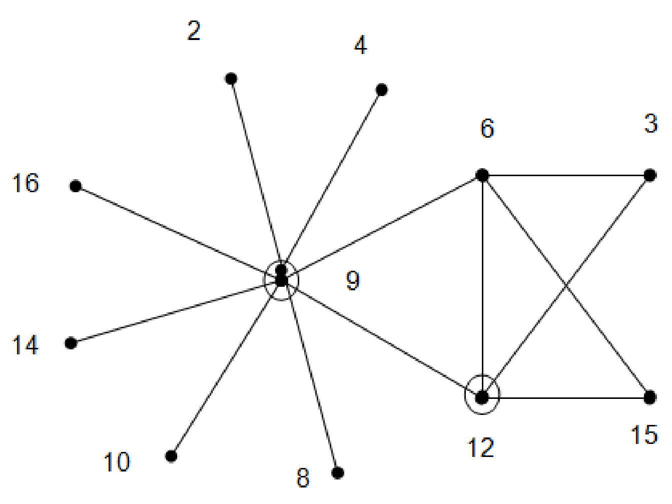
The dominating sets are D_1= {7} and D_2= {3},The domination number of each component is 1.The domination number of $\Gamma(\bar{Z}_{21})$ is 2. (i.e) $\gamma(\Gamma(\bar{Z}_{21})) = 2$

Definition:2.7[2]p^2q Zero divisor graphs and its complement graphs In the p^2q case, the center of the p^2q non-zero zero divisor graph will be $(p^2q)/2$. The vertices connected to $(p^2q)/2$ will be multiples of pq, these multiples will form a complete bipartite graph The tail vertices are qr where r is relatively prime to p. The head vertices are rp where r is relatively prime to q and the center node is p^2. The gill vertices are of pq.It is denoted by $\Gamma(Z_{18})$.

Example: 2.8

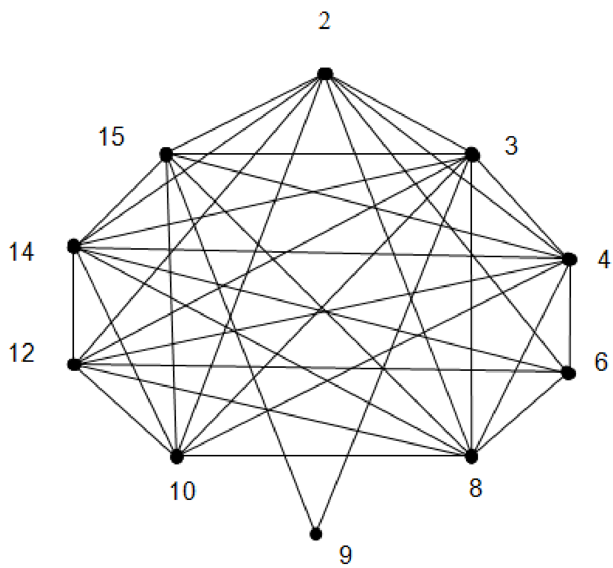
Consider Z_{18} = {0,1,2,3,4,5,.....,17}

Let $Z_{3^2.2}= Z_{18}$ be the set of congruence modulo 18 and $\Gamma(Z_{18})$ = {(9,2), (9,4), (9,6), (9,8), (9,10), (9,12), (9,14), (9,16),(6,3), (6,12), (6,15),(12,3), (12,15)} be the set of non-zero zero divisor graph.



Dominating set D= {9,12},Dominating number $\gamma(\Gamma(Z_{18}))$= 2The complement of $\Gamma(Z_{18})$ is a disconnected graph with two components and it is denoted by

Γ ($\bar{Z}_{18}$), which is shown below. Γ ($\bar{Z}_{18}$) = {(2,3), (2,4), (2,6), (2,8), (2,10), (2,12), (2,14), (2,15),(3,4), (3,8), (3,9),
(3,10), (3,14), (3,15),(4,6), (4,8), (4,10), (4,12), (4,14), (4,15),(6,8), (6,10),
(6,14),(6,15),(8,10),(8,12),(8,14),(8,15),(9,3),(9,15),(10,12),(10,14),
(10,15),(12,14),(14,15)}

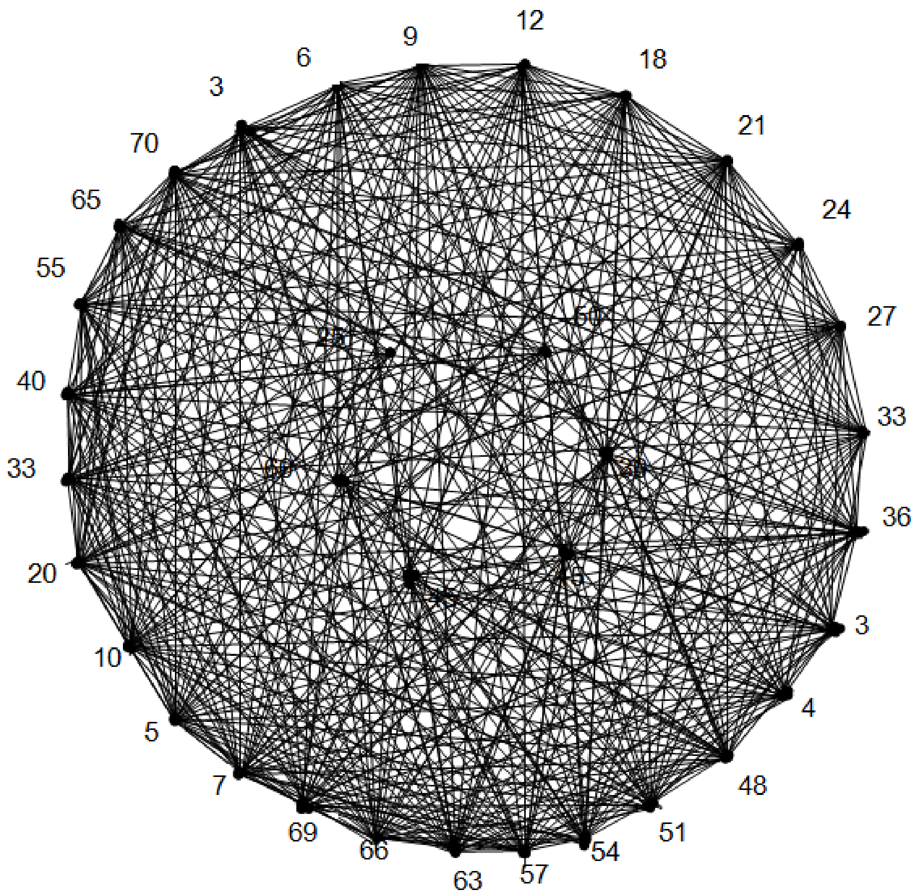


The dominating sets are D_1= {2,9},∴The domination number of Γ ($\bar{Z}_{18}$) is 2. i.e) γ(Γ ($\bar{Z}_{18}$)) = 2

Example:2.9
Consider, Z_{75} = {0,1,2,3,4,5,6,7,……..,74}$Z_{5^2.3}$=Z_{75} be the set of congruence modulo 75 and
Γ (Z_{75})=
{(25,3),(25,6),(25,9),(25,12),(25,15),(25,18),(25,21),(25,24),(25,27),(25,30),(25,33),(25,36),(25,39),(25,42),(25,45),(25,48),(25,51),(25,54),(25,57),(25,60),(25,63),(25,66),(25,69),(25,72),(50,3),(50,6),(50,9),(50,12),(50,15),(50,18),(50,21),(50,24),(50,27),(50,30),(50,33),(50,36),(50,39),(50,42),(50,45),(50,48),(50,51),(50,54),(50,57),(50,60),(50,63),(50,66),(50,69),(50,72),(5,15),(5,130),(5,45),(5,60),(10,15),(10,30),(10,45),(10,60),(15,2),(15,30),(15,35),(15,40),(15,55),(15,65),(15,70),(20,15),(20,30),(20,45),(20,60),(30,35),(30,40),(30,55),(30,65),(30,70),(35,15),(35,30),(35,45),(35,60),(40,15),(40,30),(40,45),(40,60),(45,55),(45,60),(45,65),(45,70),(55,60)} be the set of non-zero zero divisor graph.

The dominating set D= {3,70}The domination numberof Γ (Z_{75}) is 2.
i.e)γ(Γ (Z_{75})) = 2 The complement graph of Γ (Z_{75}) has single component and it is denoted by Γ ($\bar{Z}_{75}$),
Γ ($\bar{Z}_{75}$) ={(3,6),(3,9),(3,12),…,(3,72),(6,9),(6,12),(6,15),…..,(6,72),(9,12),(9,15),(9,18)….,(9,72),(12,15),…,(12,72),(15,18),…,(15,72),(18,21),…,(18,72),(21,24),…,(21,72),(24,27)….,(24,72),(27,30),…,(27,72),(30,33),…,(30,72),(33,36),…,(33,72),(36,39),…,(36,72),(39,42),…..,(39,72),(42,45),…,(42,72),(45,48),…..,(45,72),(48,51),…,(48,72),(51,54),..,(51,72),(54,5),.., (54,72),(57,60),..,(57,72),(60,63),..,(60,72),(63,66),..,(63,72),(66,69),..,(66,72),(69,72), (3,5),(6,5),(9,5), (12,5),(18,5),(21,5),(24,5),(27,5),(33,5),(36,5)(39,5),(42,5),(48,5),(51,5), (54,5),(57,5),(63,5),(66,5),(69,5),(72,5),(6,10),(9,10),(12,10),(18,10),(21,10),(24,10), (27,10), (33,10),(36,10),(39,10),(42,10),(48,10),(51,10),(54,10),(57,10),(63,10),(66,10),(69,10),(72,1), (3,20),(6,20),(9,20),(12,20),(18,20),(21,20),(24,20),(27,20),(33,20),(36,20),(39,20),(42,20), (48,20),(51,20),(54,20),(57,20),(63,20),(66,20),(69,20),(72,20),(3,35),(6,35),(9,35),(12,35),(18,35),(21,35),(24,35),(27,35),(33,35),(36,35),(39,35),(42,35),(48,35),(51,35),(54,35),(57,3), (63,35),(66,35),(69,35),(72,35),(3,40),(6,40),(9,40),(12,40),(18,40),(21,40),(24,40),(27,40), (33,40),(36,40),(39,40),(42,40),(48,40),(51,40),(54,40),(57,40),(63,40),(66,40),(69,40),(72,4), (3,55),(6,55),(9,55),(12,55),(18,55),(21,55),(24,55),(27,55),(33,55),(36,55),(39,55),(42,55), (48,55),(51,55),(54,55),(57,55),(63,55),

(66,55),(69,55),(72,55),(3,65),(6,65),(9,65),(12,65), (18,65),(21,65),(24,65),(27,65),(33,65),(36,65),(39,6),(42,65),(48,65),(51,65),(54,65),(57,65), (63,65),(66,65),(69,65),(72,65),(3,70),(6,70),(9,70),(12,70),(18,70),(21,70),(24,70),(27,70), (33,70),(36,70),(39,70),(42,70),(48,70),(51,70),(54,70),(57,70),(63,70),(66,70),(69,70), (72,70),(5,10),(5,20),(5,25),(5,35),(5,40),(5,50),(5,55),(5,65),(5,70),(10,20),(10,25),(10,35),(10,40),(10,50),(10,55),(10,65),(10,70),(20,25),(20,35),(20,40),(20,50),(20,55),(20,65),(20,7),(25,35),(25,40),(25,50),(25,55),(25,65),(25,70),(25,35),(25,40),(25,50),(25,55),(25,65),(25,7),(35,40),(35,50),(35,55),(35,65),(35,70),(35,35),(35,40),(35,50),(35,55),(35,65),(35,70), (35,40), (35,50),(35,55),(35,65),(35,70),(40,50),(50,55),(50,65),(50,70),(65,55),(65,70)}



The dominating set D_1= {3,70} The dominating number of $\Gamma(\bar{Z}_{75})$ is 2.i.e) $\gamma(\Gamma(\bar{Z}_{75}))$ = 2

3. Relation Between the Domination Number of Different Types of Zero Divisor Graphs and Its Complement Graphs

3.1 Introduction

In this section we discussed some theorems with examples.

3.2 Theorems on Zero divisor graph

Theorem:3.2.1

Let $\Gamma(Z_n)$ and $\Gamma(\bar{Z}_n)$be a non-zero divisor graph and complement of a non-zero zero divisor graph respectively, If n=pq where p,q are prime numbers and p>q. Then the domination number of non-zero zero divisor is given by

1. If n= even,

Then $\gamma(\Gamma(Z_n)) < \gamma(\Gamma(\bar{Z}_n))$

$= \gamma(\Gamma(Z_n)) + 1$

2. If n= odd,

Then $\gamma(\Gamma(Z_n)) = \gamma(\Gamma(\bar{Z}_n)) = 2$

Proof:

Let n = pq

Where p,q are primes and q>p

i) If n = even

We get the 2p zero divisor graphs.Because '2' is the only even prime number.

"If n=2p where p is an odd prime then the domination number of non-zero zero divisor graph is 1 "$\therefore \gamma(\Gamma(Z_n)) = 1$. ____(1)

Similarly,

If we find the complement of this graph ,Then we get the disconnected graph and it has 2 components which are complete graphs.The domination number of each component is 1 andThe domination number of complement graph is 2.

(i.e) $\gamma(\Gamma(\bar{Z}_n)) = 2$ _____(2)

From (1) and (2)

$\gamma(\Gamma(Z_n)) < \gamma(\Gamma(\bar{Z}_n))$ where n= even number.

ii)If n= odd ,We get the 3p,5p,7p,……. non-zero zero divisor graph. These graphs are complete bipartite graphs.The domination number of complete bipartite graph is 2. And hence

$\gamma(\Gamma(Z_n)) = 2$ _______(1)

Similarly,If we obtain the complement of this graphs then we get the disconnected graph and it has 2 component which are complete graphs.The domination number of each component is 1.$\therefore$ The domination number of complement graph is 2.

$(i.e)\gamma(\Gamma(\bar{Z}_n)) = 2$. _____(2)

From (1) and (2)

$\gamma(\Gamma(Z_n)) = \gamma(\Gamma(\bar{Z}_n)) = 2$. Where n is odd number

Hence we get from eqns 1&2 For all even number n,

$\gamma(\Gamma(Zn)) < \gamma(\Gamma(\bar{Z}_n))$

$= \gamma(\Gamma(Z_n)) + 1$

For all Odd number n,

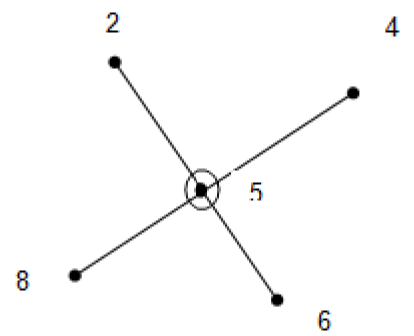
$\gamma(\Gamma(Z_n)) = \gamma(\Gamma(\bar{Z}_n)) = 2$.

Example:3.2.1

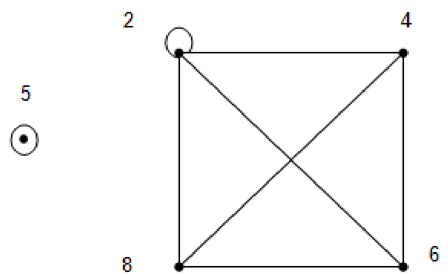
i)p=even number

$Z_{2.5} = Z_{10}$be the set of congruence modulo 10 and

$\Gamma(Z_{10}) = \{(5,2), (5,4), (5,6), (5,8)\}$ be the set of non-zero zero divisor graph.

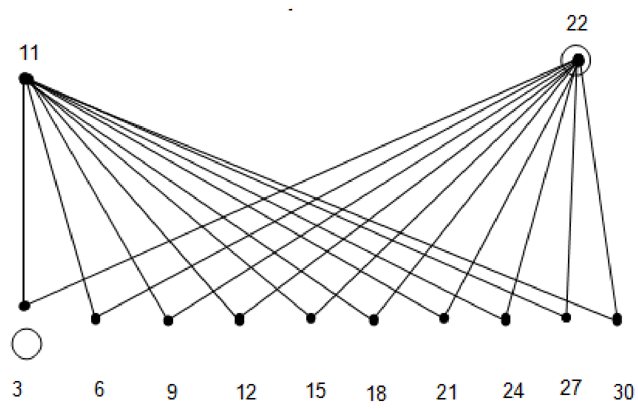


The dominating set D={5},The minimum dominating number of Γ (Z_{10}) is 1
i.e)γ(Γ (Z_{10}))= 1.The complement of Γ (Z_{10}) is a disconnectedgraph with two components and it is denoted by Γ ($\bar{Z}_{10}$), which is shown below.
Γ ($\bar{Z}_{10}$)={5} and {(2,4), (2,6), (2,8), (4,6),(4,8),(6,8)}



The dominating sets are D_1= {5} and D_2= {2}The domination number of each component is 1.The domination number of Γ ($\bar{\boldsymbol{Z}}_{\mathbf{10}}$) is 2. i.e) γ(Γ ($\bar{Z}_{10}$)) = 2.

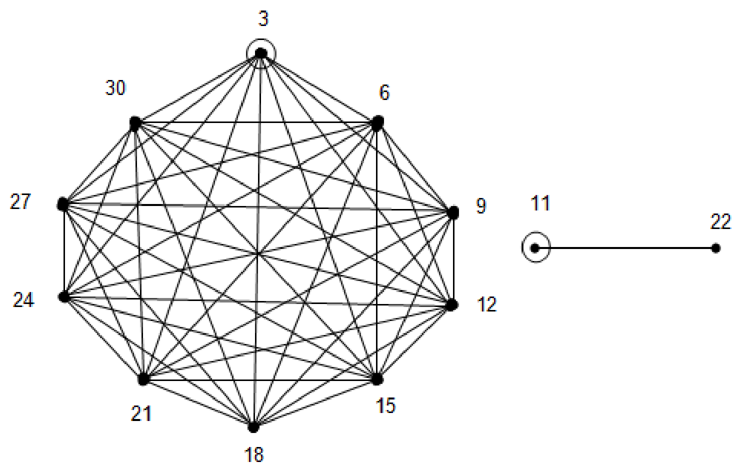
ii)p=odd number
$Z_{11.3}$ = Z_{33}be the set of congruence modulo 33 and
Γ (Z_{10}) ={(11,3),(11,6),(11,9),(11,12),(11,15),(11,18),(11,21),(11,24),(11,27),(11,30),(22,3),(22,6),(22,9),(22,12),(22,15),(22,18),(22,21),(22,24),(22,27),(22,30)}be the set of non-zero zero divisor graph.



The dominating set D={22,3}The dominating number of Γ (Z_{33}) is 2 i.e) γ(Γ (Z_{33}))= 2

The compement of $\Gamma(Z_{33})$ is a disconnected graph with two componentsand it is denoted by $\Gamma(\bar{Z}_{33})$,which is shown below.

$\Gamma(\bar{Z}_{33})$={(11,22)} and {(3,6),(3,9),(3,12),(3,15),(3,18),(3,21),(3,24),(3,27),(3,30),(6,9),(6,12), (6,15),(6,18),(6,21),(6,24),(6,27),(6,30),(9,12),(9,15),(9,18),(9,21),(9,24),(9,27),(9,30),(12,15),(12,18),(12,21),(12,24),(12,27),(12,30),(15,18),(15,21),(15,24),(15,27),(15,30),(18,21),(18,24),(18,27),(18,30),(21,24),(21,27),(21,30),(24,27),(24,30), (27,30)}



The dominating sets are D_1= {11} and D_2= {3}The domination number of each component is 1.The domination number of $\Gamma(\bar{Z}_{33})$ is 2. i.e) $\gamma(\Gamma(\bar{Z}_{33})) = 2$.

Theorem:3.2.2

Let $\Gamma(Z_n)$ and $\Gamma(\bar{Z}_n)$ be a non-zero divisor graph and complement of a non-zero zero divisor graph respectively,

If n=7p where p is prime number. Then the domination number of non-zero zero divisor is given by

i. If p= even prime then $\gamma(\Gamma(Z_n)) < \gamma(\Gamma(\bar{Z}_n))$ (or) $\gamma(\Gamma(\bar{Z}_n)) = \gamma(\Gamma(Z_n))+1$
ii. If p= odd prime then $\gamma(\Gamma(Z_n)) = \gamma(\Gamma(\bar{Z}_n)) = 2$
iii. If p= even then $\gamma(\Gamma(Z_n)) < \gamma(\Gamma(\bar{Z}_n)) = 6$

Proof: Let $\Gamma(Z_n)$ and $\Gamma(\bar{Z}_n)$ be a non-zero divisor graph and complement of a non-zero zero divisor graphs respectively,

Case (i) when n= 7p,Assume that p= even prime number.Then the non-zero zero divisor graph is a star graph.The minimum dominating set of the star graph has only one element.∴ The dominating number of the non-zero zero divisor graph is 1.(i.e) $\gamma(\Gamma(Z_n)) = 1$ ____(1)

Similarly,We obtain the complement of this graph and it is the disconnected graph which has 2 components and are complete graphs.The domination number of each component is 1.∴ The domination number of complement of non-zero zero divisor graph is 2.

(i.e) $\Gamma(\bar{Z}_n) = 2$ ________(2)

From (1) and (2)

$\gamma(\Gamma(Z_n)) < \gamma(\Gamma(\bar{Z}_n))$ (or) $\gamma(\Gamma(\bar{Z}_n)) = \gamma(\Gamma(Z_n))+1$.

case(ii)If p= Odd prime number then the vertex set V= $\mathbb{Z}(R^*)$= {7,21,35,......(7p.7).3p.5p.......}

We get the $\Gamma(\mathbb{Z}_{pq})$ non-zero zero divisor graph.We know that all the pq- zero divisor graphs are complete bipartite graph.The dominating number of the complete bipartite graph is 2.∴ The domination number of $\Gamma(\mathbb{Z}_{7p})$ non-zero zero divisor graph is 2.

(i.e) $\gamma(\Gamma(Z_n)) = 2$ where p= odd prime ____(1)

Similarly,We obtain the complement of this graph.The complement of $(\Gamma(Z_n))$ is disconnected and it has 2 component which are complete graphs.∴The domination number of complement of non-zero zero divisor graph is 2. (i.e) $\gamma(\Gamma(\bar{Z}_n)) = 2$ _______(2)

From (1) and (2) we have $\gamma(\Gamma(Z_n)) = \gamma\,\Gamma(\bar{Z}_n) = 2$

case(iii)If p=7 then $\gamma\,\Gamma(Z_n) = \gamma\,\Gamma(Z_{p^2})$

Where n=7p. We know that all the p^2 non-zero zero divisor graphs are complete graphs.The domination number of p^2 non-zero zero divisor graph is 1.

(i.e) $\gamma(\Gamma(Z_n) = 1$,where p=7.

Similarly,We obtain the complement of this graph.

The complement of p^2 non-zero zero divisor graph is disconnected and since no two vertices are adjacent and we get the different singleton set .The domination number of each component is 1.∴The domination number of the complement $\Gamma(Z_n)$ graph is increased based on the vertices of the graph.We get, $\gamma(\Gamma(Z_n)) < \Gamma(\bar{Z}_n)$where p=7

From cases (i),(ii) and (iii)

(i) $\gamma(\Gamma(Z_{7p})) < \gamma(\Gamma(\bar{Z}_{7p})$ (or) $\gamma(\Gamma(\bar{Z}_{7p})) = \gamma(\Gamma(Z_{7p}))+1$,

(ii) If p is even $\gamma(\Gamma(Z_{7p})) = \gamma(\Gamma(\bar{Z}_{7p}))$

(iii) If p is odd $\gamma(\Gamma(Z_{7p})) < \gamma(\Gamma(\bar{Z}_{7p}))=6$

Example: 3.2.2

When p= even

Consider, Z_{14} = {0,1,2,3,4,5,6,7,8,.......,48}

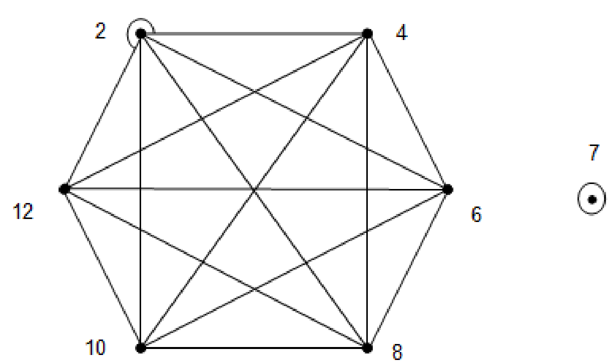
Let $Z_{7.2}=Z_{14}$ be the set of congruence modulo 14 and

$\Gamma(Z_{14})$= {(7,2),(7,4),(7,6),(7,8),(7,10),(7,12)} be the set of non-zero zero divisor graph

The dominating set D= {7}The dominating number of $\Gamma(Z_{14})$ is 1

The complement of $\Gamma(Z_{14})$ is a disconnected graph with two components and its denoted by $\Gamma(\overline{Z}_{14})$, which is shown below.

$\Gamma(\overline{Z}_{14})$= {(2,4),(2,4),(2,6),(2,8),(2,10),(2,12),(4,6),(4,8),(4,10),(4,12),(6,8),(6,10),(6,12), (8,10),(8,12),(10,12)} and {7}



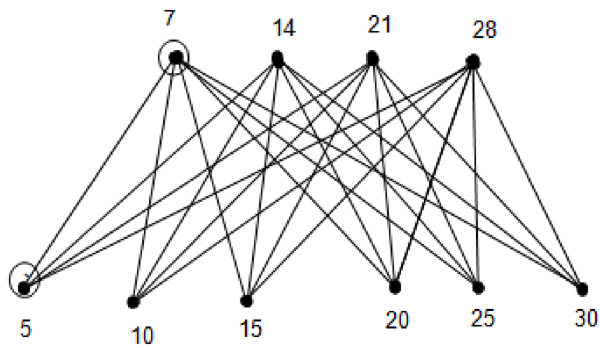
The dominating sets are D_1= {2} and D_2= {7}The domination number of each component is 1.The domination number of $\Gamma(\overline{Z}_{14})$ is 2

ii) n= odd prime

Consider, Z_{35} = {0,1,2,3,4,5,6,7,8,........,34}

Let $Z_{7.5}$ =Z_{35}be the set of congruwnce modulo 35 and

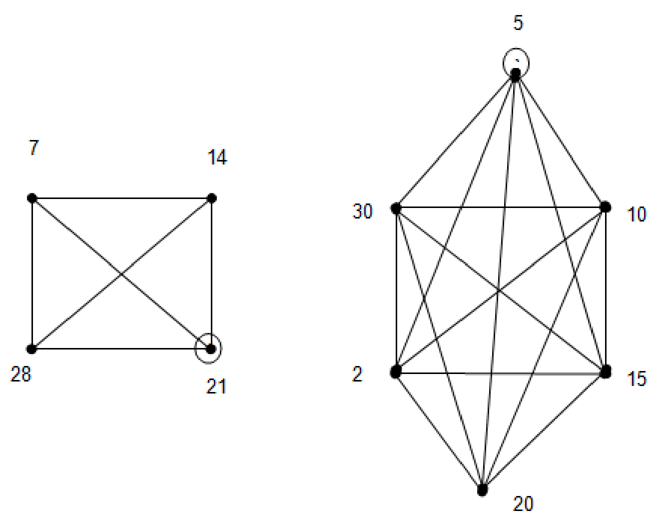
Γ (Z_{35})={(7,5),(7,10),(7,15),(7,20),(7,25),(7,30),(14,5),(14,10),(14,15),(14,20),(14,25),(14,30),(21,5),(21,10),(21,15),(21,20),(21,25),(21,30),(28,5),(28,10),(28,15),(28,20),(28,25),(28,30)} be the set of non-zero zero divisor graph.



The dominating set D= {7,5}Dominating number of Γ (Z_{35}) is 2

The complement of Γ (Z_{35}) is a disconnected graph with two components and it is denoted by Γ (Z_{35}), which is shown below.

Γ (Z_{35})= {(7,14),(7,14),(7,21),(14,21),(14,28),(21,28)} and {(5,10),(5,15),(5,20),(5,25),(5,30),(10,15),(10,20),(10,25),(10,30),(15,20),(15,25),(15,30),(20,25),(20,30),(25,30)}

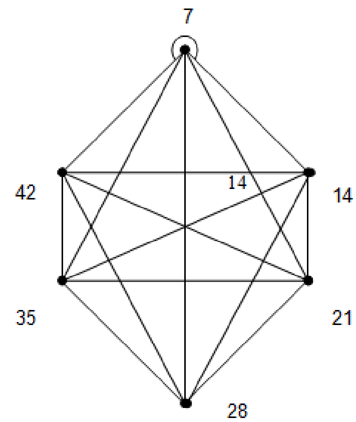


The dominating sets are D_1= {21} and D_2= {5}The domination number of each component is 1.The domination number of Γ (Z_{35}) is 2. —

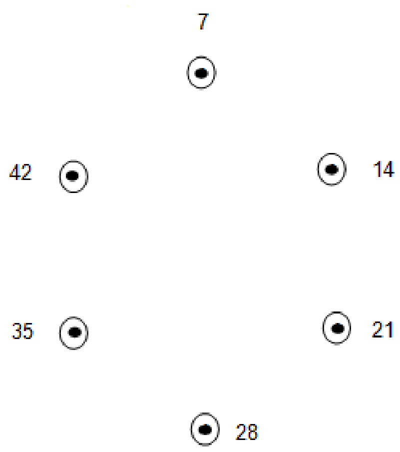
iii)p=7

Let $Z_{7.7}$=Z_{49} be the set of congruence modulo 49 and

Γ (Z_{49}) = {(7,14),(7,21),(7,28),(7,35),(7,42),(14,21),(14,28),(14,35),(14,42),(21,28), (21,35),(21,42),(28,35),(28,42),(35,42)} be the non-zero zero divisor graph.



Dominating set D_1= {7}Dominating number of (Γ (Z_{49}))=1
The complement of Γ (Z_{49})is a disconnected graph with six components and it is denoted by Γ ($\underline{Z}_{49}$), which is shown below.
Γ (Z_{49}) = {7} and {14} and {21} and {28} and {35} and {42}



The dominating sets are D_1= {7} and D_2= {14}D_1= {21} and D_2= {28}D_1= {35} and D_2= {42}The domination number of each component is 1.The domination number is 2.

Theorem: 3.2.3
Let Γ (Z_{p^2q}) and Γ ($\bar{Z}_{p^2q}$) be a non-zero divisor graph and complement of a non-zero zero divisor graph respectively,
If n=p^2q, for all primes p and q and p<q. Then the domination number of Γ (Z_{p^2q}) non-zero zero divisor is given by
γ (Γ (Z_{p^2q})) = γ(Γ ($\bar{Z}_{p^2q}$)) =2

Proof:
Let n = p^2qWhere p and q are both odd and even prime numbers and also p<q.
By definition of Γ (z_{p^2q}) non-zero zero divisor graph all the vertices are combination of the primes p and q and all the vertices are adjacent to either p or q.And therefore by the definition of domination we get the dominating set D={p,q} which forms a minimum dominating set with cordinality 2and hence dominating number of the Γ (Z_{p^2q}) non-zero zero divisor graph is 2.(i.e) γ (Γ (Z_{p^2q})) =2 _____(1)

The complement of the $\Gamma(Z_{p^2q})$ non-zero zero divisor graph is not disconnected and we get a single component.In the complement graph, the vertices which are adjacent to p in the original graph are adjacent to q.Similarly the vertices which are adjacent to q in the original graph are adjacent to p.Hence we get the same dominating set D_1={p,q}Therefore the dominating number is 2.(i.e) $\gamma(\Gamma(\bar{Z}_{p^2q}))$ =2 ______(2)

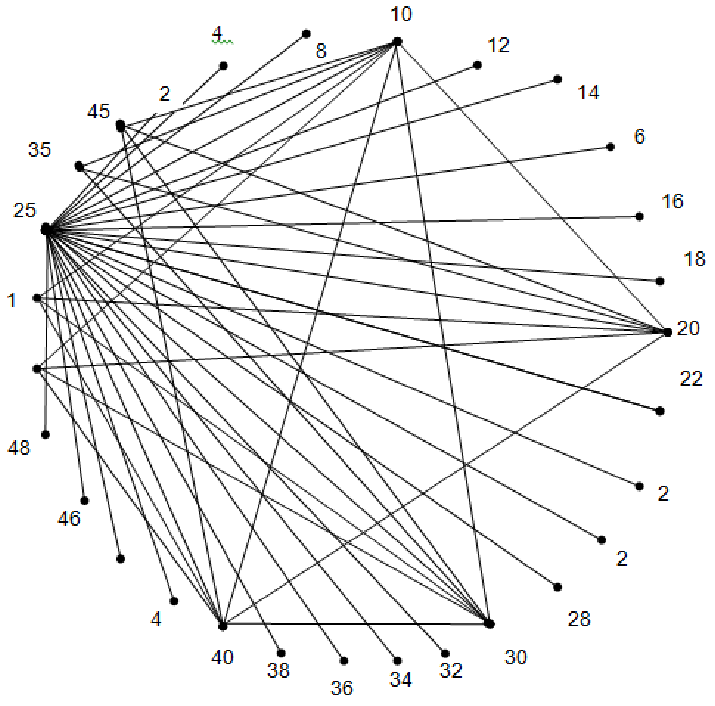
From (1) and (2) we get $\gamma(\Gamma(Z_{p^2q})) = \gamma(\Gamma(\bar{Z}_{p^2q}))$ =2 $\forall$ p and q and p<q.

Example:3.2.3

Consider, Z_{50} = {0,1,2,3,4,5,6,7,8,9,………,49}

Let, $Z_{5^2.2}$= Z_{50}be the set of congruence modulo 50 and

$\Gamma(Z_{50})$ = {(25,2),(25,4),(25,6),(25,8),(25,10),(25,12),(25,14) (25,16), (25,18),(25,20),(25,22),(25,24),(25,26),(25,28),(25,30),(25,32),(25,34),(25,36),(25,38),(25,40),(25,42),(25,44),(25,46),(25,48),(5,10),(5,20),(5,30),(5,40),(10,15),(10,20),(10,25),(10,30),(10,35),(10,40),(10,45),(15,20),(15,30),(20,25),(20,30),(20,35), (20,40), (20,45),(30,35),(30,40),(30,45),(35,40),(40,45)} be the set of non-zero zero Divisor graph.

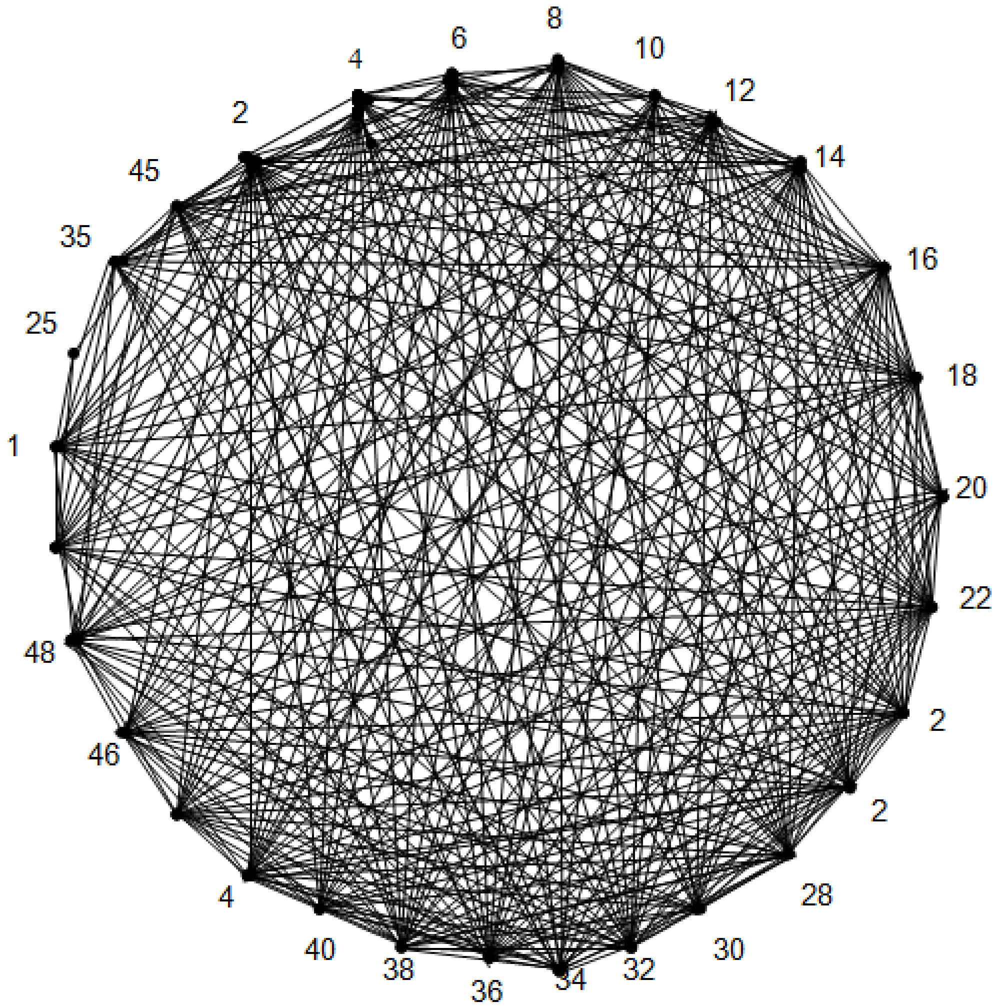


The dominating set D= {25,10}The dominating number of $\Gamma(Z_{50})$ is 2i.e) $\gamma(\Gamma(Z_{50}))$ =2

The complement of $\Gamma(Z_{50})$ has single component and it is denoted by $\Gamma(\bar{Z}_{50})$, which is shown below.

$\Gamma(\bar{Z}_{50})$={(2,4),(2,6),(2,8),(2,10),(2,12),(2,14),(2,16),(2,18),(2,20),(2,22),(2,24),(2,26),(2,28),(2,30),(2,32),(2,34),(2,36),(2,38),(2,40),(2,42),(2,44)(2,46),(2,48),(2,5),(2,10),(2,15),(2,20),(2,25),(2,30),(2,35),(2,40),(2,45),(4,6),(4,8),(4,10),(4,12), (4,14),(4,16),(4,18),(4,20),(4,22),(4,24),(2,26),(4,28),(4,30),(4,32),(4,34), (4,36),(4,38),(4,40),(4,42),(4,44),(4,46),(4,48),(4,5),(4,15),(4,35),(4,45),(6,8),(6,10),(6,12),(6,14),(6,16),(6,18),(6,20),(6,22)(6,24),(6,26),(6,28),(6,30),(6,32),(6,34),(6,36),(6,38),(6,40),(6,32),(6,34),(6,38),(6,40),(6,42),(6,44),(6,46),(6,48),(6,5),(6,15),(6,25),(6,35),(6,45),(8,32),(8,34),(8,36),(8,38),(8,40),(8,42),(8,44),(8,46),(8,48),(8,5),(8,1),(8,35),(8,45),(10,12),(10,14),(10,16),(10,18),(10,22),(10,24),(10,26)(10,28),(10,32),(10,34),(10,36),(10,38),(10,42),(10,44),(10,46),(10,48),(12,14),(12,16),(12,18),(12,20),(12,22),(12,24),(12,26),(12,28),(12,30),(12,32),(12,34),(12,36),(12,38),(12,40),(12,42),(12,44),(12,46),(12,48),(12,5),(12,15),(12,35),(12,45),(14,16),(14,18),(14,20),(14,22),(14,24),(14,26),(14,28),(14,30),(14,32),(14,34),(14,36),(14,38),(14,40),(14,42),(

14,44),(14,46),(14,48),(14,5),(14,15),(14,35),(14,45),(16,18),(16,20),(16,22),(16,24),(16,26),(16,28),(16,30),(16,32),(16,34),(16,36),(16,38),(16,40),(16,42),(16,44),(16,46),(16,48),(16,5),(16,15),(16,35),(16,45),(18,20),(18,22),(18,24),(18,26),(18,28),(18,30),(18,32),(18,34),(18,36),(18,38),(18,40),(18,42),(18,44),(18,46),(18,48),(18,5),(18,15),(18,35),(18,45),(20,22),(20,24),(20,26),(20,28),(20,32),(20,34),(20,36),(20,38),(20,42),(20,44),(20,46),(20,48),(22,24),(22,26),(22,28),(22,30),(22,32),(22,34),(22,36),(22,38),(22,40),(22,42),(22,44),(22,46),(22,48),(22,5),(22,15),(22,35),(22,45),(24,26),(24,28),(24,30),(24,32),(24,34),(24,36),(24,38),(24,40),(24,42),(24,44),(24,46),(24,48),(24,5),(24,15),(24,35),(24,45),(26,28),(26,30),(26,32),(26,34),(26,36),(26,38),(26,40),(26,42),(26,44),(26,46),(26,48),(26,5),(26,15),(26,35),(26,45),(28,30),(28,32),(28,34),(28,36),(28,38),(28,40),(28,44),(28,46),(28,48),(28,5),(28,15),(28,35),(28,45),(30,32),(30,34),(30,36),(30,38),(30,44),(30,46),(30,48),(32,34),(32,36),(32,38),(32,40),(32,42),(32,44),(32,46),(32,48),(32,5),(32,15),(32,35),(32,45),(34,36),(34,38),(34,40),(34,44),(34,46),(34,48),(34,5),(34,15),(34,35),(34,45),(36,38),(36,40),(36,42),(36,44),(36,46),(36,48),(36,5),(36,15),(36,35),(36,45),(38,40),(38,42),(38,44),(38,46),(38,48),(38,5),(38,15),(38,35),(38,45),(40,42),(40,44),(40,46),(40,48),(42,44),(42,46),(42,48),(42,5),(42,15),(42,35),(42,45),(44,46),(44,48),(44,5),(44,15),(44,35),(44,45),(46,48),(46,5),(46,15),(46,35),(46,45),(48,5),(4,15),(48,35),(48,45),(5,15),(5,25),(5,35),(5,45),(15,25),(15,35),(15,45),(25,35),(25,45),(35,45)}



,

The dominating sets are D_1= {2} and D_2= {25}

∴The domination number of Γ ($\bar{Z}_{50}$) is 2.

i.e) $\gamma(\Gamma(\bar{Z}_{35})) = 2$

4. Conclusion

In this Paper, we have established the domination number of pq non-zero zero divisor graph and its complement graph,7p non-zero zero divisor graph and its complement graph,p^2q non-zero zero divisor graph and its complement graph and relation between the domination number of different types of zero divisor graphs and its complement graphs. And we presented some example of pq,7p, p^2q non-zero zero divisor graphs.

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Prime Labeling for Some Sunshine Related Graphs

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ABSTRACT: *A Graph G with vertex set V will have a prime labeling if their vertices are labeled with distinct integers 1,2,3,...... such that edge xy the labels assigned to x and y are relative prime integers. For a graph if prime labeling is done then the graph is said to be prime graph. In this paper we define sunshine graph and we study prime labeling for some sunshine related graphs. We also discuss prime labeling in the context of same graph operation like fusion, duplication and Key words : Prime labeling, fusion, duplication.*

Keywords: *xyz, word.*

1. Introduction

In this paper we consider only finite simple undirected graph. The graph G has a vertex set V=V(G) and edge set E=E(G). The set of vertices adjacent to a vertex u of G is denoted by N(u). for notations and terminology we refer J.A.Bondy and U.S.R. Murthy [1]. In the present work Sn denotes a sunshine graph with 2n-1 vertices. Adequate literature surveys are available in printed as well as electronics form on different types of graph labelling and more than 1000 research papers have been published in the part four decades. A current survey of various graphs labeling problem can be found in [4] [J.Gallian, 2009].

Following are the common features of any graph labeling problems.

- A set of numbers from which vertex are labeled
- A rule that assigns value to every edge
- A condition that there value has to satisfy.

The notion of prime labeling was introduced by roger entringer and it is discussed in a paper by A.Tout [1982 p 3650368] [7]. Many researchers have studied prime graph for example in H.C.Fu[1994 P 181-186] [3] have proved that path in on n vertices is a prime graph.

T.O. Dertsky [1991 P 359-369] [2] have proved that the Cn of n vertices is a prime graph. S.M.Lee [1998 P 59-67] [5] have proved that wheel Wn is a prime graph iff n is even. The prime labeling for planner grid is investigated by M.Sundaram [2006, P 205-209] [6]. In [8] S.K. Vaidhya and K.K. Kanmanihave proved that the prime labeling for some cycle related graphs.

Definition 1.1: If the vertices of a graph are assigned values subject to certain conditions then it is known a graph labeling.

Definition 1.2: Let G={V(G)   {1,2,........n} is called a prime labeling if for each edge e=ur, gcd {f(u), f(v)} = 1

Definition 1.3: A graph which admits prime labeling is called a prime graph.

Definition 1.4: The sunshine graph Sn is the graph obtained from a cycle cn attaching two edges with vertex $V_{i,\,i+1}$from V_n to $V_{n,i}$ and n Figure 1: A sunshine graph Sn

Definition 1.5: Let u and v be two distinct vertices of a graph G. A new graph G1 is constructed by fusing [identifying] two vertices u and v by a single vertex x in G1, such that every edge which was incident with either u or v is G is now incident with x in G1.

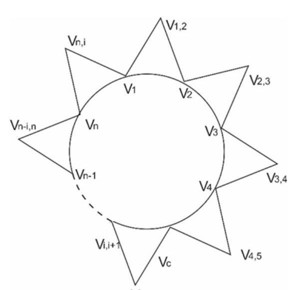


Figure 1: A sunshine graph Sn

Definition 1.6: Duplication of vertex Vk of a graph G producers a new graph G1 by adding a vertex Vk' with N(Vk')=N(Vk). In other words a vertex Vk' is said to be duplication of Vk if all the vertices which are adjacent to Vk are now adjacent to Vk'.

Definition 1.7: A vertex switching Gu of a graph G is obtained by taking a vertex u of G, removing all the edges that are incident with u and adding edges form u to every vertex that ae not incident to u in G.

2. Main Results

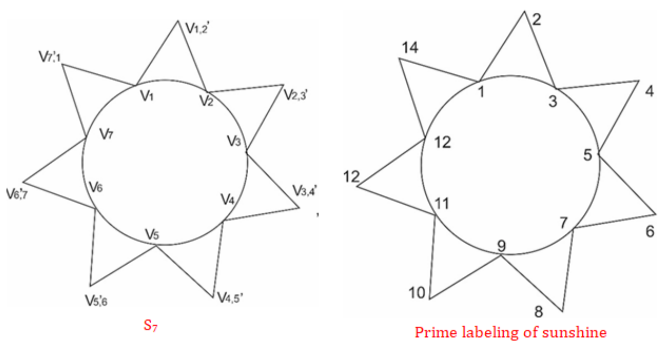
Proposition 2.1

The sunshine graph Sn is a prime graph.

Proof

Let Sn be a sunshine graph with 2n-1 vertices and
V(Sn) = $\{v_1, v_2, ...v_n, v_{12}', v_{23}', ...v_{n-1n}', v_{ni}'\}$
E(Sn) = {Viv1+1 / 1≤n-1} U {VnV1} U {Vi Vii+1' / 1≤i≤n-1}
U {Vi+1 Vii'+1, 1<i<n-1} U {Vn1 Vi,1} U {Vi Vn,i}
Here | V(Sn)| = 2n-1 if n is even or odd
f(vi) = 2i-1, 1≤i≤ n and
f(vi,i+1) = 2i, 1≤i≤n-1
Since each dege e = vivi+1 Eg, Gcd {f(vi), f(vi+1)} = 1
e = vi+1 vi, i+1 Eg, Gcd {f(vi+1), f(vi,i+1)} = 1
e = vnvn,iEg, Gcd {f(vn), f(vi,i)} = 1
f admits prime labeling e = v1 vn,iGcd {f(vi), f(vn,i)} = 1
Thus Sn is a prime graph

Example

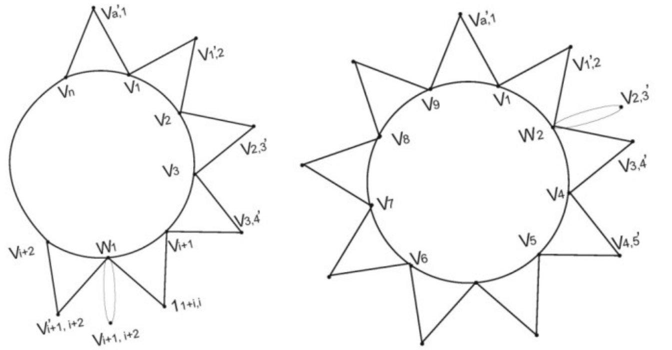


Graph S_7

Proposition 2.2

The graph obtained by fusing any two consecutive vertices in a sunshine graph Sn is a prime graph.
Proof : Let Sn be a sunshine graph, consider the fused graph Sn' fusing two consecutive vertices
Case i)
Let Vi and Vi+1 be the consecutive vertices such that
f(vi)= 2+1, f(vi+1) = 2i+1 and let it be fused to
wi for ay 1<i< n-1, Let f(wi) = max { (f(ui), f(u1+i) }
= max {2i-1, 2i+1} = 2i+1. Consider gcd {f(wi), f(1/i-1))}
= gcd {2i+1, 2i-3} =1
andgcd{f(wi), f(vi+2)} = gcd {2i-1, 2i+3} = 1

Thus Sn' is a prime graphs

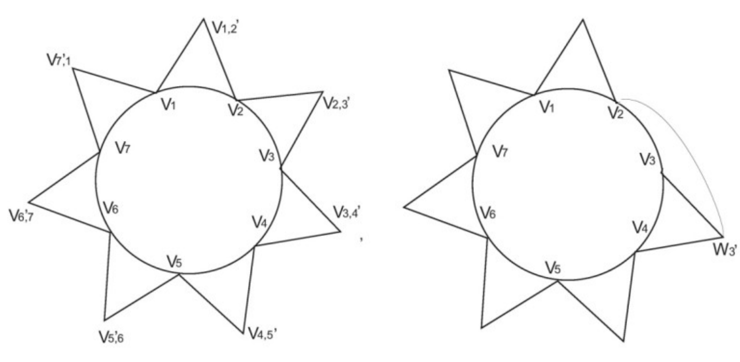


Vc, Vi+1 fused as wi in Sn'
If Vn and V1 are fused gcd {f(Vn), f(V1)}
gcd{2n-1,1} = 1

Case ii)
Let vi and Vi,i+1 be fused as wi for 1≤i≤ n-1
with f(wi') = max {f(Vi), f(vi+i)}
= max {2i-1, 2i} = 2i
consider for the vertex vi+1 1 ≤i≤ n-1
gcd { f(wi') , f(vi+1)}
= gcd {2i, 2i+1} = 1
gfVn and Vn'1 are fused as Wn' with
f(wn') = max {f(Vn), f(vn,i)} = max {2n-1,2n} = 2nm then
considergc {f(Wn'), f(Vn-1} gcd {2n,2n-3}
and gcd {f(wn'), f(vi)} = gcd {2n,1}=1
Thus Sn' the fuse graph of Sn is a prime graph

Proposition 2.3
The graph obtained by fusion two non – Consecutive vertices in a sunshine graph is not a prime graph.

Proof
Let us prove this with an counter example, consider a sunshine graph S7. If the vertices V2,3' and V3,4' as w'3,4 then gcd {W3', V2}=2 which prove that fused graph of fusion non – consecutive vertices is not a prime graph.

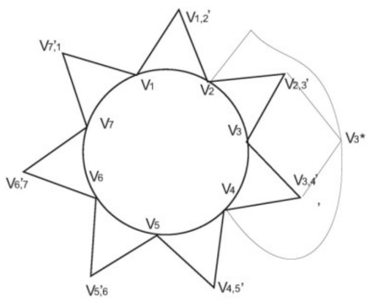


Duplication
The graph obtained by duplicating a vertex Vk in s sunshine graph Sn is not a prime graph.
Proof:
Let Sn be a sunshine graph with 2n-1 vertices
V(Sn) = {V1, V2,.......Vn, $V_{1',2}$, $V_{2',3}$,$V_{3',4}$,.....$V_{1',2}$,$V'_{n-1,n}$,$V'_{n,1}$}

E(Sn) = {Vivi+1 / 1≤i≤ n-1} U {Vnv1} U {Vi Vi,i+i / 1≤i≤ n-1}
U {Vi+1 Vi,i'+1, 1≤i≤n-1} U {Vn,Vn,i,} U {Vi Vn,i}
Let G* be the graph by duplicating the vertex vi in rim of Sn and Vi* be the new vertex. Define a labeling f:V(Gk) {1,2,........2n+1)

such that
f(vi) = 2i-1 1 ≤i≤ n
f(vi, i+1) 2i 1 ≤i≤ n

and f(vi*) = 2n+1 then G* is not a prime graph



3. Conclusion
As every graph does not admits prime labeling we investigated the graph families of sunshine graph which admitted prime labeling and which does not admits prime labeling. In future we can also talk about switching and within of two pieces of sunshine graph, whether they admits prime labeling.

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Hexy – Substitution Cipher

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ABSTRACT: *With the motivation of the idea from The Polybious square, a Substitution cipher which enciphers the plain text into the numerals/combination of numerals. In this paper we have introducing a new substitution cipher key which is much complicated than the polybious square key unless otherwise the key is known. Hexy key is a further extended form of the polybious square key. Hexy is a 6 X 6 square substitution key. Hexy is the term derived from the Hexagonal square key.*

Keywords: *Polubious Square, Hexagonal Square Key*

1. Introduction

2.1. Plain Text
The message we want to send is known as Plain Text.

2.2. Cipher Text
The Plain text in a disguised form is known as Cipher Text.

2.3. Enciphering/ Encryption
The process of converting the Plain Text into the Cipher text is known as Enciphering or Encryption.

2.4.Deciphering/Decryption
The process of converting the Cipher Text into the Plain Text is known as Deciphering or Decryption. Simply to say the reverse process of the Enciphering.

2.5. Polubious Square
The polybious Square is an ancient Greek invention, discovered by a scholar named Polybious Which consists of a 5 X 5 grid which contain the English alphabets A through Z one in each cell with the alphabets i and j share the same cell. This is used for Enciphering the alphabets into the combination of numerals

	1	2	3	4	5
1	A	B	C	D	E
2	F	G	H	IJ	K
3	L	M	N	O	P
4	Q	R	S	T	U
5	V	W	X	Y	Z

Polybious Square

2.6. Defining the Hexy
Hexy is an Hexagonal Square which contains a specific key for Substitution cipher. Hexy – The Hexagonal Square Key, consists of 6X6 grids (36 cells). Each cell is contained with alphabet or special characters or symbols. Here "_" and "/" were referred as symbols and others as special characters.

	1	2	3	4	5	6
1	A	B	C	D	E	F
2	G	H	IJ	K	L	M
3	N	O	P	Q	R	S

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4	T	U	V	WX	Y	**Z**
5	!	@	#	$	%	^
6	&	*	(	)	/	_

HEXAGONAL SQUARE KEY

3. Description of the usage of the Hexagonal Square Key

PLAIN TEXT → CIPHER TEXT → PLAIN TEXT

Hexy **Hexy**

3.1. The alphabet "A" is encrypted as the combination of the numbers as 11, and the numeral "11" can be decrypted as A. Similarly B as 12
("11" is the position of the alphabet in the Hexagonal Square Key)
Similarly all other alphabets can be encrypted and decrypted.
In general the alphabets "J" , "X" and "Z" were not used frequently, so they have been combined with the previously occurring alphabet. While decrypting the appropriate alphabet has to be used. And while encrypting the same code can be used for both the alphabets occurring in the same cell.

Examples 3.1.1

D → 14 → D

Hexy - Encryption **Hexy – Decryption**

W or X → 44 → appropriately Wor X

Hexy - Encryption **Hexy – Decryption**

3.2 The each special character has been defined with the specific meaning where as follows, "!" has to be considered as or meant as "OF" while coding. Similarly, "@" , "#" , "$" , "%" , "^", "&" , "*" , "(" , ")" has to be considered as or meant as "AT" , "NUMBER" , "CURRENCY" , "WITH" , "UNDER" , "AND" , "IMPORTANT" , "FROM" , "TO" respectively while coding. And every word has to be decoded only as the appropriate special characters mentioned as above.

3.2.1 Example

(→ FROM → (

Hexy - Encryption **Hexy – Decryption**

3.3. The symbols "_" and " /" have the special roles in the key.
"_" stands for space between the words.
If a special character is preceded by the symbol "/" then the special character has to be considered as a number corresponding to that special character. The correspondence of the special characters to the numerals is considered from the keyboard used with the computers. "!" for "1" , "@" for "2" , "#" for "3" , "$" for "4" , "%" for "5" , "^" for "6" , "&" for "7" , "*" for "8" , "(" for "9" and ")" for "0".

3.3.1.Example

/*	→	8	→	/*
	Hexy - Encryption		**Hexy – Decryption**	

4. Problem

Use the Hexy substitution key to encipher the message
"SHIFT 29 ANIMALS AND 30 BIRDS TO ZOO NUMBER 350"

Solution

Using the substitution key Hexy we may construct the table of plain text and the corresponding cipher text.

PLAIN TEXT	DESCRIPTION OF USAGE OF KEY	CIPHER TEXT
S	Direct Substitution (using 3.1)	36
H		22
I		23
F		16
T		41
_		66
2	a number – corresponding special character to be preceded by "/" (using 3.3)	/@
9	a number – corresponding special character to be preceded by "/" (using 3.3)	/(
_	Direct Substitution (using 3.1)	66
A		11
N		31
I		23
M		26
A		11
L		25
S		36
_		66
A	"AND" can be encrypted as "&" (Using 3.2)	&
N		
D		
_	Direct Substitution (using 3.1)	66
3	a number – corresponding special character to be preceded by "/" (using 3.3)	/#
0	a number – corresponding special character to be preceded by "/" (using 3.3)	/)
_	Direct Substitution (using 3.1)	66
B		12
I		23
R		35
D		14
S		36
_		66
T	"TO" can be encrypted as ")" (Using 3.2)	)
O		
_	Direct Substitution (using 3.1)	66
Z		46

O		32
O		32
_		66
N	"NUMBER" can be encrypted as "#" (Using 3.2)	#
U		
M		
B		
E		
R		
_	Direct Substitution (using 3.1)	66
3	a number – corresponding special character to be preceded by "/" (using 3.3)	/#
5	a number – corresponding special character to be preceded by "/" (using 3.3)	/%
0	a number – corresponding special character to be preceded by "/" (using 3.3)	/)

The Enciphered message is
"362223164166/@/(661131232611253666&66/#/)6646323266#66/#/%/)"

5. Conclusion

With this Hexy the substitution cipher will be much complicated to decipher without the knowledge of the key. Also this key will be much convenient because without specifically mentioning the numbers in the key we can use those numbers also. The special characters which have pre defined is much helpful to encipher and decipher shortly.

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Grayscale-Image Encryption Using Random Hill Cipher Associated With Discrete Wavelet Transformation

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ABSTRACT: *Cryptography is the study of methods of sending a message in a distinguished form so that only the intended people can receive the original message. Hill cipher algorithm is a technique for symmetric key algorithm in which we use the matrix from key for the encrypting the data. Images are also a matrix of pixels and each pixel has its intensity value. Using this concept we generate a function which select a random key matrix and then encrypt the image using the key matrix. For the decryption we again use this key matrix to get the original image.*

Keywords: *Encipher, Decipher, Discrete Wavelet Transformation*

1. Introduction

Cryptography is one of the methods to ensure confidentiality and integrity of information. It is from the Greek word **"Krypto's"** which means **"hidden"**. Cryptography is the art and science of making message unintelligible. It serves as a secret communication mechanism and can be traced back till thousands of years ago. Caesar's cipher is one of the earliest known cryptosystem which was used by Julius Caesar to convey secret messages to Marcus Cicero. All of these ciphers are the foundation for modern cryptography.

Hill Cipher is invented by **Lester S. Hill** in 1929, the Hill cipher is polygraphic substitution cipher based on linear algebra. Hill used matrices and matrix multiplication to mix up the plaintext. A Hill cipher uses linear algebraic operations on an n × n matrix to encrypt and decrypt messages. Hill ciphers were the **rstpolygraphic ciphers** where it was practical to operate on more than three symbols at once.

In our construction we are taking random hill cipher on grayscale – image of size m×m. The pixel of grayscale – image is divided into equal blocks. It is called as block matrix (sub image) of the original image the sub image size is as same as key size of hill cipher.

Terminology of cryptography:

Plain Text

Plain Text is the original intelligible message or data that is fed into the algorithm as input. It is information that the senders transmit to the receiver. The plaintext is simply text in the language of the communicating parties.

It is used as input to an encryption algorithm. The plaintext is the normal representation of the data before it can be hide by some action. This includes the electronic representation of text such as email, word processor document, music, pictures, videos, ATM, and credit card transaction etc. This can used to hide information others.

Cipher text

Cipher text is the scrambled message, produced as output. It depends on the plain text and secret key. For a given message, two different keys will produce two different cipher texts. The cipher text is an encryption algorithm. The cipher text message contains all the information of plain text apparently random stream of data and is unintelligible. It is a message text, got from message. But it is not in a format readable by a human or computer without the proper Mechanism to decrypt it. To transform the plain text into cipher text or cipher text to plain text an individual needs to know both the algorithm and the key. The cipher authentication text has same length of the plain text. Cipher text is the output of an encryption process and the input of a decryption process.

Secret key

The **secret key** is also fed as input to the encryption algorithm. The key value independent of the plaintext and of the algorithm. The algorithm will produced a different output depending on the specific key being used at the time. The exact substitutions and transformation performed by the algorithm depend on the key. The key is a variable parameter of the algorithm and the algorithms are widely distributed and the key are known by the sender and receiver which give access to the data encrypted.

Encryption

Encryption is a process of converting plaintext into cipher text. Cryptography uses the encryption technique to send confidential messages through an insecure channel. The process of encryption requires two things an encryption algorithm and a key. An encryption algorithm means the technique that has been used in encryption. Encryption has long been used by militaries and government to facilitate secret communication. It is now used in protecting information in the civilian systems such as computers, networks, wireless microphone etc. Encryption by itself products the confidentiality of message, but other techniques are still needed to product the integrity and authenticity of message.
f(p)=p+k mod 26

Decryption

Decryption is the process of cipher text into plaintext. A reverse process of encryption is called as decryption. Cryptography uses the decryption technique at the receiver side to obtain the original message from nom readable message. The process of decryption requires two things –a decryption algorithm and a key. A decryption algorithm means the technique that has been used in decryption. Generally the encryption and decryption algorithm are same.
f(c) = p-k mod 26

Random hill cipher keys

The hill cipher keys are considered from S L_n(F) domain such that n divides m. The procedure of block creation (in particular block size 4$\times$4 and 2×2) for grayscale image. Suppose user chooses such type of block matrix (sub image) and if order of block matrix does not divide order of original image matrix then to add some redundant rows or columns or both in the original image matrix. S L_n(F) is the set of all n×n matrixes that contains those elements of G L_n(F) whose determinant is 1 over field F.

The Mathematical definition of S L_n(F) as follows:

¥ S L_n(F) = {A€ G L_n(F)/ det (A) =1} **(1)**

Where G L_n(F) is general linear group over field F. because S L_n(F) contains those elements from G L_n(F). So the order of S L_n(F) is very large.
Since determinant of every element of S L_n(R) is one then inverse of hill cipher key (K^{-1}) is same as adjoint of hill cipher key (K).

Because $K^{-1} = \frac{adj(k)}{\det(k)}$.

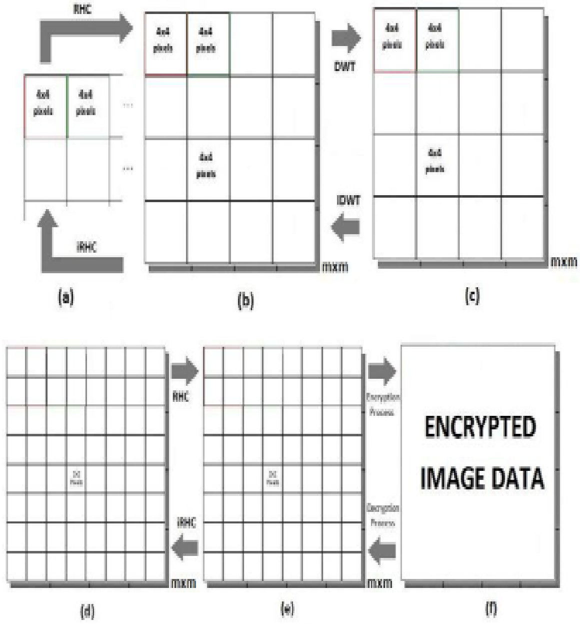
Formulation for random hill cipher:

The Formulation for random hill cipher of a block matrix (sub image) of size n×n is given as:

C=B.K (mod P) **(2)**

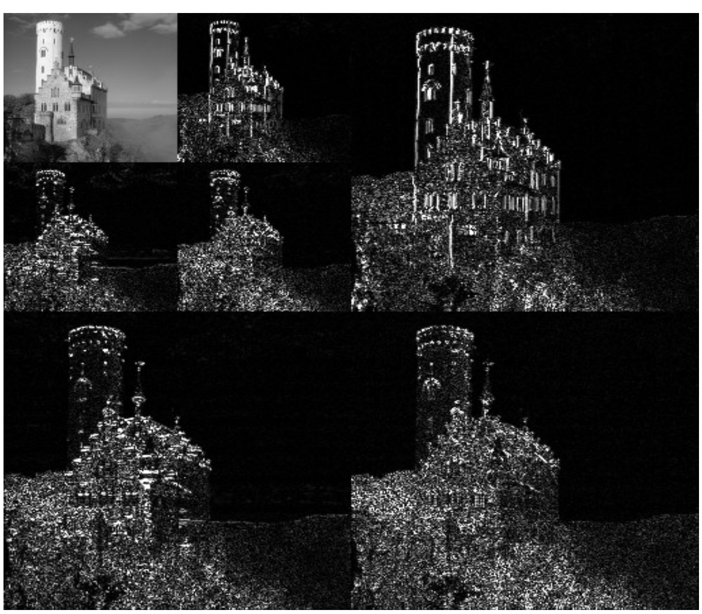
Where B be an n×n block matrix (sub image) of the grayscale image of size m×m, K is an n×n key matrix from linear group and C be a cipher block of size n×n. Formulation for inverse random hill cipher of block matrix (sub image) of size n×n is given as,

B=C.adj (K) (mod P) **(3)**



Where adj (K) is adjoint of key matrix K€ S L_n (F) and P be unit representation.
Similarly the same process is applied for remaining block matrix (sub image) of the original grayscale – image.

Discrete Wavelet Transformation (DWT):
In our proposed approach also used wavelet transformation as keys. In the DWT domain the general features and the details of a signal and image can be analyzed. Two dimensional DWT is carried out by performing in the row direction and column direction separably. When two dimensional DWT is carried out for two dimensional data such as image, four domains with different frequency characteristics are generated. Each of them is generated marked as A, H, V and D domain according to their frequency characteristic.
Because the four sub-bands are down sampled in the row and column direction, the total size of the four sub-bands is equal to the size of the original image.



DWT of an image f(X, Y) of size N×M is defined as follows:

$$W_{\varnothing} = (j_0, n, m) = \frac{1}{\sqrt{NM}} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y)\, \varnothing_{j_0,n,m}(x, y) \quad (4)$$

$$W_{\varphi}^{i} = (j, n, m) = \frac{1}{\sqrt{NM}} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y)\, \varphi_{j,n,m}^{i}(x, y),\ i = \{H, V, D\} \quad (5)$$

For $j \geq j_0$ and

$$f(X,Y) = \frac{1}{\sqrt{NM}} \sum_n \sum_m W_{\varnothing}^{i}(j,n,m)\varnothing_{j,m,n}^{i}(x,y) + \frac{1}{\sqrt{NM}} \sum_{i=H,V,D} \sum_{j=j_0} \sum_n \sum_m W_{\varphi}^{i}(J, n, m)\varphi_{j,n,m}^{i}(x, y) \quad (6)$$

Where j_0 is an arbitrary starting scale and

$$\varnothing_{j,n,m}(X, y) = 2^{j/2}\varnothing(2^j x - n, 2^j y - m), \quad (7)$$

$$\varphi_{j,n,m}^{i}(X, y) = 2^{j/2}\varphi(2^j x - n, 2^j y - m),\ i = \{H, V, D\} \quad (8)$$

Here index I identifies the directional wavelets that assumes values of H, V and D.
Equation 7-8 defines the scaled and translated basis function f(x, y), $\varnothing_{j,n,m}$ (x, y) and $\varphi_{j,n,m}^{i}$ (X, y) are functions of the discrete variables x=0, 1, 2…N-1 and y=0, 1, 2…M-1.The co-efficient defined in equation 4 and 5 are usually called Approximation and detail co-efficient respectively.

$W_{\varnothing}$(j, n, m) co efficient define an approximation of f(x, y) at scale j_0 .W_{φ}^{i}(j,n,m) co efficient and horizontal, vertical and diagonal details for scales $j \geq j_0$.Let j_0=0 and select M=N=2^j such that j=0,1,2, ….j-1 and n,m=0,1,2……2^j-1.equation (6)show that f(x,y) is obtained the inverse DWT for given $W_{\varnothing}$ *and* W_{φ}^{i} of equation(4) and(5)

Procedure of encryption and decryption using TSRHS associated with DWT:

Two stage Random Hill Cipher associated with DWT is applied on Grayscale-Image. The encryption and decryption process are

In the first stage we have entire one option for random hill cipher which is applied over grayscale-image. Similarly in the next stage one option is also available for random hill cipher for image. Therefore the total number of choices of random hill cipher applied on Grayscale-image in combining both stages. These stage options mentioned is also called Arrangement of Random hill cipher.

The procedure of encryption for Grayscale-image is given the following figure (4).In the first stage applied K_1 key for RHC then DWT is applied over partially encoded image. Finally K_2 key is applied for second stage RHC.The same procedure is applied for decryption process figure (5).

We choose key for inverse random Hill Cipher (iRHC) denoted as K_2^{-1}

(Inverse of K_2 key is applied on the partially decoded image. The finally original

Image is recovered. This approach uses only three keys-two keys for random hill cipher and one key for discrete wavelet. The size of hill cipher key depends on the size of the block matrix (sub images) of the original image which ultimately depends on the choice of encoder. But some earlier approaches such as uses fixed size of hill cipher key.

In this approach decoder have the correct keys. But does not have the information about the correct arrangement of RHC, decoder cannot recover the original image figure (6).so our proposed approach not only depends on keys. But also depends on arrangement of random hill cipher with wavelet.

Demonstration of the Procedure

The procedure is applied on Grayscale-image of size 256 ×256 pixels. The encrypted Grayscale-image with the following keys, the RHC key in the first stages is (before applying DWT):

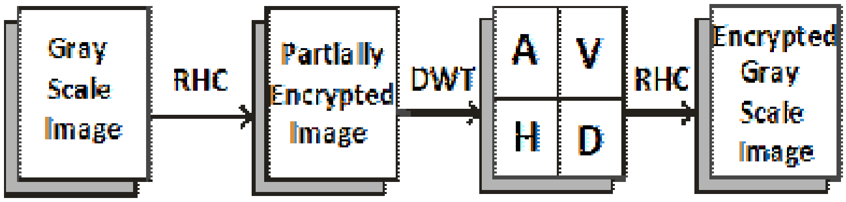


Fig. 4: Encryption process for grayscale-image.

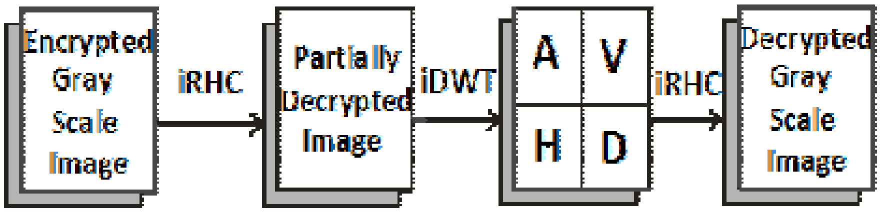


Fig. 5: Decryption process for grayscale-image.

$$K_1 = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 7 & 4 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The RHC key for partially decrypted Grayscale-image in second stage is (after applying DWT):

$$K_2 = \begin{bmatrix} 11 & 7 \\ 3 & 2 \end{bmatrix}$$

With 'db4' wavelet.

The correctly decrypted Grayscale-image with exact keys and correct arrangement.
The inverse RHC key in the first stage is (before applying DWT):

$$K_2^{-1} = \begin{bmatrix} 2 & -7 \\ -3 & 11 \end{bmatrix}$$

And the inverse RHC key for the partially decrypted Grayscale-image is second stage is (after applying DWT):

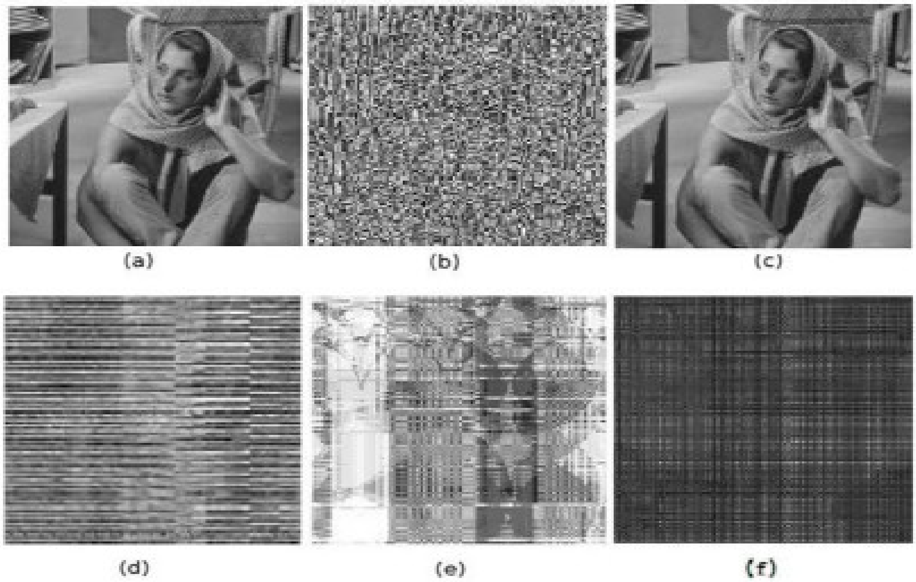


Figure-6

$$K_1^{-1}=\begin{bmatrix}1 & -3 & 16 & -123\\ 0 & 1 & -7 & 52\\ 0 & 0 & 1 & -8\\ 0 & 0 & 0 & 1\end{bmatrix}$$

With ‘db4’ wavelet.

It represents incorrectly decrypted Grayscale-image with approximation RHC key wavelet as well as arrangement is same as in the correct decryption process.
It represents incorrectly decrypted Grayscale-image with wrong WT and incorrectly arrangement of RHC keys. The RHC key in the first stage is (before applying DWT):

$$K_1^{-1}=\begin{bmatrix}1 & -3 & 16 & -123\\ 0 & 1 & -7 & 52\\ 0 & 0 & 1 & -8\\ 0 & 0 & 0 & 1\end{bmatrix}$$

And RHC keys for the partially decrypted Grayscale-image in second stage is (after applying DWT)

$$K_2^{-1}=\begin{bmatrix}2 & -7\\ -3 & 11\end{bmatrix}$$

It represents incorrectly decrypted Grayscale-image with correct Random hill cipher keys and wavelet but without knowing the correct arrangement of Random hill cipher keys and DWT.

Conclusion

This paper gives the effective way to encrypt an image. So grayscale–image is encrypted using random hill cipher associated with discrete wavelet transformation by using of bit rotation.

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Structure of Analysis With Their Preliminaries

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ABSTRACT: *In this paper, we propose a procedure to find the various definitions for understanding the analysis. The basic idea is to develop the knowledge of students to explores in the analysis. Further, we discuss about real numbers, complex numbers, convergent, divergent and cantor sets in new trending information.*

Keywords: *Analysis, Real number, Complex number, Sequence, Cantor set,*

1. Introduction

This paper gives a procedure to defined the various definitions in the analysis. Analysis is the part of Mathematics . The aim of any analysis course is to do some analysis. Some important and interesting facts dicuss about this analysis. This analysis is used in the study of real and complex numbers. It is usually derived from calculus, which gives a fundamental concepts of analysis. The eminent Cantor set is a subset of closed interval [0, 1]. It is uncountable.

Real Numbers

The Real Numbers Up to this point in budding the real numbers we have encountered only arithmetic operations. The progression from IN to Z to Q is just algebraic. All this algebra may have been a load to the weaker students in the lower grades, but conceptually the steps are easy to grip with a bit of familiarity necessary for all calculus students, is to build up the still larger system of real numbers, denoted as R. We frequently refer to the real number system as the real line and think about it as a geometrical object, even though nothing in our definitions would seem at first sight to allow this. Mainly calculus students would be hard pushed to say exactly what these numbers are. They distinguish that R includes all of IN, Z, and Q and also many new numbers, such as √ 2, e, and π. This come within reach of is adequate for applications of calculus and is a constructive way to avoid doing any hard mathematics in introductory calculus courses.
Furthermore, the set R of real numbers provide such an example. We shall assume that every real number x can be written in decimal expansion.

$X = b.a_1a_2a_3.... = b_1 + \frac{a1}{10} + \frac{a2}{10} + ...$

Where a_i are integers, $0 \leq a_i \leq 9$. This expansion is unique except for such cases x = ½ which can be expanded as

$\frac{1}{2} = 0.50000...$and $\frac{1}{2} = 0.49999.$

Complex Numbers

A complex number can be write in the figure a + b*i* where a and b are real numbers (including 0) and *i* is an imaginary number.
Therefore a complex number contain two 'parts':

- one that is real
- and another part that is imaginary

For Example : 2+3i and u-iv

Sequences

A sequence (of real numbers, of sets, of functions, of anything) is simply a list. There is a **first** element in the list, a second element, a third element, and so on continuing in an order forever. In mathematics a limited listing is not called a sequence; a sequence should continue without intermission.

For a more formal definition notice that the natural numbers are playing a key role here. Every item in the sequence (the list) can be labeled by its position; label the first item with a "1," the second with a "2," and so on. Seen this way a sequence is merely then a function mapping the natural numbers IN into some set. We state this as a definition. More generally, A sequence is a list of things (usually numbers) that are in order.

Convergent

A series is convergent if the sequence of its partial sum tends to a limit, that means that the partial sums becomes closer and closer to a given number when the number of terms increases.

Example

You preserve see it like a sort of entertainment. Suppose we have the sequence
12,23,34,…,nn+1,….12,23,34,…,nn+1,….
Suppose you state: "This sequence converges to the number 1." Suppose I state: "No it doesn't!" So we play a diversion to see what happen. Since I return it doesn't converge to one, I'm going to try to give you a number *so small*, that you will never get that close to the number 1. For instance, I give you ϵ=11000000ϵ=11000000. later than that you affirm, "Oh, no problem -- if you just look at a1000000a1000000, you'll see that indeed a1000000a1000000 is within distance ϵϵ of 11. That is, |a1000000−1|<ϵ."|a1000000−1|<ϵ."
If you can always win this game, no matter how small of an ϵϵ I give you, that is starting to sound an awful lot like the sequence converges to 1, right?
There is one subtle difference: consider the sequence 0,1,0,1,0,1,0,…0,1,0,1,0,1,0,…. The way I described the game above, you could prove that this sequence converges to 0: it is easy to find terms an an that are within distance ϵϵ of 00: any 00 term in the sequence will do. Convergent sequences shouldn't just approach a value, they should *stay there* too. So rather than saying, "a1000000a1000000 is very close (within epsilon) to aa", you say, "starting at 10000001000000, the sequence stays very close (within ϵϵ) to aa". That's where the ∀n≥N∀n≥N comes from.

Divergent

A few series that is not convergent is said to be divergent

Example:1.6.1

Un=n and Vn=(−1)n
Un=n :
(Un)n∈N diverges because it increases, **and it doesn't admit a maximum :**
limn→+∞Un=+∞
Vn=(−1)n :
This series diverges whereas the series is bounded :
−1≤Vn≤1
Why?
A series converges if it has a limit, ***single*** !
And Vn can be decompose in 2 sub-sequences :
V2n=(−1)2n=1 and
V2n+1=(−1)2n+1=1⋅(−1)=−1
Then : limn→+∞V2n=1
limn→+∞V2n+1=−1
A series converges if and only if every sub-sequences converges **to the same limit**.
But limn→+∞V2n≠limn→+∞V2n+1
Therefore Vn doesn't have a limit and so, diverges.

Cantor Set

Consider a line segment of unit length. Remove its middle third. At this moment eliminate the middle thirds from the remaining two segments. Now eliminate the middle thirds from the remaining four segments. Now eliminate the middle thirds from the remaining eight segments. Now eliminate…well you get the idea. If you might go on this construction through infinitely many steps, what would you have left?

What leftovers after infinitely many steps is a remarkable subset of the real numbers called the Cantor set, or "Cantor's Dust."

At first look one may reasonably wonder if there is *anything* left. After all, the lengths of the intervals we removed all add up to 1, exactly te length of the segment we started with:

$$\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\frac{8}{81}\ldots=\frac{1}{3}\sum_{k=0}^{\infty}\frac{2^K}{3^K}$$

$$=\frac{1}{3}\left(\frac{1}{1-\frac{2}{3}}\right)$$

$$=\frac{1}{3}\text{x}3$$

$$= 1$$

Yet, astonishingly, we can show that there are just as many "points" remaining as there were before we began! This amazing fact is only one of the many surprising properties exhibited by the Cantor set.

Earlier than we begin to expose these properties, it is important to be quite precise about this construction. Let us concur that the segments we remove at each stage of the construction are open intervals. That is, in the initial step we remove all of the points between 1313 and 2323, but leave the end points, and similarly for each successive stage. A little sign will convince you that these endpoints we leave behind never get removed, since at each stage we are only removing parts that lie strictly between the endpoints left behind at the previous stage. Thus we see that our Cantor set cannot be empty, since it contains the points 0,1, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{2}{9}$, $\frac{7}{9}$, and so on.

But in fact there is much more that remains. To see this, remember that we may choose any number base to represent real numbers. That is, there is not anything needed or even special about our common use of base ten; we can just as easily represent our numbers using base two, or base three, or any other base:

When a number is written in base two it is said to be in *binary* notation, and when it is written in base three it is said to be in *ternary* notation. Let's spotlight on the ternary representations of the decimals between 0 and 1. Since, in base three, $\frac{1}{3}$ is equal to 0.1, and $\frac{2}{3}$ is equal to 0.2, we see that in the first step of the construction (when we removed the middle third of the unit interval) we actually removed all of the real numbers whose ternary decimal representation have a 1 in the first decimal place, except for 0.1 itself. (Also, 0.1 is the same as 0.0222… in base three, so if we choose this representation we are removing all the ternary decimals with 1 in the first decimal place.) In the same way, the second step of the construction removes all those ternary decimals that have a 1 in the second decimal place. The third step removes those with a 1 in the third decimal place, and so on. (Convince yourself that this is so. Begin by noticing that $\frac{1}{9}$ is equal to 0.01 and $\frac{2}{3}$ is equal to 0.02 in base three.)

Thus, after the whole thing has been removed, the numbers that are left—that is, the numbers making up the Cantor set—are precisely those whose ternary decimal representations consist entirely of 0's and 2's. What numbers do this contain, besides the ones already noted above? How many are there?

Lots. Consider 1/4. This is not one of the finish points (those all have powers of three in the denominator), but it is not hard to show that 1/4 is in the Cantor set: in ternary entry its decimal expansion is 0.0202…. in view of the fact that it consists entirely of 0's and 2's it was never removed during the construction of the Cantor set, so it's still there—somewhere.

Construction of Cantor type sets $K^{9}_{[a,b]^2}$ and computation of its area in a closed and bounded square of area A = $(b - a)^2$ **:**

Step 1: Divide square $[a, b] \times [a, b] = [a, b]^2$ of area A in nine equal parts then remove middle open square $D_{1,1}$ of area $\frac{A}{9}$.This will give eight closed squares $S_{1,1}, S_{1,2}, \ldots S_{1,8}$ each of area $\frac{A}{9}$.

Step 2: Divide each of these eight squares $D_{2,1}, D_{2,2}, \ldots D_{2,8}$ in nine equal parts, that is in all 72 squares each of area $\frac{A}{9^2}$. This will gives sixty four (8^2) closed squares $S_{2,1}, S_{2,2}, \ldots S_{2,8^2}$ each of area $\frac{A}{9^2}$.

Step 3 : Divide each of these sixty four squares in nine equal parts that is in all $8^2 \times 9 = 576$ squares and remove the middle open squares $D_{3,1}, D_{3,2}, \ldots D_{3,8^2}$ each of area $\frac{A}{9^3}$.This will gives (8^3) closed squares $S_{2,1}, S_{2,2}, \ldots S_{2,8^3}$ each of area $\frac{A}{9^3}$.

Step 4: Continue this process. At n^{th} step remove the middle open squares $D_{n,j}$, j = 1,2, ...,8^{n-1} of area $\frac{A}{9^n}$ from $S_{n,j}$, j=1,2,..., 8^{n-1} respectively then it will gives 8^n closed squares
$S_{n,j}$, j=1,2,..., 8^n each of area $\frac{A}{9^n}$.

Step 5: Define $P_n = \bigcup_{j=1}^{8^n} S_{n,j}$, ∀n.

Step 6: Define $K^9_{[a,b]^2} = \bigcap_{n=1}^{\infty} P_n$.

This is our Cantor type place in $[a,b]^2$.

Property

Cantor type set $K^9_{[a,b]^2}$ is of area zero.

Proof: By construction of Cantor type set

$$G = (K^9_{[a,b]^2})^c = \bigcup_{n=1}^{\infty} \bigcup_{j=1}^{8^{(n-1)}} D_{n,j}$$

is open set created by countable number of disjoint open squares where
$A(I_{n,j}) = m(D_{n,j}) = \frac{A}{9^n}$,∀j = 1,2, 3, ... , $8^{(n-1)}$.

Thserefore

$$m(G) = \sum_{n=1}^{\infty} \sum_{j=1}^{8^{(n-1)}} m(D_{n,j})$$

$$m(G) = \sum_{n=1}^{\infty} \sum_{j=1}^{8^{(n-1)}} \frac{A}{9^n}$$

$$m(G) = A \sum_{n=1}^{\infty} \frac{8^{(n-1)}}{9^n}$$

$$m(G) = \frac{A}{8} \cdot \frac{8}{9} \sum_{n=0}^{\infty} \frac{8^n}{9^n}$$

$$m(G) = \frac{A}{8} \cdot \frac{8}{9} \cdot \frac{1}{1-\frac{8}{9}}$$

$$m(G) = A$$

Now, $m(G^C) = m\, K^9_{[a,b]^2} = m[a,b]^2 - m(G) = A - A = 0$

Thus Cantor type set $K^9_{[a,b]^2}$ is of area zero.

Conclusion

From this paper, we discussed about the analysis and based on definitions with examples. It is the new fascinating way to understand the student knowledge. We also can construct cantor set in easy way. We study their properties related with cantor set.

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Propositional Laws for Logical Connectives

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ABSTRACT: *In this paper we will propose a fuzzy interpretation for the connectives (conjunction,disjunction, negation, biconditional) such that the set of tautologies is exactly the set of classical tautology. The aim of this paper to give a simple look to the development of logical connectives for fuzzy logic.*

Keywords: *Biconditional, Tautology, Logical Connectives*

1. Introduction

A logical connective is a symbol or a word which is used to connect two or more sentences.Each logical connective can be expressed as a truth function.

Definition

Conjunction

It is a truth-functional connective similar to "and" in English and is represented in symbolic logic with the dot " . " and it is denoted by " Λ ".

Example

The conjunction two statements A and B is
A:Sharmila went to the school
B:Jothi went to the school, then
AΛB:Sharmila and jothi went to the school.
The truth table is given as follows

A	B	AΛB
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

It is a truth-functional connective similar to " or " in English and is represented in symbolic logic with plus "+" and it is denoted by "V".
Example:-
The disjunction of two statements A and B is
A: There is something wrong with the parents.
B: There is something wrong with the children.
AVB: There is something wrong with the parents or with the children.
The truth table is given as follows

A	B	AVB
T	T	T
T	F	T
F	T	T
F	F	F

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Biconditional

The logical biconditional is the logical connective of two statements. For any two statements A and B, the statement A<=>B is called as biconditional.This is read as "A if and only if B" and abbreviated as "A iff B". This is also called "A" is necessary and sufficient for B". A<=>B has the truth value T whenever both A and B have identical truth values.
The truth table is given as follows

A	B	A<=>B
T	T	T
T	F	T
F	T	T
F	F	F

Tautology:

A tautology is a statement that always gives a true values.

Example:

The two or more statements are equivalent if their truth values are the same.
The following propositional laws which are true for the logical connectives.

1.Commutative Laws:-

- A∨B<=> B∨A

A	B	A∨B	B∨A	A∨B<=> B∨A
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

- AΛB<=> BΛA

A	B	AΛB	BΛA	AΛB <=>BΛA
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

2. Associative Law

- AV(BVC) <=> (AVB)VC

A	B	C	AVB	BVC	AV(BVC)	(AVB)VC	AV(BVC) <=>(AVB)VC
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

- AΛ(BΛC) <=> (AΛB)ΛC

A	B	C	AΛB	BΛC	AΛ(BΛC)	(AΛB)ΛC	AΛ(BΛC) <=> (AΛB)ΛC
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

3. Distributive Law

- AΛ(BVC) <=>(AΛB)V(AΛC)

A	B	C	BVC	AΛ(BVC)	(AΛB)	(AΛC)	(AΛB)V(AΛC)	AΛ(BVC) <=> (AΛB)V(AΛC)
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	T	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

- AV(BΛC) <=> (AVB)Λ(AVC)

A	B	C	BΛC	AV(BΛC)	(AVB)	(AVC)	(AVB)Λ(AVC)	AV(BΛC) <=> (AVB)Λ(AVC)
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

4. IDEMPOTENT LAW:-

- (AΛA) <=> A

A	A	(AΛA)	(AΛA) <=> A
T	T	T	T
F	F	F	T

- (AVA) <=> A

A	A	(AVA)	(AVA) <=> A
T	T	T	T
F	F	F	T

5. ABSORPTION LAW:-

- AV(AΛB) <=>A

A	B	(AΛB)	AV(AΛB)	AV(AΛB) <=>A
T	T	T	T	T
T	T	F	T	T
F	F	F	F	T
F	F	F	F	T

- AΛ(AVB) <=>A

A	B	AVB	AΛ(AVB)	AΛ(AVB) <=>A
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	F	T

Conclusion

The main contribution of this paper was to proved that the classical logic,when seen as the set of tautologies in the truth table,can be also modified by fuzzy connectives.
Finally, you have seen a first glance of connectives and propositional logic.

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A Modified Logistic Growth Model Using Ordinary Differential Equations

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ABSTRACT: *This paper two logistic growth models have been used namely constant harvesting and periodic harvesting. Logistic growth model is appropriate for population growth. To compare the results obtained between the two strategies. The best harvesting strategy for the selected fish form is periodic harvesting.*

Keywords: *Logistic Growth Model, Modified Logistic Growth Model*

1. Introduction

Let us consider that the proportionality factor r, measuring the population growth rate in equation (3.01), is now a function of the population f(Y). As the populace increases and gets closer to carrying capacity C, the rate r decreases. One simple sub model for r is the linear

$f(Y)=\left(1-\frac{Y}{C}\right)$ ------- 1

Substituting this function into equation (1) leads to the modified logistic growth model

$\frac{dY}{dt}= rY\left(1-\frac{Y}{C}\right)$ -------2

where, the variable Y can be interpreted as the size of the fish population in tones.
C is referred to as the carrying capacity of the environment and the parameter r is called the growth rate.

Equation (3) is a more realistic model that describes the growth of species subject to constraints of space, food supply, and competitors/predators. This equation is called the modified logistic population growth model by P.F Verhulst.

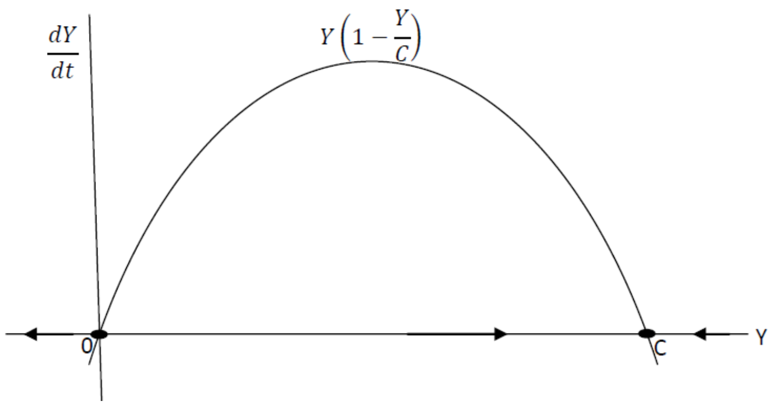

Figure 1: Graph of a modified logistic growth model

The modified logistic growth graph, displayed in Figure 1, crosses the *Y*-axis at the two points *Y* = 0 and *Y* = C, and represent critical points.

For $0<Y<C$, we have

$\frac{dY}{dt} > 0.$

Hereafter slopes are positive at any point and solutions must increase in this region.

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When *Y<0 or Y>C*, we have

$$\frac{dY}{dt} < 0$$

and so solutions must decrease. One way we can analyses the predictions of modified logistic model thus, equation (3) is to solve it. The following section describes the solution process.

Solution Of The Modified Logistic Growth Model

The following is the process of solving equation (3)

Given

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right)$$

Let

$$a = \frac{Y}{C}$$

By the separation of variables, we have

$$\frac{dY}{Y(1-a)} = rdt$$

By partial fraction resolving, and integrating we have

$$\ln\frac{Y}{(1-a)} = rt + K$$

Replacing *a* and simplifying, we have

$$Y(t) = \frac{CKe^{rt}}{C+Ke^{rt}}$$

Evaluating with the initial condition $Y(0)=Y_0$, we found that

$$K = \frac{CY_0}{C-Y_0}$$

Using this, the solution of equation (3) is given as

$$Y(t) = \frac{CY_0}{Y_0+(C-Y_0)e^{-rt}} \quad \text{--------- } 4$$

$Y_0 > 0, C>0$

Investigation Of Solution Of The Modified Logistic Growth Model:

In fishery fabrication, if the initial population is less than the environmental carrying capacity, the fish will grow quickly to fill the living space (carrying capacity).

For y_0 if < *C*,

The population will monotonically increase toward the carrying capacity C and remain there (figure 1).

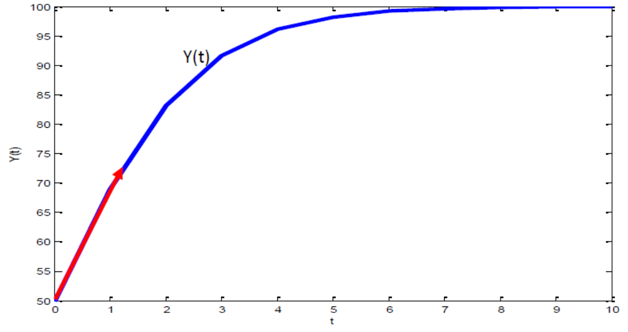

Figure 2: Logistic growth model with initial population less than carrying capacity (y_0 < C)

From the logistic curve in figure 1, we observed that as time increases from t =0 to t =7 months, fish population Y(t) increases to the carrying capacity C. The population remains there as time increases (t >7).

This implies that in fishery production; if the pond surface area is very large and few f fingerlings were put in it, the fingerlings would grow and reproduce quickly to fill the pond.
For $y_0 > C$,
the population will monotonically decrease toward the carrying capacity C and remain there (figure 2).

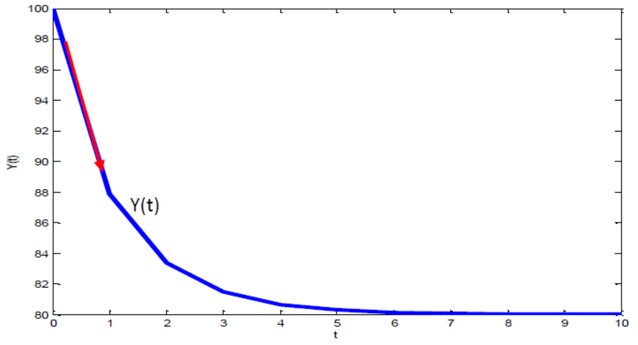

Figure 3 : Logistic growth model with carrying capacity more than initial population ($y_0 > C$)

Similarly, figure 3 shows that at time t = 0, the fish population Y(t) = 100, but after six month the population Y(t), decreases to carrying capacity C = 80 and remains constant at that level as time increases. Hence, we can conclude that if fish population is more than the pond surface area, some will die off, until the population is at the level of the pond carrying capacity.
Another way, we could, analyses equation (3) is to find the equilibrium solutions.

Modified Logistic Growth Model With Harvesting:
In this section, the modified logistic growth model would be adjusted to take into account harvesting of the population. This will enable fish harvesters to determine what frequency of harvesting rate their population can tolerate. These harvesting models are the constant harvesting and seasonal (periodic) harvesting.

Modified Logistic Growth Model With Constant Harvesting:
Constant harvesting is where a fixed number of fish were removed from the stock at constant time rate. We assume that a constant number, H, of tilapia fish are removed from the population. Hence, the model is given by

$\frac{dY}{dt} = rY\left(1 - \frac{Y}{C}\right) - H,\ H > 0$ --------- 5

where, the variable $\boldsymbol{Y}$ can be interpreted as the size of the fish population in tones, r is called the rate of fish survival at maturity stage , $\boldsymbol{C}$ is referred to as the carrying capacity of the environment, $\boldsymbol{H}$ is constant number of fish harvested each time.

Equilibrium Solutions Of Constant Harvesting:
By setting equation (5) equal to zero, we have two equilibrium solutions. These occurs when the growth rate of the fish population is equivalent to the harvest rate, thus,

$rY\left(1 - \frac{Y}{C}\right) - H = 0$

$rY\left(1 - \frac{Y}{C}\right) = H$

$\left(rY - \frac{rY^2}{C}\right) = H$

$rY^2 - rCY + CH = 0$

By quadratic formula, we have the equilibrium solutions as follows:

$Y_{1,2} = \frac{-Cr \pm \sqrt{(Cr)^2 - 4r(CH)}}{2r}$ -------- 6

For maximum sustainable harvesting rate, we let the expression under the radical sign equal zero, as follows:

$(Cr)^2 - 4r(CH) = 0$

$C^2r - 4CH = 0$

$Cr - 4CH = 0$

$H = \frac{rC}{4}$ ------ 7

Hence, equation (7) is the maximum rate of harvesting and this gives us the maximum Sustainable yield (MSY). The value H = $\frac{rC}{4}$ is called the bifurcation point.

Figure 3, is the graph of functions $h(Y)$in the three cases of the harvesting rate:

$H < \frac{rC}{4}$:$H = \frac{rC}{4}$,and $H > \frac{rC}{4}$

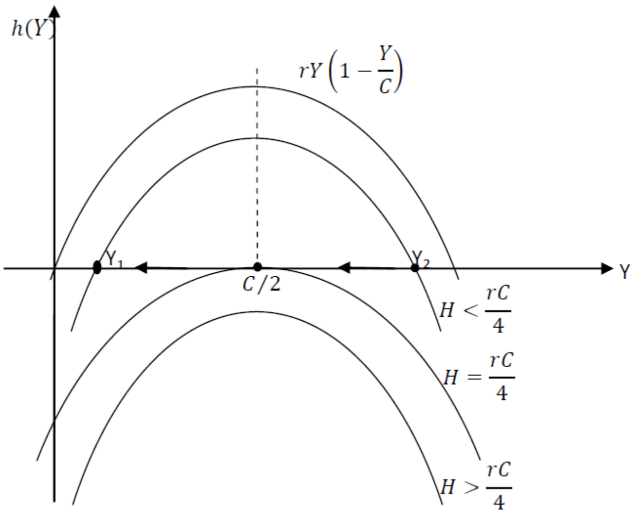

Figure 4

We can see that for a harvesting rate that is not too large ($0<Y<\frac{rC}{4}$) there exist two equlilbria solutions (0 and C) as in Figure 3.8. The lower equilibrium solution $Y=0$ is unstable. Thus, if for overfishing or disease outbreak, the size of the population Y drops below zero and the population eventually die out in a short time. The upper equilibrium solution $Y=C$ is stable. This is the steady state toward which the population approaches a constant harvesting C.

For H > $\frac{rC}{4}$,

there is no critical point or equilibrium solution, and the entire population will be harvested in a short time.

For H = $\frac{rC}{4}$,

there is one critical point or equilibrium state that is semi-stable. Thus, it is mathematically possible to continue harvesting indefinitely at such rate if the initial population is sufficiently large. However, any small change in the equilibrium size of the population will lead to a complete harvest of the population in a short time.

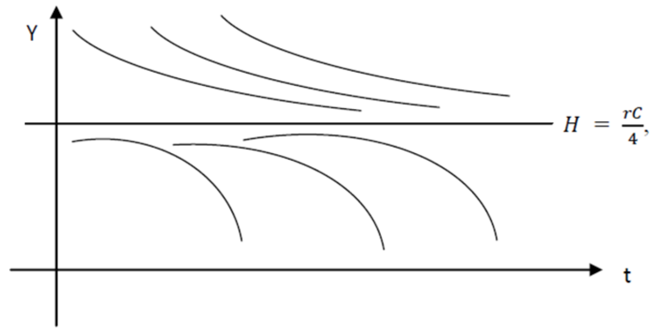

Figure 5: Solution curves for maximum sustainable harvest H = $\frac{rC}{4}$,

+It turns out however, that the harvest can be organized so as to obtain in a stable manner a harvest at the rate $\frac{rC}{4}$, for one unit time and more than this can be achieved since $\frac{rC}{4}$, is the maximal reproductive rate of the un harvested population.
For H < $\frac{rC}{4}$, we have two critical points Y_1 and Y_2. Y_2 is the carrying capacity with human intervention and Y_1 is the extinction level that means if the population is less than Y_1, it will lead to extinction. From figure 3.8 above, as the harvesting rate H increases Y_1 increases and Y_2 decreases.

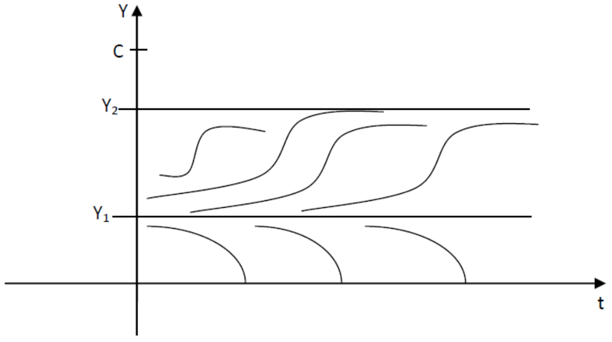

Figure 6: Solution curves 0 <Y<C

Conclusions
In this work, we studied the sustainable harvesting strategies of the tilapia fish population in a pond. Management of fish populations to sustain catches and abundance levels can be based on several alternative means of strategic catch regulation. This thesis is intended to explore harvesting strategies that optimizes catch while still maintaining a sustainable tilapia fishing industry and two logistic growth models have been used namely constant harvesting and periodic harvesting. We extend the modified logistic growth model developed by P.F Verhulst in 1838 by incorporating two types of harvesting strategies into the model and investigate how the demise (catch) of certain number of tilapia fish will affect the total population of tilapia fish in the pond.

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A New Method of Evaluating Reduction Formula for Definite Integral of Powers of Sine With Limits 0 to π

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ABSTRACT: *In this paper presents a easier and shorter method of evaluating integrals of powers of sine with limits 0 to π. The reduction formula for sine is repeatedly applied to the integral of the n^{th} powers of sine until generalized formulas are derived.*

Keywords: *Integration, Powers of sine, Reduction formula, Trigonometric identities.*

1. Introduction

Evaluating integrals of powers of trigonometric function is always a part of the study of integral calculus. Integrals of powers of sine are usually evaluated using trigonometric identities and the solution depends on whether the power is odd or even.

For odd powers the Integral is transformed by factoring out one sine and the remaining even powered sine is converted into cosine using the identity $sin^2x = 1 - cos^2x$.

The integral is then evaluated using power formulas with the factored sine used as the differential of cosine.

For even powers, the double angle identity $sin^2x = \frac{1-cos2x}{2}$ is used to reduce the power of sine into an expression. Where direct integration formulas can already be applied.

Another method used to evaluate powers of sine is by reduction formula. A reduction formula transforms the integral into an integral of the same or similar expression with a lower integer exponent. It is repeatedly applied until the powers of the last term are reduced to two or one and the final integral can be evaluated.

2. Derivation of formulas

Evaluate $\int sin^n xdx$, where n is any positive integer

Using the reduction formula

$$I_n = \int sin^n xdx = \frac{-sin^{n-1}xcosx}{n} + \frac{n-1}{n}\int sin^{n-2}xdx$$

$$I_n = \frac{-sin^{n-1}xcosx}{n} + \frac{n-1}{n} I_{n-2} \qquad (1)$$

Find I_{n-2}

Put $n = n-2$ in (1)

we get $I_{n-2} = \frac{-sin^{n-3}xcosx}{n-2} + \frac{n-3}{n-2} I_{n-4}$ (2)

Find I_{n-4}

Put $n = n-4$ in (1)

we get $I_{n-4} = \frac{-sin^{n-5}xcosx}{n-4} + \frac{n-5}{n-4} I_{n-6}$ (3)

Substitute (2), (3) in (1),

$$I_n = \frac{-sin^{n-1}xcosx}{n} - \frac{n-1}{n(n-2)} sin^{n-3}xcosx + \frac{(n-1)(n-3)}{n(n-2)}\left[\frac{-sin^{n-5}xcosx}{n-4} + \frac{n-5}{n-4} I_{n-6}\right]$$

$$I_n = \frac{-sin^{n-1}xcosx}{n} - \frac{n-1}{n(n-2)} sin^{n-3}xcosx - \frac{(n-1)(n-3)}{n(n-2)(n-4)} sin^{n-5}xcosx + \frac{(n-1)(n-3)}{n(n-2)(n-4)} I_{n-6}$$

Continuing the same manner until the last term becomes even or odd.

2.1 Derivation of powers of sine with limits 0 to π

Let $I_n = \int_0^{\pi} sin^n\, xdx$ (4)

Using the reduction formula

$I_n = \int sin^n xdx = \frac{-sin^{n-1}xcosx}{n} + \frac{n-1}{n} I_{n-2}$

Applying the limits (0 to π),

$I_n = \int_0^{\pi} sin^n xdx = \left[\frac{-sin^{n-1}xcosx}{n}\right]_0^{\pi} + \frac{n-1}{n} I_{n-2}$

$I_n = \frac{n-1}{n} I_{n-2}$ (5)

Find I_{n-2}

Put $n = n-2$ in (5)

$I_{n-2} = \frac{n-3}{n-2} I_{n-4}$ (6)

Similarly,

$I_{n-4} = \frac{n-5}{n-4} I_{n-6}$ (7)

Substitute (6), (7) in (5)

$I_n = \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4} I_{n-6}$

Continuing the same manner until the last term becomes even or odd.

2.2 Odd powers

$I_n = \int_0^{\pi} sin^n xdx$

$= \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-3} \dots \dots \dots \dots \dots I_3 I_1$ (8)

Find I_3

Put $n = 3$ in (5)

$I_3 = \frac{2}{3} I_1$ (9)

Find I_1

Put $n = 1$in (4)

$I_1 = \int_0^{\pi} sinxdx$

$= (-cosx)_{0=-(-1)-(-1)}^{\pi}$

$I_1 = 1 + 1 = 2$ (10)

Substitute (9),(10) in (8)

$I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \dots \dots \dots \dots \dots \dots \dots \frac{2}{3} \cdot 2$, when n is odd.

2.3 Even powers

$I_n = \int_0^{\pi} sin^n\, xdx$

$= \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \dots \dots \dots \dots I_2 . I_0$ (11)

Find I_2,

Put $n = 2$ in (5)

$I_2 = \frac{1}{2} I_0$ (12)

Find I_0,
Put $n = 0$ in (4)

$I_0 = \int_0^{\pi} sin^0\, xdx$

$= \int_0^{\pi} dx$

$I_0 = \pi$ (13)

Substitute (12), (13) in (11)

$I_n = \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4} \dots \dots \dots \frac{1}{2}\pi$, when n is even.

3. Comparison between the old and the new method

Evaluate $\int_0^{\pi} sin^3 xdx$

Using the old method

$\int_0^{\pi} sin^3xdx = \frac{1}{4}\int_0^{\pi}(3sinx - sin3x)\, dx$

$= \frac{1}{4}\left[-3cosx + \frac{cos3x}{3}\right]_0^{\pi}$

$= \frac{1}{4}\left[\left(-3(-1) - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right]$

$= \frac{1}{4}\left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right]$

$= \frac{1}{4}\left[6 - \frac{2}{3}\right]$

$\int_0^{\pi} sin^3\, xdx = \frac{4}{3}$

Using the new method

For n is odd, using the reduction formula,

$I_n = \int_0^{\pi} sin^n xdx = \frac{n-1}{n}.\frac{n-3}{n-2} \dots \dots \dots \dots \dots \frac{2}{3}.2$

Put $n = 3$,
We get

$I_3 = \int_0^{\pi} sin^3xdx = \frac{2}{3}.2$

$\int_0^{\pi} sin^3xdx = \frac{4}{3}$

Evaluate $\int_0^{\pi} sin^5 xdx$

Using the old method

$\int_0^{\pi} sin^5xdx = \int_0^{\pi} sin^2xsin^3xdx$

$= \int_0^{\pi}\left(\frac{1-cos2x}{2}\right)\left(\frac{3sinx - sin3x}{4}\right) dx$

$= \frac{1}{8}\int_0^{\pi}(3sinx - sin3x - 3cos2xsinx + cos2xsin3x)xdx$

$= \frac{1}{8}\int_0^{\pi}\left[3sinx - sin3x - \frac{3}{2}(sin3x - sinx) + \frac{1}{2}(sin5x + sinx)\right] dx$

$= \frac{1}{16}\int_0^{\pi}[6sinx - 2sin3x - 3sin3x + 3sinx + sin5x + sinx]\, dx$

$= \frac{1}{16}\int_0^{\pi}[10sinx - 5sin3x + sin5x]dx$

$= \frac{1}{16}\left[-10cosx + \frac{5cos3x}{3} - \frac{cos5x}{5}\right]_0^{\pi}$

$= \frac{1}{16}\left[\left(10 - \frac{5}{3} + \frac{1}{5}\right) - \left(-10 + \frac{5}{3} - \frac{1}{5}\right)\right]$

$= \frac{1}{16}\left[20 - \frac{10}{3} + \frac{2}{5}\right]$

$= \frac{1}{16}\left[\frac{300-50+6}{15}\right]$

$\int_0^{\pi} sin^5 xdx = \frac{16}{15}$

Using the new method

For n is odd, using the reduction formula,

$I_n = \int_0^{\pi} sin^n xdx = \frac{n-1}{n}\frac{n-3}{n-2} \dots \dots \dots \frac{2}{3}.2$

Put $n = 5$

$I_5 = \int_0^{\pi} sin^5 xdx = \frac{5-1}{5}.\frac{5-3}{5-2}.2$

$= \frac{4}{5}.\frac{2}{3}.2$

$I_5 = \int_0^{\pi} sin^5 xdx = \frac{16}{15}$

Evaluate $\int_0^{\pi} sin^6 xdx$

Using the old method

$\int_0^{\pi} sin^6 xdx = \int_0^{\pi} sin^3 xsin^3 xdx$

$= \int_0^{\pi} \left(\frac{3sinx - sin3x}{4}\right)\left(\frac{3sinx - sin3x}{4}\right)dx$

$= \frac{1}{16}\int_0^{\pi}(9sinxsinx - 3sin3xsinx - 3sin3xsinx + sin3xsin3x)dx$

$= \frac{1}{16}\int_0^{\pi}(9sin^2 x - 6sin3xsinx + sin^2 3x)dx$

$= \frac{1}{16}\int_0^{\pi}\left[\frac{9}{2}(1 - cos2x) - 6(cos2x - cos4x) + \frac{1}{2}(1 - cos6x)\right]dx$

$= \frac{1}{32}\int_0^{\pi}(9 - 9cos2x - 12cos2x + 12cos4x + 1 - cos6x)dx$

$= \frac{1}{32}\left[9x - \frac{9sin2x}{2} - \frac{12sin2x}{2} + \frac{12sin4x}{4} + x - \frac{sin6x}{6}\right]_0^{\pi}$

$= \frac{1}{32}[9\pi + \pi]$

$= \frac{10\pi}{32}$

$\int_0^{\pi} sin^6 xdx = \frac{5\pi}{16}$

Using the new method

For n is even, using the reduction formula,

$\int_0^{\pi} sin^n xdx = \frac{n-1}{n}\frac{n-3}{n-2} \dots \dots \dots \frac{2}{3}.\pi$

Put $n = 6$,

$\int_0^{\pi} sin^6 xdx = \frac{5}{6}\frac{3}{4}\frac{1}{2}.\pi$

$\int_0^{\pi} sin^6 xdx = \frac{5\pi}{16}$

4. Conclusion

The new method is simpler and easier to use, since the powers of sine is higher it is more complicated to evaluate that type of problems using older method. It is very helpful since integrals of powers of sine are always encountered in higher Mathematics courses like Differential equation and Advanced engineering Mathematics and even in Physics and Mechanics.

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A Study on Fuzzy Matrix Theory

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ABSTRACT: *The purpose of present this paper is to introduce the basic concept of fuzzy matrix theory. Moreover, we shall consider the operations defined on these matrices for further treatment of determinant and adjoint theory of square fuzzy matrices. First of all, we shall give fuzzy matrix theory and some operations defined on fuzzy matrices.*

Keywords: *Fuzzy Matrix theory, Row matrix, column matrix, square matrix, Diagonal matrix, Scalar, transpose of a fuzzy matrix.*

I Introduction

The theory of fuzzy set was first Introduced Professor Loffi A. Zadeh laid. Fuzzy matrix relation explained Ovehinnikov. And the Fuzzy Matrices were introduced first time by Thomson. We did not know until 1965 how the Vagueness arising from Subjectivity which is inherent in human thought processes can be modelled and analyzed. . However the book of Paul Horst, "Matrix Algebra for Social Scientists". Would be a boon to social scientists who wish to make use of matrix theory in their analysis.

II Fuzzy Matrix Theory

A fuzzy matrix is a matrix which has its elements from [0, 1] called fuzzy unit interval.

Definition: 2.1

Consider a matrix A + $[a_{ij}]_{mxn}$ where a_{ij} Ç [0, 1], $1 \le i \le m$ and $1 \le j \le n$. Then A is fuzzy matrix.

III Types of fuzzy Matrices

(a) (i) Fuzzy Rectangular Matrix :

Let A = $[a_{if}]_{mxn}$ (m≠n) where a_{ij} ε [0, 1], $1 \le i \le m$ and $1 \le j \le n$.
Then A is a fuzzy rectangular matrix.

For example: $\begin{bmatrix} 0 & 1 & 0.2 \\ 0.1 & 0.5 & 0.3 \end{bmatrix}$ is a 2 x 3 fuzzy rectangular matrix.

(ii) Fuzzy square Matrix :

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{ij} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{21} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{bmatrix}$$

where a_{ij} ε [0, 1], $1 \le i, j \le n$. Then A is a fuzzy square matrix.

(iii) Fuzzy Row Matrix :

Let a = [a1 a2 ......an], a_{ij} ε [0, 1] ; j = 1, 2, ..., n.
Then A is called a_{1xn} fuzzy row matrix or fuzzy row vector.
For example: [0.3 0.7 0.05 1] is a 1 x 4 fuzzy row matrix.

(iv) Fuzzy column Matrix :

Let $A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$

Where $a_i \in [0, 1]$; i = 1, 2, ..., m.
Then A is called a_{mx1} fuzzy column matrix.

For example : $\begin{bmatrix} 0 \\ 0.4 \\ 0.5 \end{bmatrix}$

is a_{3x1} fuzzy column matrix or fuzzy column vector.

(v) Fuzzy Diagonal Matrix :
A fuzzy square matrix $A = [a_{ij}]_{nxn}$ is said to be fuzzy diagonal matrix if $a_{ij} = 0$. when $i \neq j$, where $a_i \in [0, 1]$, $1 \leq i, j \leq n$.

For Example: $\begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}$ is a fuzzy diagonal matrix of order 3

This diagonal matrix is also denoted by [0.4, 0.3, 0.9].

(vi) Fuzzy Scalar Matrix :
A fuzzy diagonal matrix is said to be fuzzy scalar matrix, if all its diagonal entries are equal. Thus, a fuzzy square matrix $A = [a_{ij}]_{nxn}$ is said to be fuzzy scalar matrix if

$$\begin{cases} a_{ij} = 0 \text{ when } i \neq j \\ a_{ij} = \alpha \text{ when } i \neq j \end{cases} \quad \text{where } \alpha \in [0, 1], 1 \leq i, j \leq n.$$

For example : [0.3] and $\begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}$ are fuzzy scalar matrices of order 1 and 2

Respectively.
Usual identity matrix and zero matrix are fuzzy matrices as their entries are from
the fuzzy crisp set {0, 1}.

If the entries in upper triangular matrix and lower triangular matrix are from fuzzy unit interval [0. 1], then these matrices are said to be fuzzy upper triangular and fuzzy lower triangular matrices respectively.

IV Equality of Fuzzy Matrices:
Two fuzzy matrices of the same type are said to be equal iff their elements in the corresponding positions are equal.

V Operations On Fuzzy Matrices
5.1 Operations of Maximum and Minimum:
We shall define the following three operations on fuzzy matrices:
(a) Maximum of matrices
(b) Minimum of a matrix by a scalar
(c) Max min of matrices

(a) Operation I Maximum of Matrices
If two matrices are of the same type, then they are said to be confortable for addition. But the question arises that when we add two fuzzy matrices, then the resultant matrix is not a fuzzy matrix. So in case of funny matrices of same type, max operation is defined.

Definition 5.2

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ be two fuzzy matrices.

Then their sum, denoted by A+B, is defined as

$A + B = \max\{A, B\}$

i.e., $[a_{ij} + b_{ij}]_{m\times n} = [\max(a_{ij}, b_{ij})]_{m\times n}$; for $1 \le i \le m$, $1 \le j \le n$

For example : Let $A = \begin{bmatrix} 0 & 0.3 & 0.9 \\ 0.4 & 0.3 & 0.1 \\ 1 & 0.8 & 0.4 \\ 0.5 & 0.2 & 0.6 \end{bmatrix}$ and $B = \begin{bmatrix} 0.6 & 0.7 & 0 \\ 0.9 & 0.5 & 0.5 \\ 0.7 & 1 & 0 \\ 0.6 & 0.8 & 0.5 \end{bmatrix}$

Then

$$A + B \max\{A, B\} = \begin{bmatrix} \max(0, 0.6) & \max(0.3, 0.7) & \max(0.9, 0) \\ \max(0.4, 0.9) & \max(0.3, 0.5) & \max(0.1, 0.5) \\ \max(1.0, 0.7) & \max(0.8, 1.0) & \max(0.4, 0.0) \\ \max(0.5, 0.6) & \max(0.2, 0.8) & \max(0.6, 0.5) \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & 0.7 & 0.9 \\ 0.9 & 0.5 & 0.5 \\ 1 & 1 & 0.4 \\ 0.6 & 0.8 & 0.6 \end{bmatrix}$$

In a similar way, we can define the difference of two fuzzy matrices of same type as the max operation. Thus, in case of fuzzy matrices of same type,
A-B = max {A, B} = A + B

(b) Operation II Maximum of a matrix by a scalar:
Definition: 5.3

Let $A = [a_{ij}]_{m\times n}$ be any fuzzy matrix and $k \in F$, where $F = [0,1]$ is a fuzzy unit interval. Then scalar multiple of A by k, denoted by kA or Ak is given by

$$kA = Ak = [ka_{ij}]_{m\times n} = [\min(k, a_{ij})]_{m\times n};\ a_{ij} \in [0,1],\ 1 \le i \le m,\ 1 \le j \le n.$$

Thus kA is the matrix obtained when each entry of A is multiplied by k.

For example : $0.3 \begin{bmatrix} 0.4 & 0.5 & 1 \\ 0.2 & 0.8 & 0.6 \end{bmatrix}$

$$= \begin{bmatrix} \min(0.3, 0.4) & \min(0.3, 0.5) & \min(0.3, 1) \\ \min(0.3, 0.2) & \min(0.3, 0.8) & \min(0.3, 0.6) \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & 0.3 & 0.3 \\ 0.2 & 0.3 & 0.3 \end{bmatrix}$$

(c) Operation III max min of matrices:

If we wish to find the product AB of two fuzzy matrices A and B where A and B are compatible under multiplication i.e., number of columns of A = number of rows of B; still we may not have the product AB to be a fuzzy matrix. So in case of fuzzy matrices compatible under multiplication, max min operation is defined.

Definition 2.3.3 : Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$ be two fuzzy matrices. Then their product, denoted by AB, is defined to be the fuzzy matrix $[C_{ik}]_{m\times p}$. Where $C_{ik} = \sum_{j=i}^{n} a_{ij} b_{jk} = \max\{\min(a_{ij}, b_{jk};\ 1 \le i \le m,\ 1 \le k \le p\}$ for $j=1,2,\ldots\ldots n$.

Remark:

If the fuzzy product AB is defined then BA may not be defined.

For example : Let $A = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.4 & 0 & 0.6 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} 0 & 0.3 & 0.7 & 0.5 \\ 0.4 & 0.7 & 0.4 & 1.0 \\ 0.6 & 0.3 & 1 & 0.1 \end{bmatrix}_{3\times 4}$

Since no. Of columns of A = no. Of rows in B
Then the product AB is defined and is given by

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$$AB = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{bmatrix}_{2\times4}$$

When C11 = max {min(0.3, 0), min(0.2, 0.4), min(0.5,0.6)}
= max {0, 0.2, 0.5}
= 0.5

C12 = max {min(0.3, 0.3), min(0.2, 0.7), min(0.5,0.3)}
= max {0.3, 0.2, 0.3}
= 0.3

C13 = max {min(0.3, 0.7), min(0.2, 0.4), min(0.5,1)}
= max {0.3, 0.2, 0.5}
= 0.5

C14 = max {min(0.3, 0.5), min(0.2, 1), min(0.5,0.1)}
= max {0.3, 0.2, 0.1}
= 0.3

C21 = max {min(0.4, 0), min(0,0.4), min(0.6,0.6)}
= max {0, 0, 0.6}
= 0.6

C22 = max {min(0.4, 0.3), min(0,0.7), min(0.6,0.3)}
= max {0.3, 0, 0.3}
= 0.3

C23 = max {min(0.4, 0.7), min(0,0.4), min(0.6,1)}
= max {0.4, 0, 0.6}
= 0.6

C24 = max {min(0.4, 0.5), min(0,1), min(0.6,1)}
= max {0.4, 0, 0.1}
= 0.4

$$\text{Thus AB} = \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.3 \\ 0.6 & 0.3 & 0.6 & 0.4 \end{bmatrix}$$

But since, no. Of columns of B ≠ no. Of rows of A. Thus the product BA is not defined.
Using the max-min function, we can find the positive integral powers of a square fuzzy matrix.

VI Transpose of fuzzy matrix:
Definition : 6.1

Let $A = [a_{ij}]_{m\times n}$ be any fuzzy matrix. Then the transpose of A, denoted by A′ or A^t or A^T, is n × m fuzzy matrix obtained from A by interchaing its rows and columns.

i.e., $A' = [b_{ij}]_{n\times m}$ where by $b_{ij} = a_{ji} \in [0,1]$; for $1 \le i \le n$ and $1 \le j \le m$.

Remarks

1. The transpose of a fuzzy row matrix is a fuzzy column matrix and vice-versa.
2. The product AA' and $A'A$ of two fuzzy matrices are always defined.

For example : Let $A = \begin{bmatrix} 0.2 \\ 0.6 \\ 0.7 \end{bmatrix}_{3\times1}$ be a fuzzy column matrix

3. Then $A' = [0.2 \;\; 0.6 \;\; 0.7]_{1\times3}$ is a fuzzy row matrix.

Observe that the products AA' and $A'A$ of two fuzzy matrices A and A' are defined.

$$\text{Now } A'.A = [0.2 \;\; 0.6 \;\; 0.7] \begin{bmatrix} 0.2 \\ 0.6 \\ 0.7 \end{bmatrix}$$

$$= \max\{0.2, 0.6, 0.7\}$$

$$= 0.7$$

Then $A'.A$ is a singleton fuzzy matrix.

$$\text{and } A.A' = \begin{bmatrix} 0.2 \\ 0.6 \\ 0.7 \end{bmatrix} = [0.2 \;\; 0.6 \;\; 0.7]$$

$$= \begin{bmatrix} \min(0.2, 0.2) & \min(0.2, 0.6) & \min(0.2, 0.7) \\ \min(0.6, 0.2) & \min(0.6, 0.6) & \min(0.6, 0.7) \\ \min(0.7, 0.2) & \min(0.7, 0.6) & \min(0.7, 0.7) \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.6 \\ 0.2 & 0.6 & 0.7 \end{bmatrix}$$

Then $A.A'$ is a symmetric fuzzy matrix as the matrix obtained by interchanging its rows and columns is the matrix itself and the elements of the matrix belong to fuzzy unit interval [0,1].

Conclusion

Consists of fuzzy matrix theory involving the types and equality of matrices. Moreover, operations on these matrices including transpose of these matrices are explained.

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Addition, Multiplication Theorems of Probability and Boole's Inequality for Mathematical Statistics

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ABSTRACT: *In this paper, we define Random inquiry, sample space, trail, event, probability and its important probabilities axioms is given. The relations between addition and multiplication theorems are established. Boole's inequality theorem is derived.*

Keywords: *S: Sample space*
H: Head
T: Tail
P: Probability
A, B: Events.

1. Introduction

The theory of mathematical probability has it's in the 17thcentury.There are three different approaches of measuring probabilities. They are classical probability, relative frequency of thin gamy and axiomatic probability.

The word "probability" or "chance" is very commonly used I day-to-day conversation and generally people have an imprecise opinion about its meaning.

Galileo (1564-1642), an Italian mathematician, was first man to attempt quantitative measure of probability while dealing with some problems related to the theory of dice in gamble.

The 19thcentury, required precise knowledge about the risk of loss in order to calculate premium. In this chapter we shall study the classical probability and the axiomatic approach.

Definition: Random Inquiry

An inquiry is known as Radom inquiry, if when conducted oft underneath. Essentially homogeneous condition the result is not unique but may be anyone of the diverse probable impact.

Sample Space

Out of the more possible outcomes of a random inquiry, one and only one can takes the place in a trail. The set of all probable impact is called a sample space and it is denoted by S.

Example

If coin is tossed the probable impact head and tail.
Sample space, S= {H, T}

Defiition

TRAIL : Performing of a random inquiry is trail.
EVENT: Combination of outcomes are termed as event.

Examble-1

- If a coin is tossed oft the result is not unique get any one of the two faces.(Head, Tail)
- Tossing a coin is a random experiment is trail and getting (Head, Tail) is an event.

Example-2

- Throwing a dice is a trail and getting a unique of phase is (1, 2, 3, 4, 5, 6) event.

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Types of Event

- Exhaustive event.
- Favorable event
- Mutually exclusive event
- Equally likely event
- Independent and dependent event.

Definition: Probability

The probability for the occurrence of an event A is defined as the ratio betwixt the number of favorable outcomes for the thin gamy of the event and the total number of possible outcomes.

Ie) Probability of an event== $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

Axioms of Probability

Given a sample space of random inquiry, the probability of occurrence any event A is defined as a set of function P (A) satisfying the following steps.

1. P (A) is defined as real number and non-negative.

I.e.) 0≤P (A) ≤1

2. The probability of the entire sample space is 1

I.e.) P(S) =1

3. If A1, A2, A3 …An are any finite or sequence of the joint evets of "S".

P ($\bigcup_{i=1}^{n} A_i$) =$\sum_{i=1}^{n} p(Ai)$ or

p ($\bigcup_{i=1}^{\infty} A_i$)=$\sum_{i=1}^{\infty} p(Ai)$

4. If A and B are mutually exclusive events.

P (AUB) =P (A) +P (B).

Note

P ($^{A}/_{B}$) = p ($^{A \cap B}/_{B}$) B≠0

P ($^{B}/_{A}$) = p ($^{A \cap B}/_{A}$) B≠0

P (A)= $\frac{n(A)}{n(S)}$

P (B)= $\frac{n(B)}{n(S)}$

P ($^{A}/_{B}$) = $\frac{n(A \cap B)}{n(S)}$

Theorems of Probability

There are two important theorems of probability

I. The Addition Theorem
II. The Multiplication Theorem.

Addition Theorem of Probability

Statement

If A and B are two events.ThenP (AUB) =P (A) +P (B)-P (A∩B)

Proof

From the Venn diagram we have,
$(A\cup B)=A\cup(\bar{A}\cap B)$ ------------------------------------ (1)
$B=(A\cap B)\cup(\bar{A}\cap B)$ ------------------------------------ (2)

Taking probability on both side (1) and (2)
Taking probability on both side
$P(A\cup B)=P[A\cup(\bar{A}\cap B)]$ ------------------------------ (a)
$P(B)=P[(A\cap B)\cup(\bar{A}\cap B)]$
$P(B)=P(A\cap B)+P(\bar{A}\cap B)$ ---------------------------- (b)

(a) => $P(A\cup B)=P[A\cup(\bar{A}\cap B)]$
$=P(A)+P(\bar{A}\cap B)$

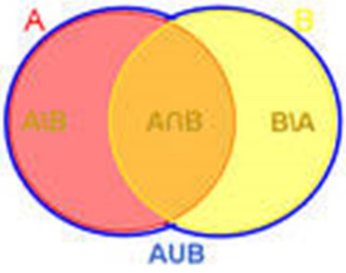
Adding and subtracting $P(A\cap B)$ on L.H.S we have,
$P(A\cup B)=P(A)+P(\bar{A}\cap B)+P(A\cap B)-P(A\cap B)$-------------(3)
(b) Equation sub (3) in we get,

$$P(A\cup B)=P(A)+\frac{P(\bar{A}\cap B)+P(A\cap B)}{P(B)}-P(A\cap B)$$
$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

Hence the addition theorem is proved.
Multiplication Theorem of Probability

Statement
If A and B are two events. Then $P(A\cap B)=P(A).P(B/A)$, $P(A)>0$.

Proof



We know the notation, we have,

$P(A)=\frac{n(A)}{n(S)}$ --------------------------------- (1)
$P(B)=\frac{n(B)}{n(S)}$ --------------------------------- (2)
$P(A\cap B)=\frac{n(A\cap B)}{n(S)}$ ------------------------- (3)

Multiply and divide n (B) in L.H.S in
(3) => $P(A\cap B)=\frac{n(A\cap B)}{n(S)}.\frac{n(A)}{n(A)}$
$=\; n(A\cap B)\bullet\frac{1}{n(S)}\bullet n(A)\bullet\frac{1}{n(A)}$
$=\frac{n(A\cap B)}{n(A)}\bullet\frac{n(A)}{n(S)}$ [by note]

$P(A\cap B=P(B/A)\bullet P(A)$ --------------- (*)

{The number of common outcomes A and B is either less or equal to the number of outcomes in any of the event.ie)P (B/A) ≤P (B).}
P(A∩B)=P(A)•P(B) or P(A∩B)=P(B)•P(A).
Hence the multiplication theorem is proved.

Boole's Inequality
Statement
If A1, A2…………….An are events in the sample space S. Then P (A1UA2UA3U………..An) ≤ P (A1) + P (A2) + ………..+ P (An).

Proof:
Given A1, A2 …….. An are n events.
Let us prove this result using mathematical induction.
We know Theorem, we have P (AUB) = P (A) + P (B) – P (AB)
Consider two events A1 and A2.
Then, P (A1UA2) = P (A1) + P (A2) – P (A1A2) -------------------- (1)
But P (A1A2) ≥0 [by note 0≤ P (A) ≤1]
I.e.), P (A1A2) =0 --- (2)
2 sub in 1 we get,
P (A1UA2) ≤ P (A1) + P (A2) -------------------------------------- (*)

The results is true for two events.
Let us now assume that the result is true for (n-1) events.
P(A1UA2U……UAn-1)≤p(A1)+p(A2)+……+p(An-1)…..-------------------------(**)
Consider n events A1, A2…An.
P (A1UA2……An) =P (BUA) ----------------------------------- (a)
[Where B=A1UA2…….UAn-1]
P (B) =P (A1UA2U…….UAn-1)
P (B) =p (A1) +p (A2) +……+P (An-1) ----------------------------- (b)
(a) => P (A1UA2U…………UAn) ≤P (BUAn)
P (A1UA2U………..UAn) ≤P (B) +P (A)
P (A1UA2U…………UAn) ≤$\frac{P(A1)+P(A2)+\cdots\ldots\ldots+P(An-1)}{P(B)}$+P (An)

Thus we establish the result is true for n events if it is true for (n-1) events .But we have proved that the result is true for two events.
It is true for n=3, 4, 5………..by induction.
I.e., it is true for all positive integer values of n.
P(A1UA2U……..UAn)≤P(A1)+P(A2)+………+P(An).
Hence Boole's inequality proved.

Conclusion
In this paper we have to study addition, multiplication theorem and types of event. We have to obtained an optimal solution for the probability. The Boole's inequality theorem is established.

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Convolution for the Laplace Transform

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ABSTRACT: *Generally, convolution will assist us in solving integral equations. This paper is to study Convolution Theorem of the form*

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

and example method to compute convolution for any functions. In addition to that we are going to prove that the Convolution can take place in solving Laplace transforms.

Keywords: *Convolution, Laplace Transform*

1. Introduction

This project is concerned with Convolution Theorem and its usage in Laplace Transform. A convolution is a mathematical operation taking two functions as input, and producing a third function as output, much like addition or multiplication operations but with a more involved definition. As we will see below, convolutions have interesting applications in connection with Laplace transforms because of their simple transforms.
The content of this project is developed in an extensive way based on the definition of Convolution and its solution methods. Convolution problems are solved in a bunch of different ways. Especially through this, we are going to see it in the context of taking Laplace transforms.

Definition: (Convolution)

The convolution of f & g is written as f * g using an asterisk (or) star. It is defined as the integral of the two functions after one is reversed and shifted. As such, it is a type of integral transform

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$$

and it is equivalent to the definition given below

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau$$

For functions f & g on only $[0,\infty)$, the integration limits can be truncated, resulting in:

$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$ for f, g: $[0,\infty) \to$ R

Compute Convolution

Let $f(t) = \sin t$, $g(t) = \cos t$

By using convolution definition,

$$(f * g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau$$

$(f * g)(t) = \int_0^t \sin(t-\tau)\cos(\tau)d\tau$ $\quad [\sin(t-\tau) = \sin(t)\cos(\tau) - \sin(\tau)\cos(t)]$

$$= \int_0^t (\sin(t)\cos(\tau) - \sin(\tau)\cos(t))\cos(\tau)d\tau$$

$$= \int_0^t (\sin(t)\cos^2(\tau) - \cos(t)\sin(\tau)\cos(\tau))d\tau$$

$$= \int_0^t \sin(t)\cos^2(\tau)d\tau - \int_0^t \cos(t)\sin(\tau)\cos(\tau)d\tau$$

$$= \sin(t)\int_0^t \cos^2(\tau)d\tau - \cos(t)\int_0^t \sin(\tau)\cos(\tau)d\tau$$

$$= \sin(t)\int_0^t \frac{1}{2}(1+\cos 2\tau)d\tau - \cos(t)\int_0^t u\,du$$

$[u = \sin(\tau), du = \cos(\tau)d\tau]$

$$= \frac{1}{2}\sin(t)\ [\tau + \frac{1}{2}\sin 2\tau]_0^t - \cos t\ [\frac{\sin^2(\tau)}{2}]_0^t$$

$= \frac{1}{2}\sin(t)\ [t + \frac{1}{2}\sin 2t] - \cos t\ [\frac{\sin^2(t)}{2}]$
$= \frac{1}{2}\,t\sin(t) + \frac{1}{4}\sin t\sin 2t - \frac{1}{2}\sin^2(t)\cos t$
$= \frac{1}{2}\,t\sin(t) + \frac{1}{2}\sin^2(t)\cos t - \frac{1}{2}\sin^2(t)\cos t \qquad [\sin 2t = 2\sin t\cos t]$
$(f * g)(t) = \frac{1}{2}\,t\sin(t)$

Result
$(f * g)(t) = \frac{1}{2}\int_0^t f(t-\tau)g(\tau)d\tau = \int_0^t \sin(t-\tau)\cos(\tau)\,d\tau = \frac{1}{2}\,t\sin(t)$

Definition: (Laplace transform of f(t))
Let $f(t)$ be a real valued function of the variable 't' defined for $t > 0$. Then the Laplace transform of $f(t)$ is defined by
L{f(t)} = $\int_0^\infty e^{-st} f(t)\ dt$
where the parameter 's' is real and $f(t)$ is denoted by the symbol $F(s)$ or L{f(t)}.
$F(s) = \int_0^\infty e^{-st} f(t)\ dt$

Laplace Transform of Convolution Theorem
If $L[f_1(t)] = F_1(s)$ and $L[f_2(t)] = F_2(s)$
then, $L\left\{\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right\} = F_1(s)F_2(s)$

Proof
According to the definition of Laplace Theorem,

$$L\left\{\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right\} = \int_0^\infty e^{-st}\left(\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right)dt$$

where the right-hand side is a double integral over the angular region bounded by the lines $\tau = 0$ and $\tau = t$ in the first quadrant of the tτ plane.
Changing the integration, we write

$$L\left\{\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right\} = \int_0^\infty f_1(\tau)\left(\int_\tau^\infty e^{-st} f_2(t-\tau)\,dt\right)d\tau$$

Let $u = t - \tau, du = dt$ we obtain

$$\int_\tau^\infty e^{-st} f_2(t-\tau)\,dt = \int_0^\infty e^{-(u+\tau)s} f_2(u)\,du$$
$$= e^{-\tau s}\int_0^\infty e^{-su} f_2(u)\,du$$

By using definition of Laplace Transform,

$$= e^{-\tau s} L[f_2(u)] = e^{-\tau s}F_2(s)$$
$$L\left\{\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right\} = \int_0^\infty f_1(\tau)e^{-\tau s}F_2(s)\,d\tau$$
$$= F_2(s)\int_0^\infty f_1(\tau)e^{-\tau s}\,d\tau$$
$$= F_2(s).F_1(s)$$

$L\left\{\int_0^t f_1(\tau)f_2(t-\tau)d\tau\right\} = F_1(s).F_2(s) \qquad [\because \textit{Commutative property} \rightarrow f * g = g * f]$

Example
Find $L[t^2 * e^t]$

Solution:
Since $L[f * g] = F(s).G(s)$
$L[t^2 * e^t] = \frac{2}{s^3}.\frac{1}{s-1}$ $\qquad\qquad \because L[t^2] = \frac{2}{s^3}, L[e^t] = \frac{1}{s-1}$

Example:
Use Convolutions to find the inverse Laplace Transform of $F(s) = \frac{3}{s^3(s^2-3)}$

Solution:
We express F as a product of two Laplace Transforms,

$$F(s) = \frac{3}{s^3(s^2-3)}$$

Multiply and divide by $2\sqrt{3}$ terms on both the sides, we get

$$F(s) = \frac{3}{2}.\frac{1}{\sqrt{3}}\left(\frac{2}{s^3}\right)\left(\frac{\sqrt{3}}{s^2-3}\right)$$

$$= \frac{\sqrt{3}}{2}.L[t^2]L\left[\sinh(\sqrt{3}t)\right]$$

$$F(s) = \frac{\sqrt{3}}{2}.L\left[t^2 * \sinh(\sqrt{3}t)\right]$$

By using Laplace transform of convolution,

$$L^{-1}[F_1(s).F_2(s)] = \int_0^t f_1(\tau)f_2(t-\tau)d\tau$$

We conclude that,

$$f(t) = \frac{\sqrt{3}}{2}\int_0^t \tau^2 \sinh\left[\sqrt{3}(t-\tau)\right]d\tau$$

CONVOLUTION THEOREM:
If $L^{-1}\{\bar{f}(s)\} = f(t)$ and $L^{-1}\{\bar{g}(s)\} = g(t)$,
then $L^{-1}\{\bar{g}(s).\bar{f}(s)\} = \int_0^t f(u)\,g(t-u)\,du = \int_0^t g(u)f(t-u)\,du$

Proof:
Let $\emptyset(t) = \int_0^t g(u)f(t-u)du$

$$L\{\emptyset(t)\} = \int_0^\infty e^{-st}\left[\int_0^t g(u)f(t-u)du\right]dt$$

$$= \int_{t=0}^\infty \int_{u=0}^\infty e^{-st}g(u)f(t-u)dudt$$

Changing the order of integrals (limits are t=u to t=∞ and u=0 to u=∞)

$$= \int_{u=0}^{u=\infty}\int_{t=u}^{t=\infty} e^{-st}g(u)f(t-u)dudt$$

$$= \int_{u=0}^\infty g(u)\left[\int_{t=u}^\infty e^{-st}f(t-u)dt\right]du$$

Put $t-u = x \Rightarrow dt = dx$ and limits are $x = 0$ to $x = \infty$

$$L\{\emptyset(t)\} = \int_0^\infty g(u)\left[\int_0^\infty e^{-s(x+u)}f(x)dx\right]du$$

$$= \int_0^\infty g(u)\left[\int_0^\infty e^{-sx}e^{-su}f(x)\,dx\right]du$$

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$$= \int_0^\infty e^{-su} g(u)\, du \int_0^\infty e^{-sx} f(x)\, dx$$

$$L\{\emptyset(t)\} = \bar{g}(s).\bar{f}(s)$$

$$\emptyset(t) = L^{-1}\{\bar{g}(s).\bar{f}(s)\}$$

$$\int_0^t f(u)\, g(t-u)\, du = L^{-1}\{\bar{g}(s).\bar{f}(s)\}$$

Example:

Evaluate using Convolution theorem $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$

Solution:

Let $\bar{f}(s) = \frac{1}{s^2+1}$ and $\bar{g}(s) = \frac{1}{s^2+9}$

$L^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t = f(t)$ and $L^{-1}\left\{\frac{1}{s^2+9}\right\} = \sin 3t = g(t)$

By convolution theorem $L^{-1}\{\bar{f}(s).\bar{g}(s)\} = \int_0^t f(u)\, g(t-u)\, du$

$$L^{-1}\{\bar{f}(s).\bar{g}(s)\} = \frac{1}{2}\int_0^t 2\sin u \sin 3(t-u)\, du$$

$$= \frac{1}{2}\int_0^t \cos(u-3t+3u) - \cos(u+3t-3u)\, du$$

$[\because 2\sin A \sin B = \cos(A-B) - \cos(A+B)]$

$$= \frac{1}{2}\int_0^t \cos(4u-3t) - \cos(3t-2u)\, du$$

$$= \frac{1}{2}\left[\frac{\sin(4u-3t)}{4} - \frac{\sin(3t-2u)}{-2}\right]_0^t$$

$$= \frac{1}{2}\left[\frac{\sin t - \sin(-3t)}{4} + \frac{\sin t - \sin 3t}{2}\right]_0^t$$

$$= \frac{1}{8}[\sin t + \sin 3t + 2\sin t - 2\sin 3t] \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$L^{-1}\{\bar{f}(s).\bar{g}(s)\} = \frac{1}{8}[3\sin t - \sin 3t]$$

$$L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\} = \frac{1}{8}[3\sin t - \sin 3t]$$

Conclusion

After seeing the evaluation method of Laplace Transform using Convolution, it is confirmed that convolution and its usage are explained as we mentioned in introduction. We also discussed about the application of convolution in different fields.

Thus, the proposed project is successfully found a solution of Laplace using convolution that is by replacing the inputs in Laplace with the functions used in Convolution theorem as given in the previous topic. For that reason, our project is titled as 'Convolution for the Laplace Transform'.

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An Extenstion of Fermat's Last Theorem in Six Dimensional Euclidean Space

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ABSTRACT: *The Fermat last theorem states that there is no integer triple (a,b,c) such that $a^n + b^n = c^n$ for n >2. And in an extension of Fermat's last theorem, they shown that $a^n + b^n + c^n = d^n$ is true for n=2,3 and in the extention of an extension of fermat's last theorem proved that $a^n + b^n + c^n + d^n = e^n$ for n=2 and now in this paper it is an attempt to show that $a^n + b^n + c^n + d^n + e^n + f^n = g^n$ for n=2*

Keywords: *Integer quadruple, integer quintuples, integer sextuples, five dimension Euclidean space, Fermat last theorem*

1. Introduction

Pierre Fermat (1601-1665) wrote a comment by the side while reading a book of Pythagoras triple that there is no integer triple(a,b,c), for which $a^n + b^n = c^n$ $for\ n > 2$ this result known as Fermat's last theorem and unsolved till 1995 when Andrew wiles in a 110-page paper was able to provide a proof [2]

Then in an extension of Fermat's last theorem it is an attempt to extend the Fermat's last theorem to integer quadruple and proved the result for $a^n + b^n + c^n = d^n$ for n=2,3

And in an extension of an extension of fermat's last theorem proved for integer quintuples $a^n + b^n + c^n + d^n = e^n$ $for\ n = 2.$

And in an extension of fermat's last theorem in five dimensional Euclidean space prove for integer sextuples $a^n + b^n + c^n + d^n + e^n = f^n$ $for\ n = 2$

Now in this paper it is an attempt to solve to solve for integer septuples
$a^n + b^n + c^n + d^n + e^n + f^n = g^n$ $for\ n = 2$

Preliminary results

We present few results on integer septuple

Result 1

If (a,b,c,d,e,f,g) is an integer septuples, then multiple of any integer n with this integer septuples is again an integer septuples (na,nb,nc,nd,ne,nf,ng)

Proof

$$(na)^2 + (nb)^2 + (nc)^2 + (nd)^2 + (ne)^2 + (nf)^2 = n^2a^2 + n^2b^2 + n^2c^2 + n^2d^2 + n^2e^2$$
$$= n^2(a^2 + b^2 + c^2 + d^2 + e^2 + f^2)$$
$$= n^2(g^2)$$

Result 2

For any septuples (a,b,c,d,e,f,g), if a is even and b,c,d,e,f are odd then g cannot be an even

Proof

Let
a =2p, b=2q+1, c=2r+1, d=2s+1 e=2t+1 f= 2l+1
for p,q,r,s,t$\in z$
if $a^2 + b^2 + c^2 + d^2 + e^2 + f^2 =$
$(2p)^2 + (2q + 1)^2 + (2r + 1)^2 + (2s + 1)^2 + (2t + 1)^2 + (2l + 1)^2$
$= 4p^2 + 4q^2 + 1 + 4q + 4r^2 + 1 + 4r + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t + 4l^2 + 1 + 4l$

$= 2(2p^2 + 2q^2 + 2r^2 + 2s^2 + 2t^2 + 2l^2 + 2q + 2r + 2s + 2t + 2l) + 5$

Which is an odd number

Result 3

For any septuples (a,b,c,d,e,f,g), if a,b are even and c,d,e,f are odd then g must be an even number

Proof :

Let a=2p, b=2q, c=2r+1, d=2s+1, e=2t+1, f=2l+1

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 4p^2 + 4q^2 + (2r+1)^2 + (2s+1)^2 + (2t+1)^2 + (2l+1)^2$

$= 4p^2 + 4q^2 + 4r^2 + 1 + 4r + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t + 4l^2 + 1 + 4l$

$g^2 = 2(2p^2 + 2q^2 + 2r^2 + 2r^2 + 2t^2 + 2s^2 + 2l^2 + 2r + 2s + 2t + 2l + 2)$

Which is an even number

Result 4

For any septuples (a,b,c,d,e,f,g),if a,b,c are even and d, e, f are odd then g must be an odd

Proof

Let a=2p, b=2q, c=2r, d=2s+1, e= 2t+1, f=2l+1

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = (2p)^2 + (2q)^2 + (2r)^2 + (2s+1)^2 + (2t+1)^2 + (2l+1)^2$

$= 4p^2 + 4q^2 + 4r^2 + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t + 4l^2 + 1 + 4l$

$g^2 = 4p^2 + 4q^2 + 4r^2 + 4s^2 + 4l^2 + 4s + 4t + 4l + 3$

An odd number

Result: 5

For any septuples (a, b, c, d, e, f ,g),if a, b, c, d are even and e,f are odd then g must be even

Proof

Let a=2p, b=2q, c=2r, d=2s, e= 2t+1, f=2l+1

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 4p^2 + 4q^2 + 4r^2 + 4s^2 + 4t^2 + 4t + 1 + 4l^2 + 1 + 4l$

$= 2(2p^2 + 2q^2 + 2r^2 + 2s^2 + 2t^2 + 2l^2 + 2t + 2l + 1)$

$g^2 = an\ even\ number$

Result 6

For any septuples (a, b, c, d, e, f,g), if a, b, c, d, e are even and f is odd then g must be odd

Proof

Let a=2p b=2q c=2r, d=2s, e= 2t, f=2l+1

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 4p^2 + 4q^2 + 4r^2 + 4s^2 + 4t^2 + 4l^2 + 1 + 4l$

$= 2(2p^2 + 2q^2 + 2r^2 + 2s^2 + 2t^2 + 2l^2 + 2l) + 1$

$g^2 = an\ odd\ number$

Result 7

For any septuples (a, b, c, d, e, f,g), if a, b, c, d, e,f are even then g must be even

Proof

Let a=2p b=2q c=2r, d=2s, e= 2t, f=2l

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 4p^2 + 4q^2 + 4r^2 + 4s^2 + 4t^2 + 4l^2$

$= 2(2p^2 + 2q^2 + 2r^2 + 2s^2 + 2t^2 + 2l^2)$

an even number

Result 8

For any septuples (a, b, c, d, e, f,g), if a, b, c, d, e,f are odd then g must be even

Proof:
a =2p+1, b=2q+1, c=2r+1, d=2s+1 e=2t+1, f=2l+1
for p, q, r, s, t $\in z$
if $a^2+b^2+c^2+d^2+e^2+f^2=$
$(2p+1)^2+(2q+1)^2+(2r+1)^2+(2s+1)^2+(2t+1)^2+(2l+1)^2$
$=4p^2+4p+1+4q^2+1+4q+4r^2+1+4r+4s^2+1+4s+4t^2+1+4t+4l^2+1+4l$
$=2(2p^2+2q^2+2r^2+2s^2+2t^2+2l^2+2p+2q+2r+2s+2t+2l+3)$
Which is an even number

Main result
Integer septuples and the six dimensional Euclidean
Here we relate the integer septuples (a,b,c,d,e,f,g) to points on six dimensional Euclidean space and a solution is obtain for the equation $x^2+y^2+z^2+u^2+v^2+w^2=1$ from we get the general solution for integer septuples is $a^2+b^2+c^2+d^2+e^2+f^2=g^2$
Let (a, b, c, d, e, f, g), be integer septuples for which $a^2+b^2+c^2+d^2+e^2+f^2=g^2$,
Divide by g^2, we get $\left(\frac{a}{g}\right)^2+\left(\frac{b}{g}\right)^2+\left(\frac{c}{g}\right)^2+\left(\frac{d}{g}\right)^2+\left(\frac{e}{g}\right)^2+\left(\frac{f}{g}\right)^2=1$
$\left(\frac{a}{g},\frac{b}{g},\frac{c}{g},\frac{d}{g},\frac{e}{g},\frac{f}{g}\right)$ *is the* Solution of the equation $x^2+y^2+z^2+u^2+v^2+w^2=1$
Here $x^2+y^2+z^2+u^2+v^2+w^2=1$ is an 6 dimensional Euclidean space whose coordinates (x, y, z, u, v,w) are rational number. Notice that it has 12 coordinates' points $(\pm1,0,0,0,0,0),(0,\pm1,0,0,0,0),(0,0,\pm1,0,0,0),(0,0,0,\pm1,0,0),(0,0,0,0,\pm1,0)$ *and* $(0,0,0,0,0,\pm1)$
Suppose we consider a vector b and the line L going through the point ($(-1,0,0,0,0,0)$ having b as its direction. The line L is given by the vector equation
$L: r=-i+tb$
Where b=$b_1i+b_2j+b_3k+b_4l+b_5m+b_6n$ the Cartesian equation of L is given by
$\frac{x+1}{b_1}=\frac{y}{b_2}=\frac{z}{b_3}=\frac{u}{b_4}=\frac{v}{b_5}=\frac{w}{b_6}$ To find the intersection of space and L, we have to solve
$x^2+y^2+z^2+u^2+v^2+w^2=1$ and $\frac{x+1}{b_1}=\frac{y}{b_2}=\frac{z}{b_3}=\frac{u}{b_4}=\frac{v}{b_5}=\frac{w}{b_6}$

From the above equation we get
Y=$\frac{b_2(x+1)}{b_1}$, z=$\frac{b_3(x+1)}{b_1}$, $u=\frac{b_4(x+1)}{b_1}$, $v=\frac{b_5(x+1)}{b_1}$, $w=\frac{b_6(x+1)}{b_1}$
Now substitute these values in Euclidean space equation we have

$$x^2+\left(\frac{b_2(x+1)}{b_1}\right)^2+\left(\frac{b_3(x+1)}{b_1}\right)^2+\left(\frac{b_4(x+1)}{b_1}\right)^2+\left(\frac{b_5(x+1)}{b_1}\right)^2+\left(\frac{b_6(x+1)}{b_1}\right)^2=1$$

$$b_1{}^2x^2+b_2{}^2(x+1)^2+b_3{}^2(x+1)^2+b_4{}^2(x+1)^2+b_5{}^2(x+1)^2+b_6{}^2(x+1)^2=b_1{}^2$$

$$\left(b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2\right)x^2+2\left(b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2\right)x+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2=b_1{}^2$$

$$\left(b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2\right)(x^2+2x+1)=2b_1{}^2(x+1)$$

$$\left(b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2\right)(x+1)^2=2b_1{}^2(x+1)$$

$$\left(b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2\right)(x+1)=2b_1{}^2$$

$$x=\frac{b_1{}^2-b_2{}^2-b_3{}^2-b_4{}^2-b_5{}^2-b_6{}^2}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2}$$

Similarly the value of y , z and w are
Y=$\frac{2b_1b_2}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2}$,

$$z=\frac{2b_1b_3}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2},\ u=\frac{2b_1b_4}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2+b_6{}^2},$$

$v = \dfrac{2b_1b_5}{b_1{}^2 + b_2{}^2 + b_3{}^2 + b_4{}^2 + b_5{}^2 + b_6{}^2}, w = \dfrac{2b_1b_6}{b_1{}^2 + b_2{}^2 + b_3{}^2 + b_4{}^2 + b_5{}^2 + b_6{}^2}$

Thus every point (x, y, z, u, v,w) on the 6 dimensional Euclidean space
$x^2 + y^2 + z^2 + u^2 + v^2 + w^2 = 1$

Is $\left(\dfrac{b_1{}^2 - b_2{}^2 - b_3{}^2 - b_4{}^2 - b_5{}^2}{b_1{}^2 + b_2{}^2 + b_3{}^2 + b_4{}^2 + b_5{}^2 + b_6{}^2}, \dfrac{2b_1b_2}{b_1{}^2 + b_2{}^2 + b_3{}^2 + b_4{}^2 + b_5{}^2 + b_6{}^2}, \dfrac{2b_1b_3}{b_1{}^2 + b_2{}^2 + b_3{}^2 + b_4{}^2 + b_5{}^2 + b_6{}^2}, \dfrac{2b_1b_4}{b_1{}^2 + b_2{}^2 + b_3{}^2 + b_4{}^2 + b_5{}^2 + b_6{}^2}, \dfrac{2b_1b_5}{b_1{}^2 + b_2{}^2 + b_3{}^2 + b_4{}^2 + b_5{}^2 + b_6{}^2}, \dfrac{2b_1b_6}{b_1{}^2 + b_2{}^2 + b_3{}^2 + b_4{}^2 + b_5{}^2 + b_6{}^2}\right)$

Now we give a result which obtain the septuples (a, b, c, d, e, f,g)

Theorem
For any integer

$a = p^2 - q^2 - r^2 - s^2 - t^2 - l^2,\ b = 2pq, c = 2pr, d = 2ps, e = 2pt, f = 2pl \text{ for } p, q, r, s, t, l \in z$ then
$g = p^2 + q^2 + r^2 + s^2 + t^2 + l^2,$ Will satisfies the equation
$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = g^2$

Proof
Now
$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = (p^2 - q^2 - r^2 - s^2 - t^2 - l^2)^2 + (2pq)^2 + (2pr)^2 + (2ps)^2 + (2pt)^2 + (2pl)^2$

$= \left(p^2 - (q^2 + r^2 + s^2 + t^2 + l^2)\right)^2 + (2pq)^2 + (2pr)^2 + (2ps)^2 + (2pt)^2 + +(2pl)^2$
$= p^4 - 2(q^2 + r^2 + s^2 + t^2 + l^2)p^2 + (q^2 + r^2 + s^2 + t^2 + l^2)^2 + 4p^2q^2 + 4p^2r^2 + 4p^2s^2 + 4p^2t^2 + 4p^2l^2$
$= p^4 + 2(q^2 + r^2 + s^2 + t^2 + l^2)p^2 + (q^2 + r^2 + s^2 + t^2 + l^2)^2$
$= (p^2 + q^2 + r^2 + s^2 + t^2 + l^2)^2 = g^2$

Conclusion
In this paper i extend the Fermat's last theorem and an attempt made to produce the result for $a^n + b^n + c^n + d^n + e^n + f^n = g^n$ for n=2, thus we called an extension of Fermat's last theorem in six dimensional Euclidean space

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An JVS Algorithm For Solving Permutation Flowshop Scheduling

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ABSTRACT: *The origin of flow shop scheduling is in the year 1954. Till date in recent engineering and industrial built-up units are facing lots of problems in a lot of aspects such as machining time, electricity, man power raw material and customer's constraints. The flow shop scheduling is a pure sequencing problem. The permutation flow shop scheduling problem is one of the most well known and well studied optimization problems which are too difficult to be solved optimally and hence heuristics are used to obtain good solutions in a reasonable time. In this paper the author solved the permutation flow shop scheduling problem in a reasonably good make span or otherwise optimal make span even though the Problem is NP-hard in nature. Comparison has been made with the algorithms available in the literature and it was found that our algorithm performs well in both the cases that of minimizing the make span and reducing the resource idle time and comparison has been made with Gupta's heuristics. Gantt chart is generated to verify the effectiveness of the proposed approach.*

Keywords: *Flow shop; make span, permutation, Gupta's method*

1. Introduction

In this chapter the author focus his attention on Permutation flow shop scheduling with the objective of minimizing the make span and resource idleness. The author developed an algorithm called JVS algorithm and tested the algorithm with the existing algorithm available in the literature namely Gupta's Heuristic. A sample problem is chosen from the literature and the results have been compared with the existing algorithm. It was found that the algorithm developed by the author performs well in both the cases namely make span is 81 and resource idleness is 54 unit time and the results are given in appendix A(JVS Algorithm) and then using Gupta's algorithm it was found that the make span is 81 unit time and the resource Idle time is 77 unit and the results are given in Appendix B(Gupta's Algorithm). The details are listed in the appendix below:

1.1 Appendix – A

1.1.1 An JVS Algorithm Rule:

Processing n jobs through 3 machines:

The problem is completely described as follows.

- There are n jobs
- Only three machines are involved.

Let them be A, B, C.

- Each job is processed in the proscribed order A, B and C.
- No passing of jobs is allowed between the machines.
- The exact of expected processing times are given.

Job	1	2	3	4	5	…….	n
A	A_1	A_2	A_3	A_4	A_5	…….	A_n
B	B_1	B_2	B_3	B_4	B_5	…….	B_n
C	C_1	C_2	C_3	C_4	C_5	…….	C_n

No general procedure is available, by which we can obtaining an optimal sequence is this case.
However, the proposed method adopted by JVS algorithm can be extended to cover the n jobs through 3 machines case is general and special cases if either or both of the conditions stipulated below is satisfied:

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Step: 1

- Minimum processing on machine A ≤ Maximum processing time on machine B(i.e.) Min $A_i \le$ max B_i
- Minimum processing time on machine C ≤ Maximum processing time on machine B (i.e.) min $C_i \le$ max B_i

If the above conditions are not satisfied, the method fails, otherwise to the next step.

Step: 2

Convert the three machines problem into two machines problem by introducing two fictitious machines G and Corresponding processing times are given by

$G_i = A_i + B_i$, $H_i = B_i + C_i$, i=1, 2 ....n

Step: 3

Determine the optimal sequence of the jobs on G and H in the order G-H with the rules given earlier [n jobs through 2 machines]. The resulting sequence will also be optimal for the original (given) problem

1.1.2 Example: Consider an 8-Jobs problem [8-Jobs, 3-machine]

Table: General 3-machine and 8-Jobs problem:

Job i	1	2	3	4	5	6	7	8
M1	5	6	2	3	4	9	15	11
M2	4	6	7	4	5	3	6	2
M3	8	10	7	8	11	8	9	13

Step: 1

Minimum processing time on M1=2
Minimum processing time on M3=7
Maximum processing time on M2=7

- Min M1 ≤ Max M2, 2 ≤ 7 (True)
- Min M3 ≤ Max M2 , 7 ≤ 7 (True)

The (i) and (ii) conditions are satisfied.

Step: 2

We can convert the problem of three machines to equivalent problem of 2 machines denoted by G and H. And the corresponding time G_i and H_i are defined by

G_i= M1+ M2 and H_i= M2+M3

M_i → processing time of job on.

Now we solve this problem with the prescribed ordering G-H using Johnson's algorithm.

The processing time for the new problem are given below.

Job i	1	2	3	4	5	6	7	8
G	9	12	9	7	9	12	21	13
H	12	16	14	12	16	11	15	15

In the above list of processing time, minimum is 7 units for Job 4 on machine G.Job4 is scheduled first

4							

Reduced set of processing time is as follows

Job i	1	2	3	5	6	7	8
G	9	12	9	9	12	21	13
H	12	16	14	16	11	15	15

We have 3 equal minimum processing times. It is 9 units for Job1, Job3 and Job5 on machine G
Therefore, Job 1 is scheduled next to Job4, Job3 is scheduled next to Job1 and
Job5 is scheduled next to Job3

4	1	3	5				

Reduced set of processing time is

Job i	2	6	7	8
G	12	12	21	13
H	16	11	15	15

Minimum processing time is 11 units for Job 6 on machine H
Job 6 is placed last in the sequence

4	1	3	5				6

Reduce set of processing time is

Job i	2	7	8
G	12	21	13
H	16	15	15

Minimum processing time is 12 units for Job2 on machine G
Job 2 is scheduled next to Job 5

4	1	3	5	2			6

Reduced set of processing time is

Job i	7	8
G	21	13
H	15	15

Minimum processing time is 13 units for Job 8 on machine G
Job 8 is scheduled next to Job 2

4	1	3	5	2	8		6

Step: 3
Thus the output optimal sequence is

4	1	3	5	2	8	7	6

Thus total processing time can be calculated as:

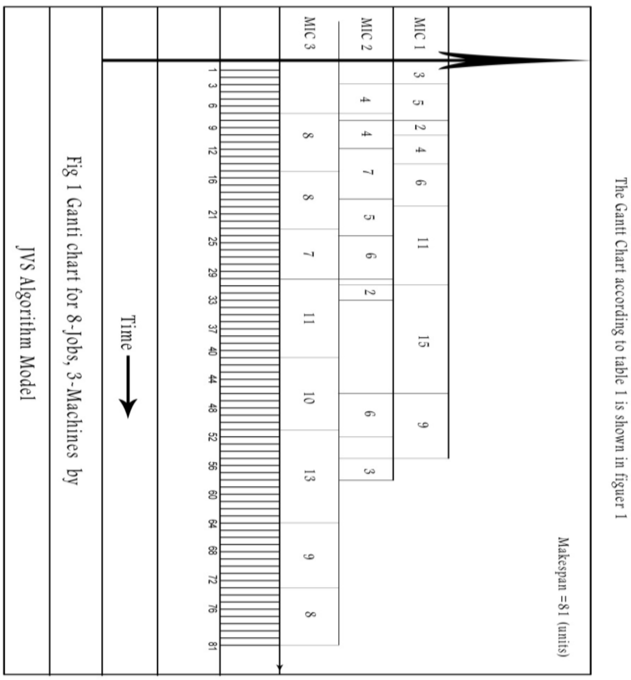
Table: 1 Total processing time for8-Jobs, 3-machines JVS Algorithm Rule model

Job i	M/C$_1$		M/C$_2$		M/C$_3$	
	Time In	Time Out	Time In	Time Out	Time In	Time Out
4	0	0+3=3	3	3+4=7	7	7+8=15
1	3	3+5=8	8	8+4=12	15	15+8=23
3	8	8+2=10	12	12+7=19	23	23+7=30

5	10	10+4=14	19	195=24	30	30+11=41
2	14	14+6=20	24	24+6=30	41	41+10=51
8	20	20+11=31	31	31+2=33	51	51+13=64
7	31	31+15=46	46	46+6=52	64	64+9=73
6	46	46+9=55	55	55+3=58	73	73+8=81

Therefore, Total processing time = 81
Total Idle time for M/C1= 81-55 = 26 (Units)
Total Idle time for M/C2= 3+1+1+13+3=21(Units)
Total Idle time for M/C3= 7+0 = 7(Units)

- **Gantt Chart:**



1.2 APPENDIX - B
1.2.1 Gupta's Heuristic Rule:
Algorithm: Gupta's Heuristic
Procedure: Gupta's Heuristic
Input: Job list I, machine m;
Output: Schedule S;

Begin
For i=1 to n
For k=1 to m-1
If $t_{i1}<t_{i2}$ then
e_i=1
Else
e_i=-1
Calculate $s_i = e_i/\min\{t_{ik} + t_i, K+1\}$;

step-1
End
Permutation schedule is constructed by sequencing the Jobs in Non- increasing order of s_i such as;

step-2
End
Output optimal sequence is obtained as schedule s.

Step-3
End

1.2.2 Example: Consider the above 8-Jobs and 3-Machine problem:

Table : General 3-machine and 8-Jobs problem:

Job i	1	2	3	4	5	6	7	8
M1	5	6	2	3	4	9	15	11
M2	4	6	7	4	5	3	6	2
M3	8	10	7	8	11	8	9	13

The solution constructed as follows:

Step-1
Set the slope index s_i for the job I as:
S_1= -1/min {9,12 }= -0.111, S_2= 1/min {12, 16}= 0.083, S_3=1/min {9, 14} =0.111
S_4=1/min {7, 12} =0.142, S_5=1/min {9, 16}=0.111, S_6=-1/min {12, 11} =-0.090
S_7=-1/min {21, 15}=-0.066, S_8=1/min {13, 15} =0.076

Step-2
Jobs are sequenced according: $S_4 \geq S_3 \geq S_5 \geq S_2 \geq S_8 \geq S_7 \geq S_6 \geq S_1$
0.142≥0.111≥0.111≥0.083≥0.076≥-0.066≥-0.090≥-0.111

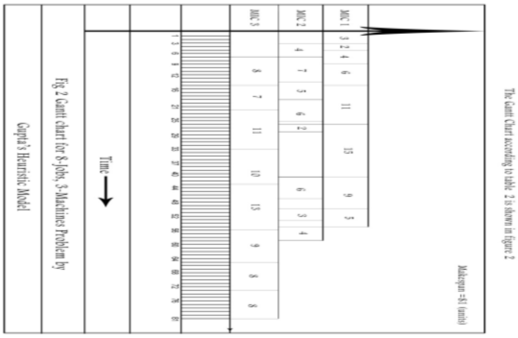
Step-3
Output optimal sequence is {4, 3, 5, 2, 8, 7, 6, 1}
Thus, total processing time can be calculated as:

Table 2. Total processing time for 1.2.2 Example
8-Jobs, 3-Machinces by Gupta's Heuristic model

Job i	M/C_1		M/C_2		M/C_3	
	Time In	Time Out	Time In	Time Out	Time In	Time Out
4	0	0+3=3	3	3+4=7	7	7+8=15
3	3	3+2=5	7	7+7=14	15	15+7=22
5	5	5+4=9	14	14+5=19	22	22+11=33
2	9	9+6=15	19	19+6=25	33	33+10=43
8	15	15+11=26	26	26+2=28	43	43+13=56
7	26	26+15=41	41	41+6=47	56	56+9=65
6	41	41+9=50	50	50+3=53	65	65+8=76
1	50	50+5=55	55	55+4=59	73	73+8=81

Therefore,
Total processing Time=81
Total Idle Time for M/C_1= 81-55=26(units)
Total Idle Time for M/C_2=3+1+13+3+2+ (81-59)=44(units)
Total Idle Time for M/C_3= 7+0 =7(units)

1.2.3 Gantt Chart:



RESULTS AND DISCUSSION

Make span and Resource Idle time for the applied heuristics rules are given as follows:

Rule	JVS Algorithm	Gupta's Heuristics
Make span	81 Units	81 Units
Idle time	54 Units	77Units

Make span is the time length from the starting of the first operation of the first demand to the finishing of the last operation of the demand**".**

From the result it was found that the JVS algorithm is superior to the other one well known heuristics algorithm not only for make span minimization but also for resource idleness. Future research is needed for increasing the number of machines and number of jobs.

Conclusion

It has been found that the JVS algorithm performs well with multiple objective that of minimizing the make span and resource idle time when compared with the Gupta's heuristics. Our main objective is to minimize the Make span initially and it was by accident that it also reduces the resource idleness. The future direction is on minimizing the tardiness and Lateness of the flow shop scheduling problem. Further investigation is needed on Meta Heuristics and lot streaming and also for parallel machine flow shop scheduling problem. Even though the flow shop Scheduling Problem, Job Shop Scheduling problem and the Open shop scheduling problem are in one family which has different both uniform and different types of scheduling problem, we have to work the same for both job shop and open shop scheduling problem. Problems with other measure of performance related to tardiness, due date and lateness can be studied further. It was found that ours is a multiple objective flow shop scheduling Problem and it attains a reasonably good schedule, compare with the other one algorithm found in the literature.

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Method for Finding Critical Path

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ABSTRACT: *In today's highly competitive business environment, project management's Performance guidelines is becoming increasingly important to obtain competitive priorities such as on-time delivery and customization. When the activity times in the project are deterministic and be a useful tool in managing projects in an efficient to meet this challenge.*

Keywords: *CPM, Predecessor*

1. Introduction

CPM is associate economical tool for programming and planning. A CPM could also be a route between a pair of or heaps of operations that minimize (or maximize) some live of performance. This could even be printed as a result of the sequence of activities that is in a position to wish greatest ancient time to accomplish i.e. throughout a vital path the sequence of activities with longest duration's unit singled out. it's called a vital path as a results of any delay in activities lying on this path would cause a delay inside the full project. The target of critical path analysis is to estimate the whole project length and to assign starting and finishing times to any or all or any activities involved inside the project. This helps in checking actual progress against the regular length of the project.

Applications of critical path method:

CPM has been successfully used in many applications including

1. Constructing dam or canal system in an exceedingly region.
2. Programming construction comes like building, and swimming pools.
3. Developing a counting and "hold" procedure for the launching of house flights.
4. Putting in a replacement automatic data processing system.
5. Planning and selling a replacement product.
6. finishing a company merger.
7. Building a ship.

Basic Definitions

Definition 1:

A network could be a graphic representation of a project's operations and consists of activities and events that has to be completed to achieve the end objective of a project, showing the look sequence of their accomplishments, their dependence and inter-relationships. The followings area unit some basic definitions used in network analysis.

Definition 2

Any operation, that utilizes resources and has a starting and an finish, is termed activity. An arrow is used to represent an activity with its head indicating the direction of progress with in the project. These are typically classified into following four classes.

(a) Predecessor activity: An activity that must be completed immediately prior to the start of another activity is called predecessor activity.

(b) Successor activity: An activity which started immediately after one or more of other activities are completed is called successor activity.

(c) Concurrent activity: An activity that can be accomplished concurrently is known as concurrent activity. It may be noted that an activity can be predecessor or a successor to an event or it may be concurrent with one or more of the other activities.

(d) Dummy activity: An activity which does not consume either any resource and/or time is known as dummy activity

Definition 3
Events in the network diagram represent project milestones, such as the start or the completion of an activity or activities, and occur at a particular instant of time at which some specific part of the project has been or is to be achieved.

The events can be further classified into following two categories:
(a) Merge Event: An event which represents the joint completion of more than one activity is known as merge event.
(b) Burst Event: An event which represents the initiation of more than one activity is known as burst event.Events in the network diagram are identified by numbers. Each event should be identified by a number higher than that allotted to its immediately preceding event to indicate progress of work.

Definition 4
The earliest starting time for an activity (i , j) is the earliest time at which an activity can possibly start without delaying the project completion .It is denoted by ESij

Definition 5: The earliest finishing time for an activity (i , j) is the earliest time at which an activity can possibly finish without affecting the project completion. It is denoted by EF_{ij} and is calculated as:

$EF_{ij} = ES_{ij} + t_{ij}$, where t_{ij} is the duration of an activity (i , j)

Definition 6
The earliest occurrence time of an event, j is the earliest time for an event to occur immediately after all the preceding activities have been completed without delaying the entire project. It is denoted by E_j and is calculated as:
E_j = Maximum $\{ES_{ij} + t_{ij}$ | for all immediate predecessor activities of node $j\}$
where, $ES_{ij} = E_i$.

Definition 7
The latest finishing time for an activity (i , j) is the latest time at which an activity must finish without delaying the project completion. It is denoted by LF_{ij} .

Definition 8
The latest starting time for an activity (i , j) is the latest possible time at
which an activity can start without delaying the project completion. It is denoted by LS_{ij}

and is calculated as:
$LS_{ij} = LF_{ij} - T_{ij}$

Definition 9
The latest allowable time of an event, i is the latest time at which an event can occur without causing a delay in already determined project's completion time. It is denoted by L_i and is calculated as:
L_i = Minimum{ $LF_{ij} - T_{ij}$ | for all immediate successor activities of node ij
where, $LF_{ij} = L_j$.

Definition 10
The float is the length of time to which an activity can be delayed or ex-tended without delaying the total project completion time. Mainly three types of floats are
defined for each activity of the project.

(a) Total Float:
The amount of time by which the completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time. Mathematically, the total float of an activity (i , j) is the difference between the latest start time and earliest start time of that activity, i.e.,
$TF_{ij} = LS_{ij} - ES_{ij}$ (or) $LF_{ij} - EF_i$

Properties

1. The earliest start time for an activity is the value of E at the beginning event
2. EF **=** ES + Duration
3. The latest finish time for an activity is the value of L at the ending event of the activity.
4. *LS = LF – Duration*
5. *TF = LF-EF (OR) LS - ES (or) = Head event L – Tail event E –Duration*

Method for Finding the Critical Path:

The FCP of a project network can be obtained by using the following steps.

1. Identify activities in a project.
2. Establish precedence relationships of all activities.
3. Estimate the activity time with respect to each activity.
4. Construct the project network.
5. Let ES = (0,0,0,0) and calculate EF
6. Let LF = ES and calculate LF
7. Calculate TS with respect to each activity in a project network.
8. Find all the possible paths and calculate CPM (P).
9. Find the CP. (All those activities for which the total float is zero constitude the critical path).
10. Find the grade of membership that the project can be completed at scheduled time.

In the literature the following methods are used to determine critical path of a given project network.

1. Method based upon forward and backward pass calculations.
2. Method based upon linear programming.

Method based upon forward and backward pass calculations

In this method, the following steps are followed to find critical path of a given project network.

(a) Forward pass calculations (For Earliest event time)

Before starting computations, the occurrence time of initial network is fixed. Then, the forward pass computation yields the earliest start and earliest finish time for each activity (i , j) , and indirectly the earliest expected occurrence time for each event. This is mainly done in the following three steps:

Step 1: For easiness, the forward pass computations start by assuming the earliest starting time of zero for the initial project event

Step 2: Calculate E_j , j = 2,3,..., n by using Definition 6, where n is the number of nodes

Step 3: Calculate EF_{ij} for each activity by using Definition 5

(b) Backward pass calculation (For Latest allowable time)

These can be computed by reversing the method of calculation used for earliest event times. This is done in the following steps:

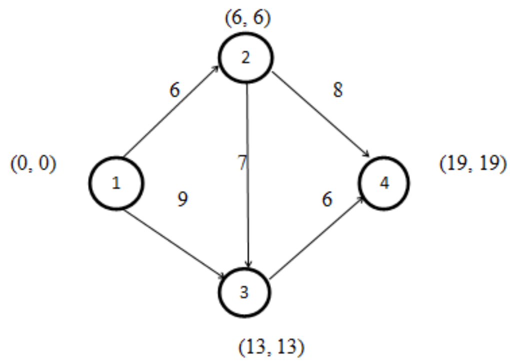
Step 1: For ending node n assume L_n = E_n

Step 2: Calculate L_j , j = n- 1, n- 2,...,1 by using Definition.9

Numerical Example

Suppose there is a project network with the set of node N={1,2,3,4}, the activity time for each activity. All of the durations are in hours. Find the CP for the network.

A project network



Activity Time for Each Activity

Activity	Duration	ES	EF	LS	LF	TF
1-2	6	0	6	0	6	0
1-3	9	0	9	4	13	4
2-3	7	6	13	6	13	0
2-4	8	6	14	11	19	5
3-4	6	13	19	13	19	0

6

The CP of the given network can be obtained by the following steps.

Step 1: Set ES_1 = (0,0) and calculate ES_j, j= 2,3, 4,
$ES_2 = ES_1 + ET_{12} = (0 + 6) = 6$
ES_3 = Maximum{ $ES_1 + ET_{13}$, $ES_2 + ET_{23}$} =Maximum{9,13}=13
ES_4 = Maximum{$ES_2 + ET_{24}$,$ES_3 + ET_{34}$} = Maximum{14,19} = 19

Step 2: Set LF_4 = 19 and calculate LF_j, j = 3,2,1,
$LF_3 = LF_4 - ET_{34} = (19 - 6) = 13$
LF_2 = Minimum {$LF_4 - ET_{24}$, $LF_3 - ET_{23}$} = Minimum{11,6} =6
LF_1 = Minimum{$LF_3 - ET_{13}$, $LF_2 - ET_{12}$} = (4,0)=0

Step 3: Calculate TF_{ij} with respect to each activity
$TF_{12} = (ES_1 - ET_{12}) =0$
$TF_{13} = (ES_1 + ET_{13}) = 4$
$TF_{23} = (ES_2 + ET_{23}) = 0$
$TF_{24} = (ES_2 + ET_{24}) = 5$
$TF_{34} = (ES_3 + ET_{34}) =$ **0**

Step 4: Find all the possible paths and calculate $CPM (P_k)$
P = {(1,2,4),(1,2,3,4),(1,3,4)}
1 . P_1(1,2,4) = CPM(P_1) = $ET_{12} + ET_{24} = 6+8=14$
2. P_2(1,2,3,4) = CPM(P_2) = $ET_{12} + T_{23} + ET_{34} = 6+7+6 = 19$
3 . P_3(1,3,4) = CPM(P_3) = $ET_{13} + ET_{34} = 9+6=15$

Step 5: Find the CP . The value of $CP(P_i)$, i = 1,2,3, can be obtained
Since CP(P_1) < CP(P_3) < CP(P_2)
The CP is P_2 and The project completion time = 19 hours

Conclusion

This method evaluation analysis, it is very often that information available for making decision is vague and uncertain. Therefore, it is rather difficult to obtain exact activity assessment data. This chapter proposes an algorithm to tackle the problem in project decision analysis. Thus by conducting activity time assessments, the decision-makers can obtain the CP automatically.

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Extension of Factorial using Properties of Permutation and Combination

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ABSTRACT: *This paper introduce the various types of factorials, among which include the sub factorial, double factorial, triple factorial, quadruple factorial. This extension of factorial applying the properties of Permutation and Combination. This concept analysed some properties results are verified.*

Keywords: *Factorial, Sub factorial , Double factorial, Triple factorial, Permutation, Combination, Factorial function.*

1. Introduction

In factorial is generally placed on its application as in Combinatorics, Calculus, Number theory, Probability theory. The factorial function is always helpful to understand optimize animal experiments and reduce animal use and economics factorial analysis.

Different types of factorial defined in Sub factorial $(!n)$, Double factorial $(n!!)$,Triple factorial $(n!!!)$, Quadruple factorial $\frac{(2n)!}{n!}$, where *n* is the positive integers. If *n* is the non-positive integer we have the negative factorial$(-n!)$. But the negative factorial is not defined in Permutation and Combination.

1. Related works

1.1 Definition of factorial

The factorial of a natural number *n* is the product of the positive integers less than or equal to *n*. This is written as $n!$ and pronounced '*n factorial*'. The factorial function is defined by

$$n! = \prod_{m=1}^{n} m \;\forall\;\; n \geq 0$$

or)

$$n! = \begin{cases} 1 & if\ n = 0 \\ (n-1)! \times n & if\ n > 0 \end{cases}$$

For example

$$n! = n(n-1)(n-2)(n-3)\ldots\ldots 1$$

$$0! = 1, 1! = 1, 2! = 2(2-1)! = 2.1 = 2$$

1.2 Definition of Sub factorial

!*n* is the number of dearrangement of *n* objects. That is the number of permutation *n* objects in order that no objects stands in its original position. This factorial function is

$!n = (n-1)(n-2)(n-3)\ldots\ldots 3.2.1$

For example

$$!1 = 1, !2 = 1, !3 = 2, !4 = 6.$$

1.3 Relation between factorial and sub factorial

$$n! = !n + 1$$

We know that

$$n! = \begin{cases} 1 & if\ n = 0 \\ n(n-1)! & if\ n > 0 \end{cases}$$

$$!n = \begin{cases} 1 & if\ n = 0 \\ (n-1)\,!(n-1) & if\ n > 0 \end{cases}$$

For example

$!n+1 = (n)!\,n$

$!n = (n-1)!\,(n-1)$

$!n = (n-1)(n-2)(n-3)!\,(n-3)$

1.4 Definition of Double factorial

A function related to the factorial is the product of all odd (even) values up to some odd (even) positive integer *n*. It is often called double factorial and is denoted by $(n!!)$

$$n!! = \begin{cases} n(n-2)(n-4)\ldots\ldots 5.3.1 & if\ n > 0 \quad odd \\ n(n-2)(n-4)\ldots\ldots 6.4.2 & if\ n > 0 \quad even \\ 1 & if\ n = -1,0 \end{cases}$$

(or)

$$n!! = \begin{cases} 1 & if\ 0 < n < m \\ [(n-m)!^{(m)} & if\ n \geq m \end{cases}$$

For example

$0!! = 1, 1!! = 1, 2!! = 2, 3!! = 3.$

1.5 Definition of Triple factorial

The triple factorial of a positive integer *n* is the product of positive integers less than or equal to *n* and congruent to *n* mod 3. It is denoted by (*n*!!!*)*.

$n!!! = n(n-3)(n-6)(n-9)\ldots\ldots..(n\ mod\ 3)$

For example

$n!!! = n(n-3)!!!$

$1!!! = 1, -2!!! = 1,\ 0!!! = 1$

1.6 Definition of Quadruple factorial

The quadruple factorial however is not the multiple factorial $n!^{(4)}$; It is a much larger number given by $\frac{(2n)!}{n!}$.

For example the quadruple factorial *n* = 0,1,2,3,4,5 and 6

1.7 Definition of Permutation

A Permutation is an arrangement of all or parts of a set of objects, with regard to the order of the arrangement. It is denoted by np_r.

$np_r = \frac{n!}{n-r!}$

2. Results

Using the sub factorial in permutation.

Property: 1

$np_r = \frac{!(n+1)}{!(n-r+1)}$

For example

$5p_3 = \frac{!(5+1)}{!(5-3+1)}$

$= \frac{!6}{!3} = \frac{(6-1)!(6-1)}{(3-1)!(3-1)}$

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$= \frac{5\times4\times3\times2\times1}{2\times1}$

$5p_3 = 60$

Property: 2

$np_0 = \frac{!(n+1)}{!(n-0+1)}$

$= \frac{!(n+1)}{!(n+1)}$

$np_0 = 1$

For example

$6p_0 = \frac{!(6+1)}{!(6-0+1)}$

$= \frac{!7}{!7}$

$6p_0 = 1$

3. Results

Using double factorial in permutation

Property: 1

$np_r = \frac{n!}{(n-r)!}$

$n! = n!! \times (n-1)!!$

$np_r = \frac{n!!\times(n-1)!!}{(n-r)!!\times(n-r-1)!!}$

For example

$5p_2 = \frac{5!!\times(5-1)!!}{(5-2)!!\times(5-2-1)!!}$

$= \frac{5!!\times4!!}{3!!\times2!!}$

$= \frac{5\times3\times1\times4\times2}{3\times1\times2}$

$5p_2 = 20$

Property: 2

$np_0 = \frac{n!!\times(n-1)!!}{(n-0)!!\times(n-0-1)!!}$

$= \frac{n!!\times(n-1)!!}{n!!\times(n-1)!!}$

$np_0 = 1$

For example

$5p_0 = \frac{5!!\times(5-1)!!}{(5-0)!!\times(5-0-1)!!}$

$= \frac{5!!\times4!!}{5!!\times4!!}$

$5p_0 = 1$

4. Result

Using sub factorial in Combination.

Definition of Combination

A Permutation is an arrangement of all or parts of a set of objects, where the order doesn't matter. Computing the number of combination of *r* objects chosen from *n*

$nc_r = \frac{n!}{r!(n-r)!}$

Property: 1

$nc_r = \frac{!(n+1)}{!(r+1)!(n-r+1)}$

For example

$nc_0 = \frac{!(n+1)}{!(0+1)!(n-0+1)}$

$= \frac{!(n+1)}{!1!(n+1)}$

$= \frac{!(n+1)}{!(n+1)}$

$nc_0 = 1$

Property: 2

$nc_r = nc_p$ then $r = p$ (or) $r+p=n$

$nc_r = \frac{!(n+1)}{!(r+1)!(n-r+1)} \dots\dots\dots\dots. (1)$

$nc_p = \frac{!(n+1)}{!(p+1)!(n-p+1)}$……………(2)

Equating (1) and (2)

$!(r+1) = !(p+1)$ then $r = p$ (or) $r+p=n$

For example

$5c_3 = \frac{!(5+1)}{!(3+1)!(5-3+1)}$

$= \frac{!6}{!4!3}$

$= \frac{5\times4\times3\times2\times1}{3\times2\times1\times2\times1}$

$5c_3 = 10$……………………..(1)

$5c_2 = \frac{!(5+1)}{!(2+1)!(5-2+1)}$

$= \frac{!6}{!3!4}$

$= \frac{5\times4\times3\times2\times1}{2\times1\times3\times2\times1}$

$5c_2 = 10$………………………(2)

From (1) and (2) $5c_3 = 5c_2$ then *3 = 2* (or) *3+2=5*

Property: 3

$nc_0 + nc_1 + nc_2 + \cdots + nc_n = 2^n$

Proof

$nc_r = \frac{!(n+1)}{!(r+1)!(n-r+1)}$

$nc_0 = \frac{!(n+1)}{!(0+1)!(n-0+1)} = \frac{!(n+1)}{!(1)!(n+1)} = \frac{!(n+1)}{!(n+1)} = 1$

$nc_1 = \frac{!(n+1)}{!(1+1)!(n-1+1)} = \frac{!(n+1)}{!2\,!n} = \frac{!(n+1)}{!n} = \frac{(n)!n}{!n} = n$

$nc_2 = \frac{!(n+1)}{!(2+1)!(n-2+1)} = \frac{!(n+1)}{!3\,!(n-1)} = \frac{n\,!n}{2\,!(n-1)} = \frac{n(n-1)!(n-1)}{2!(n-1)} = \frac{n(n-1)}{2}$

$nc_3 = \frac{!(n+1)}{!(3+1)!(n-3+1)} = \frac{!(n+1)}{!4\,!(n-2)} = \frac{n\,!n}{3(!3)\,!(n-2)} = \frac{n(n-1)(n-2)!(n-2)}{3\times2(!2)!(n-2)} = \frac{n(n-1)(n-2)}{6}$

⋮

$$nc_{n-2} = \frac{!(n+1)}{!(n-2+1)!(n-n+2+1)} = \frac{!(n+1)}{!(n-1)!3} = \frac{n!n}{!(n-1)2!2} = \frac{n(n-1)!(n-1)}{!(n-1)\times 2} = \frac{n(n-1)}{2}$$

$$nc_n = \frac{!(n+1)}{!(n+1)!(n-n+1)} = \frac{!(n+1)}{!(n+1)!1} = \frac{!(n+1)}{!(n+1)} = 1$$

Adding all we get

$$nc_0 + nc_1 + nc_2 + \cdots + nc_n = (1 + n + \frac{n(n-1)}{2!} + \frac{n(n-1)(n-2)}{3!} + \cdots + \frac{n(n-1)}{2!} + 1)$$

$$=(1+1)^n$$

$$nc_0 + nc_1 + nc_2 + \cdots + nc_n = 2^n$$

Note

[Since using the binomial formula

$$(x+y)^n = (x^n + nx^{n-1}y^1 + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \cdots + y^n)$$

We observe that x=1, y=1 we get the above values.]

Property: 4

$$nc_0 + nc_2 + nc_4 + \cdots = nc_1 + nc_3 + nc_5 + \cdots = 2^{n-1}$$

Proof

$$nc_0 + nc_2 + nc_4 + \cdots = 1 + \frac{n(n-1)}{2!} + \frac{n(n-1)(n-2)(n-3)}{4!} + \cdots$$

$$= \frac{1}{2}[(1+1)^n + (0)^n]$$

$$= \frac{1}{2}(2^n)$$

$$nc_0 + nc_2 + nc_4 + \cdots = 2^{n-1}$$

$$nc_1 + nc_3 + nc_5 + \cdots = n + \frac{n(n-1)(n-2)}{3!} + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} + \cdots$$

$$= \frac{1}{2}[(1+1)^n - (0)^n]$$

$$= \frac{1}{2}(2^n)$$

$$nc_1 + nc_3 + nc_5 + \cdots = 2^{n-1}$$

Hence $nc_0 + nc_2 + nc_4 + \cdots = nc_1 + nc_3 + nc_5 + \cdots = 2^{n-1}$

Note

Since using the binomial formula

$$\frac{1}{2}[(x+y)^n + (x-y)^n] = x^n + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)(n-3)}{4!}x^{n-4}y^4 + \cdots$$

$$\frac{1}{2}[(x+y)^n - (x-y)^n] = nx^{n-1}y^1 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}x^{n-5}y^5 + \cdots$$

Property: 5

$$nc_0 - nc_1 + nc_2 - nc_3 + \cdots + (-1)^n\ nc_n = 0$$

Proof

$$nc_0 - nc_1 + nc_2 + \cdots + (-1)^n nc_n = (1 - n + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \cdots + (-1)^n 1)$$

$$=(1-1)^n$$

$$nc_0 - nc_1 + nc_2 + \cdots + (-1)^n nc_n = 0$$

Note

Since using the binomial formula

$$(x-y)^n = (x^n - nx^{n-1}y^1 + \frac{n(n-1)}{2!}x^{n-2}y^2 - \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \cdots + (-1)^n y^n)$$

Conclusion

In this paper find out some specific properties of Permutation and Combination using Sub factorial and Double factorial.

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Applications of Probability in Real Life

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ABSTRACT: *The Research reported in this paper explores nature of students knowledge about the* ***Applications of Probability in Real Life*** *and an individual may develop an understanding of certain topic of probability. In this paper we define about probability in which models of fascinating informations in events, trial, random variables, about probability.*

Keywords: *Probability, Random Variable, Theoritical Distribution,*

1. Introduction

In this paper define about a probability and applications of probability. **"Mathematics is not a subject is a part of our life in each and every person"** at each and every persons are the part of mathematics is used in our life. This gives details about probability and its used in our fields and life with examples in various methods.

Concepts

- Probability.
- Random Variable.
- Theoritical Distribution.

Definition

Probability

Probability is measure likelihood that an event will occur.A number expressing the likelihood of the occurrence of a given event, Especially, a fraction expressing how many times the event will happen in a given number of test or experiments.
In otherwords, the fraction of favourable outcomes and the total no.of outcomes.
The Probability of that event is denoted by P(E)

P(E) = Favourable Outcomes / Total no. of outcomes.

The Range of Probabiltiy

0<= P(E) <= 1

Types of Events

Example

There are eight possible outcomes, {1,2,3,4,5,6,7,8}
Favorable outcomes will be both blue and even numbers.
The blue numbers that are even are {4,6}.
So, there are 2 favorable outcomes out of 8 possible outcomes.

$$\frac{2}{8} = \frac{1}{4} = 0.25 = 25\%$$

Random Variable

A is a Random variable that is subject to randomness. Which means it can take on different values. The random variable takes on different values depending on the situation. Each value of the random variable has probability.

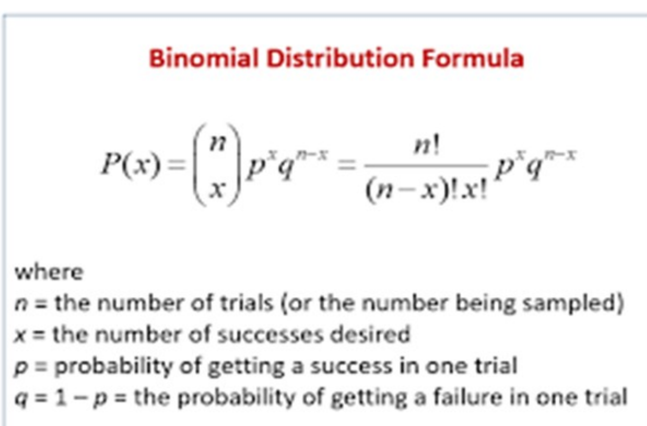
Discrete Random Variable

A discrete variable is a variable which can only take a countable number of values. In this example, the number of heads can only take 4 values (0, 1, 2, 3) and so the variable is discrete. The variable is said to be random if the sum of the probabilities is one.

Continous Random Variable

A continuous random variable is a random variable where the data can take infinitely many values. For example, a random variable measuring the time taken for something to be done is continuous since there are an infinite number of possible times

Theoritical Distribution
Binomial Distribution



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Example

If a coin is tossed thrice, find the probability of a getting head at least two times.

Solution

The probability of getting head at least two times is the sum of probabilities of getting head two times and three times.

P(X ≥ 2) = P(X=2)+P(X=3)

= 32C(0.5)2(0.5)1+33C(0.5)3(0.5)0

= 3×0.125+1×0.125=0.5

Hence, the needed probability is 0.

Poisson Distribution

Poisson Distribution Formula

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where

x = 0, 1, 2, 3, ...

λ = mean number of occurrences in the interval

e = Euler's constant ≈ 2.71828

Example

There are five students in a class and the number of students who will participate in annual day every year is a Poisson random variable with mean What will be the probability of more than 3 students participating in annual day this year?

Solution

Mean for Poisson random variable, m = 3

P(x>3;3)=P(4;3)+P(5;3)

P(4;3) = e−3 344! = 0.16803135574

P(5;3) = e−3 355! = 0.10081881344

Hence, P(x>3;3) = P(4;3)+P(5;3)=0.268850169

The probability of getting more than three students participating is 0.268850169.

Normal Distribution

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu =$ Mean

$\sigma =$ Standard Deviation

$\pi \approx 3.14159 \cdots$

$e \approx 2.71828 \cdots$

Example:

A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hr?

Solution

Let x be the random variable that represents the speed of cars. x has $\mu = 90$ and $\sigma = 10$. We have to find the probability that x is higher than 100 or $P(x > 100)$

For $x = 100$, $z = (100 - 90) / 10 = 1$

$P(x > 90) = P(z >, 1)$ = [total area] - [area to the left of z = 1]

$= 1 - 0.8413 = 0.1587$

The probability that a car selected at a random has a speed greater than 100 km/hr is equal to 0.1587

Applications of Probability in Every Field

Weather forecasting

The legendary department makes use of the concept of probability for their work. However, all peoples are also make use of the concept of probability. You observe and notice for a few days that clouds come up but it does not rain. Next time when you are about to step out, you think it over in your mind and decide there is no need to carry an umbrella. Probability are used to scientist in this field. It is very useful to the scientist.

Sports

The sports are very important in all peoples. Probability have used in this field. To decide what are the chances of winning or losing of a particular team based on their previous record. Similarly, probability is put to use in order to devise the sports strategy also. The probability are used maximum in the sports field. It is calculate win or loss for the games. It is very useful to this field.

Typing on smart devices

It does this using probability- depending on which word is how commonly used. Insurance policies are you get anything insured, you study which insurance policy would be appropriate depending on your usage of the thing or person and what are the chances of damage and what kind of damage is possible. Typing is most important in this world, because the world change into computer world.

Card Games

Traditional card game of skills 'rummy' uses the concept of probability, along with those of rearrange in different orders and process of combining. Players keep on calculating their chances of getting a particular card they require and in the process learn these concepts without making awake efforts.

Video Games and Board Games

These games to proper the player to think in terms of probability. In general, whenever we consider the chances of something happening, we are actually give effect to the mathematical concept of probability.

Tiny changes that can boost your social relations

Remembering peoples' names and birthday and any functions, smiling when you meet them and caring for them when no one else does are examples of very tiny changes you can do. Such small changes can do the theoretical entertainment affect your social life in the society.

Applications of Probability in Real Life

- ❖ Probability is used in everyday in our life, because they have important in daily life.
- ❖ Probability theory is applied in everyday life. In risk assessment and modeling. The insurance industry and markets use actuarial science to determine pricing and make trading decisions.
- ❖ Governments apply probabilistic methods in environmental regulation, entitlement analysis and financial regulation.
- ❖ The Monte Carlo Simulation is an example of a stochastic model used in finance. Stochastic modeling as applied to the insurance industry, telecommunication, traffic control etc.,
- ❖ Accordingly, the probabilities are neither assessed independently nor necessarily very rationally. The theory of behavioral finance emerged to describe the effect of such groupthink on pricing, on policy, and on peace and conflict.
- ❖ In addition to financial assessment, probability can be used to analyze trends in biology (e.g. disease spread) as well as ecology (e.g. biological Punnett squares).

- Probability is used to design games. The discovery of rigorous methods to assess and combine probability assessments has changed society.
- Another significant application of probability theory in everyday life is reliability. Many consumer products, such as automobiles and consumer electronics, use reliability theory in product design to reduce the probability of failure.
- Failure probability may influence a manufacturer's decisions on a product's warranty. The cache language model and other statistical language models that are used in natural language processing are also examples of applications of probability theory.

Conclusion

We have studied in this paper and it is very useful our life and some inventions and probability determines our life. The main aim of this paper is to present the importance of probability in various application of probability in every field and everyday life. An overview is presented especially to project the ideas of probability.

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Introduction to Stochastic Models in Human Genetics

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ABSTRACT: *Stability of spatially distributed Ecosystem, the community models with stochastic dynamic identical for all the points in the space occupied by the community usually,*
Any type of Stochastic nature, however our study is poisson point process, Study on Reproductive age specified women, longevity , fecundity, Coales Reproductive Measures using this demographic study the Speciation of Stochastic model based on Piaget's Cognititative Structure, Binnet's IQ measure may be used for choosen Community.
This study may be useful for differential mental attitude improvement exercise may be Used, to know the Extinction (speciation) of community may be studied.

Keywords: *Progeny, Net Reproduction Rate, Chebyschev's inequality, Francis GaltonBranching process, Central Limit theorem, Leslie equation.*

1. Introduction

Piage t 's after his doctorate in Zoology, he joined in Alfred Bennet's labTo construct the Pshchyometric test, where he found similar wrong answers byVarious age groups of people, in our opinion this paper is torevew the basics ofPopulation Genetics Equilibrium developed by Mathematician Hardy and physicst Weinberg, assuming panmixrandam mating population, inbreeding of biological communities. Francis Galton Braching process, Shenon diversity index for understanding diversity of Community duly applying Net Reproductive Measure, Coales measures are utilized.
In this review, we attempted describe the probability and Mathematical Statistics concepts Chebyshive's inequality, central Limit theorem, Uniquness,and inversion thereom .

2. Methods and Materials

Stability of spatially distributed Ecosystem, the community models with stochasticDynamic identical for all the points in the space occupied by the community or Ecosystem, usually using following differential equation
Dni/dt = fl(N1, N2, ……..Nn; l1,l2,…….ln) ; i= 1,2, …. N where Ni is the number (or) density of species in the community, and l1,l2,…..ln are parameter, where the scoring Points communities with migration between points, dnt/dt= FiNi+gii=1,2,…nHere N-n(x,y, t) is the density of specimens at the point (x,y)

2.1 Population Growth and Regulation

Life tables show how survival and reproduction rates vary with age, Size, life expectancy, fecundity rate is as follows
Nx: Number of individuals alive at age x (or) decreas from at age x
Sx: age specified survival rate , which is the chance that an individual of age x will survive At the age x+1
Lx: Survivorship, Which is the process of individual the survive from birth(age) at year.
Fx: Fecundiy the average number of offspring produced by a female child at the age of x
Survivorship Curve(lx) to plot the number of individividual from hypothetical chohart that will increase, decrease, or constant of population dynamics could be understood.

2.2 Speciation: Shenon's Diversity Index:

As Ecologists, defines the biogenesis, an ecosystem, an biological community, the Mathematical definition of stability and it is precisely for this reason more rich in comprising species are more stable. Differently adopted to varations in environment, such that a variety of may respond with more success to differential reproduction, motivate the use of such indices
D= -pilnpl, Pi = Ni/N, N= Ni

Where n is the number of speies in a community , Ni is the population of ith species

Di = 1- Ni(Ni-1)/N(N-1)

Where Di is the probability that two individuals randomly sampled from collection of N, willnot belong to the same specices, For big N and fairly uniform distribution in species

Di = D, Di = N/N-1(1- pl2) = 1- Pi - Pi(Pi-1) - pi logPl

It would be logical to suppose that in equilibrium state, the community is not
Most stable , and hence should exhibit maximal diversity but as it may be readily, show In this case the community structure is such that specification of any species occur with the same frequency (Max Di is attained at Ps=1/n), that is all species are equally piabundant.

2.3 Trinomial distribution; Hardy-weinbergEqulibrium
Let AA, Aa, aa its offspring probability distribution is ,p,q,rrespectively,Its mean and variance can be statistically to be measureable, such that any population Genetic is studied using this model Assumption. Violation leads to genetic drift and selection

3. Analsysis
However, in our assumption o f population Genetics, Hardy-Weinberg Law is violated, as consequtenly Genetic Drift and Natural selection forces are inevitable in studying Population dynamics.

3.1 Significance ofChebysheve's in equality in Speciation
P([X-U}>kO< 1/k

(or) P(x-u}<kO>- 1-1/k

Notice, However that in the numerous laboratory and non laboratory communities in the early stages of their evolution towards and equiliburium state, an increase in diversity(l.e. of Di) clearly to a certain extent , this measure still characterizes N community, moreover, this measure in all probability can even characterize its stability but only in the early stages of evolution.
For observed set of measurements, standard deviation can provide us information about the way the probability accumulates in intervals increase. As Chebyschev's inequality states , if the standard deviation is small, greater probability, concentrate near the mean and the standard deviation is large the probability spreads out more.

(i) The speciation, the percentages of total probability that lies in an interval centred at the Mean P([X-U) > KO) < 1/ K

The probability assigned to values of X, outside the interval;{u-k , U+k} is atmost
Increase in Diversity (i.e Di= 1- Ni(Ni-1)/N(n-1)

Where N= Ni; N/N= Pl

Among ecologist, it is taken atomost an axion, that communities, which ate more Complex in studies more rich in comparing species are necessarily more Studied.

3.2 Central Limit theorem
The central Limit theorem(CLT) in the mathematical theory of probability may be expressed as follows. If Xi (i=1,2,.. n) be independent random variable such that
E(Xi) =U and V(Xi) =c, then it can be proved that under very general condions, the random variables sn= X1+x2+….Xn is asymptotically normal with mean U and standard deviation

3.3 Uniqueness theorem:
IF Mx=My each distribution will uniquely express identically

The Moment generating function of a distribution if it exists, uniquely determines the distribution. This implies that correspondingly to a given probabaility distribution theire is only one m.g.f, there is only one probability distribution. X and y are identically distributed.

3.4 Inversion thereom: Characteristics function of decay, constant, increase in population its speciation distribution will give explicit characteristic function.

3.4 Trinomial Distribution
Each richness of speies are, in particular, we choose, three mutually Exculsive events AA, Aa, aa (P1+P2+p3)n = 1

Conclusion
Branching process arise from a particular model of population growth, dealing with problems in genetics, Consider Bienayme Watson Branching process arising from the following model, consider a population in which the individual of the N+1 the generation are produced by those of the n the generation in a such a way that each individual can, independent of others, produce 0,1,2, successor (direct descendent of offspring) with probabilities ko, k1, k2, … . If xn denotes number of individuals in the n th generation (n=0,1,.)
We x0=1, for X0=i(>1) the ith process started by the I ancestors are o copies of the same problem. For 0<s<1, denotes, Fn(S)=E(SXn) (n=0,1,)
We have F0(S) =s, F1(S)= E(Sx1) = Kjsj =K(S)
A special class of Markov Chain is a branching process, using biological terminology consider a situation in which each organison of one generation produces a random numbers of offspring to form the next generation.
Given the probability distribution of the number of offspring produced by an organism, one is interested in characteristics of such as the distribution(Mean and Variance), we shall use biological terms in organisim and offspring
Suppose that at the end of its life time an organism producea random nuber of S of iffspring with probability distribution P(s=j)=Pjwechan use Chebyschev'sienqulatiy
P{Xn-Exn}>E V(Xn)/E2
As n when u <1 ,w we also have
P{xn-0]>e } 0
P(Xn=0)-1, when U >1 , the probability of the exting of the probability is not easy to obtain;
In what way do the random fluctuationos of E influence the population dynamics? Suppose the e values at different times are mutually 0dt i8ndependent. Thus if e(t) at each instant t are normally distributed with mean e and 9tvariance o , that are independent of time then according to the central limit theorem e(t[r)ft id is also normally distributed with mean et and variance and since N(t)/no =expe(t)dt}. The quantity N/no must have a logarithmatically normal distribution with the same m[e-) it ean and variance with the distribution density
This distribution has one maximum at Nmax=N0exp it follows at oncethat if E , then Nmax 0 as t A and distribution Mode shift to the left(Type II) censorinf while for e> it shifts to the right(Type I) censoring data which leads to survival data analysis.

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MHD Flows of UCM Fluids above Porous Stretching Sheets By Fad Method using Mathematica

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ABSTRACT: *The system of two non-linear ODE with the specified boundary conditions that describes the UCM fluids and heat transfer over a non-linearly stretching sheet has been developed and its solution is obtained by FAD method. FAD method proved its capability to solve a large class of non-linear problems efficiently, accurately and easily with approximations convergence vary rapidly to solutions and its variations are calculated using MATHEMATICA.*

Keywords: *Non-Linear Ordinary Differential Equation, Boundary Conditions, Fuzzy A Domain Decomposition Method, Mathematica.*

1. Introduction

Agassant et al. (1991) have investigated stretching sheet is an important operation in polymer, Production of plastic sheets and foils, for example, involves extrusion of molten polymers through a slit die with the extradite collected by a wind-up roll upon solidification. This process is normally accompanied with both heat and momentum transfer aspects. But there are also cases in which a plastic sheet may be stretched with no heat transfer involved.

Cold drawing of plastic sheets, for example, is an important operation in which a plastic sheet is elongated in certain direction in order to improve its mechanical properties in that direction. In all such operations with or without heat transfer involved the force required to pull the sheet is one of the most important design criteria. This force is known to depend to a large extent on the extensional viscosity of the sheet material. It also depends on the physical or rheological properties of the fluid surrounding the sheet.

While traditionally Newtonian fluids are used for this purpose for example water or air but, in recent years it has been shown that there might be some advantages if the fluid surrounding the sheet can be made viscoelastic, say through the use of polymeric additives Alternatively, one may resort to injection or suction in order to modify flow kinematics provided the sheet is porous at the first place. And in cases where the fluid surrounding the sheet is electrically conducting, one may equally well rely on applying a sufficiently strong magnetic field to modify flow kinematics. Conceivably, one might also envisage cases in which a combination of all these methods might be involved simultaneously to achieve the best result Among the techniques mentioned above to control flow kinematics, the idea of using magnetic fields appears to be the most attractive one both because of its ease of implementation and also because of its non-intrusive nature. The idea is not new and has shown its effectiveness for both Newtonian and non-Newtonian fluids a like by T. Hayat et al and Kandasamy et al (2005, 2006). Also they have been extensive efforts in the past, in both theoretical and experimental areas alike, to better understand such magneto hydrodynamic (MHD) flows.

For Newtonian fluids, theoretical results are in favor of the experimental observations. For non-Newtonian fluids, in contrast, theoretical results are not always in support of experimental findings. This is not surprising realizing the fact that most studies carried out in the past on MHD flows of non-Newtonian fluids were concerned primarily with simple rheological models such as second-order model, or third-order model Presented by Bird (1987) . To this should be added the fact that these simple rheological models are known to be good only for fluids of low-elasticity in slow and/or slowly-varying flows.

In practice, however, such restrictive conditions may not prevail. For reasons like these, the reliability of theoretical results obtained using such simple rheological models are in serious doubt.

Hayat et al. (2006) tried to investigate MHD flow of a more realistic viscoelastic fluid model, i.e., the so-called upper-converted Maxwell model, above a porous stretching sheet. Also a study on MHD flows of UCM fluids above porous stretching sheets using FHAM purposed by Sadeghy et al. (2009) and the application of the homotopy perturbation method in nonlinear problems has been devoted by scientists and engineers, because this is to continuously deform a simple problem easy to solve into the difficult problem under study.

Fuzzy Homotopy techniques were applied to find all roots of nonlinear equations investigated by He and Liao (1997, 1998 and 2001).

In this paper we use the Fuzzy homotopy perturbation method and Fuzzy Adomian decomposition method to solve the highly nonlinear ODE to derive an approximate analytical solution. FHPM is an analytical procedure for finding the solutions of problems which is based on the constructing a homotopy with an embedding parameter p that is considered as a small parameter. The purpose of this paper is to apply homotopy perturbation method which is useful for finding the approximate analytical solution of MHD flows of UCM fluids above porous stretching sheets.

II . Equations of Motion

The constitutive equation for a Maxwell fluid is

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S} \tag{2.1}$$

where **T** is the Cauchy stress tensor and the extra stress tensor **S** satisfies

$$\mathbf{S}+\lambda\left(\frac{ds}{dt} - LS - SL^T\right) = \mu A_1, \tag{2.2}$$

where □ is the viscosity, □ is the relaxation time and the Rivlin-Ericksen tensor A_1 is defined through

A_1 □ □ □□□□V□^{T} (2.3)

For the steady two-dimensional flow, the equations of continuity and momentum for the magneto hydrodynamic flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.4}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \sigma B_0^2 u, \tag{2.5}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial S_{yx}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \tag{2.6}$$

where □ is the fluid density, □ is the electrical conductivity, B_0 is the constant applied magnetic field in the y-direction and S_{xx} ,S_{xy}, S_{yx} and S_{yy} are the components of the extra stress tensor. Using the following boundary layer approximations (Schlichting and Sadeghy et al. (1964, 2005))

$$u=o(1),\ v=o(\delta),\ x=o(1),\ y=o(\delta), \tag{2.7}$$

$$\frac{T_{xx}}{\rho} = o(1),\ \frac{T_{xy}}{\rho} = o(\delta)\quad \frac{T_{yy}}{\rho} = o(\delta^2) \tag{2.8}$$

the flow in the absence of the pressure gradient is governed by Eq. (2.4) and

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left[u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x\partial y}\right] = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u, \tag{2.9}$$

where □ being the boundary layer thickness. The appropriate boundary conditions on the flow are

u □ Bx, v □ V0 at y □ 0, (2.10)
u □ 0 as y □ □ □

where $V_0 > 0$ is the suction velocity and $V_0 < 0$ gives the injection velocity. Introducing

$\eta = \sqrt{\frac{B}{\nu}}\,y$, u=Bx$f'(\eta)$, v=-$\sqrt{\nu B}$ (2.11)

The governing problem is t ransformed to

f''' $M^2 f'$ f'^2 ff'' $2ff'f''$ $f^2 f'''$ 0 (2.12)

f R, f ' 1 at   0,

$f' \rightarrow 0$ as η→ ∞ (2.13)

Here $M^2 = \frac{\sigma B_0^2}{\rho B}$, $\beta = \lambda B$, R=$\frac{V_0}{\sqrt{\nu B}}$ where R>0 corresponds to suction nd R <0 for injection is nonlinear differential equation which can be solved analytically by FADM and FHPM.

III. Fuzzy Homotopy Perturbation Method

Now we will use the method in order to obtain the solution of Eq. (2.12). Assuming *u=f*, Eq. (2.12) can be written in the following form u''' $F(u)$ 0, (2.14)
in which

$F(u)$ M $^2 u'$ u'^2 uu'' $2uu'u''$$u^2 u'''$ (3.1)

According to the homotopy perturbation method (He, 1998) we construct a homotopy in the form
$u''' - \alpha^2 u' + p[F(u) + \alpha^2 u'] = 0$ (3.2)

with the initial conditions

$u(0)$ R, $u'(0)$ 1, $u'($ $)$ 0. (3.3)

When p=0 Eq. (3.16) becomes a linearized equation, u''' 2u' = 0,

where is an unknown parameter to be further determined when p=1, it turns out to be the original one. The embedded parameter p monotonically increases from zero to unit as the trivial problem, u''' 2u' 0is is continuously deformed to the original problem, Eq. (2.14). According to the homotopy perturbation method, we assume that the solution to Eq. (1.16) may be written as a power series in p:

u=$u_0 + pu_1 + p^2 u_2 + \cdots$ (3.4)

Substituting Eq. (1.18) into Eq. (1.16) and equating the terms with the identical powers of p, we have
$p^0: u_0''' - \alpha^2 u_0 = 0, u_0(0) = R, u_0'(0) = 1, u_0'(\infty) = 0,$ (3.5)

$p': u_1''' - \alpha^2 u_1' - (u_0')^2 + u_0 u_0'' - \beta(2\,u_0 u_0' u_0'' - u_0^2 u''') + (\alpha^2 - M^2) u_0' = 0,$ (3.6)

$u_1(0) = 0$, $u_1'(0) = 0$ $u_1'(\infty) = 0$

The solution of Eq. (3.4) can be readily obtained, which reads

$u_0(\eta)$=R+$\frac{1}{\alpha}$(1-exp(-$\alpha\eta$)). (3.7)

Substituting Eq. (1.21) into Eq. (1.20) results in
$u_1''' - \alpha^2 u_1' = (M^2 + R\alpha + \beta R^2 \alpha^2 + 2\beta R\alpha + \beta - \alpha^2 + 1)e(-\alpha\eta) - \beta \exp(3\alpha\eta)$
(3.8)

In case R ^{2} (2 □ □ 1)^2 □ (□ R ^{2} □ 1)(4 M^2 □ 3□ □ 4) □ 0 we can solve u_1 from Eq. (3.8) under the initial or boundary conditions $u_1(0) = 0, u_1'(0)=0$ and $u_1'(\infty)=0$ and obtain the following solution with case:

$$u_1(\eta) = -\frac{\beta}{24\alpha^3} + \frac{M^2+R\alpha+\beta R^2\alpha^2+2\beta R\alpha+\beta-\alpha^2+1}{2\alpha^2}\eta \exp(-\alpha\eta) + \frac{\beta}{24\alpha^3}\exp(-3\alpha\eta), \quad (3.9)$$

Which

$$\alpha = \frac{-R(2\beta+1) - \sqrt{R^2(2\beta+1)^2 - (\beta R^2 - 1)(4M^2+3\beta+4)}}{2(\beta R^2 - 1)} \quad (3.10)$$

Therefore, we obtain the first-order approximate solution for

R ^{2} (2 □ □ 1)^2 □ (□ R ^{2} □ 1)(4 M^2 □ 3□ □ 4) □ 0 , which reads

$$u(\eta) = u_0(\eta) + u_1(\eta) \quad (3.11)$$

IV . Results and Discussion

Table 1: Comparison of the values of □ $f''(0)$ obtained by FHPM and FADM with HAM at R =0.3 for the Case of suction.

□	M	FHPM (Present Results)	FADM (Present Results)	HAM Aliakbar et al.(2009)
0	0	1.161187	1.06187	1.15897
	5	5.251225	5.25125	5.2515
	10	10.20995	10.2995	10.2195
5	0	7.633621	7.68921	7.033621
	5	11.22855	11.02855	10.99855
	10	17.30534	17.40534	17.30534
10	0	64.418655	64.4525	64.418655
	5	68.023447	68.23447	68.023447
	10	77.157946	77.0946	77.157946

Table 2: Comparison of the values of □ $f''(0)$ obtained by FHPM and FADM with HAM at R =-0.3 for the Case of injection

□	M	FHPM (Present Results)	FADM (Present Results)	HAM Aliakbar et al.(2009)
0	0	0.861187	0.865213	0.861506
	5	4.951225	4.895623	4.95122
	10	9.900995	10.2995	9.901
5	0	3.804673	3.81234	0.700263
	5	5.575188	4.78456	4.74886
	10	11.40379	11.40534	11.0275
10	0	6.051008	6.044525	0.269701
	5	6.198820	4.23447	4.15231
	10	14.520849	13.0946	13.5784

Tables 1 and 2 shows that the homotopy perturbation and Adomian decomposition methods are in a good agreement with each other. But our results are more reliable than results obtained by Sadeghy et al. (2009) because in the all of cases wall shear stress decreases when the value of □ increases.

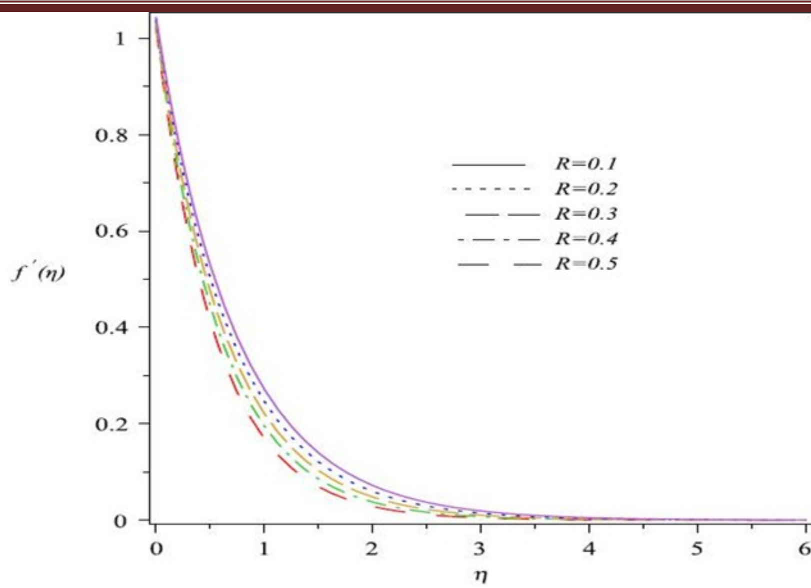

Fig. 1: Effects of suction velocity R on FADM and FHPM of f'(?)with ? =0.3 and M = 0.5.

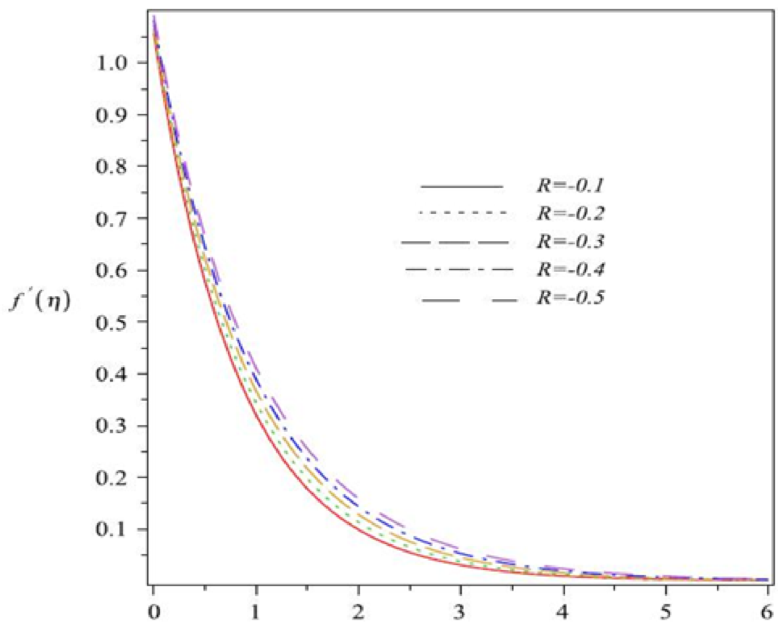

Fig. 2: Effects of injection velocity R on FADM and FHPM off'(?) with ? =0.3 and M = 0.5.

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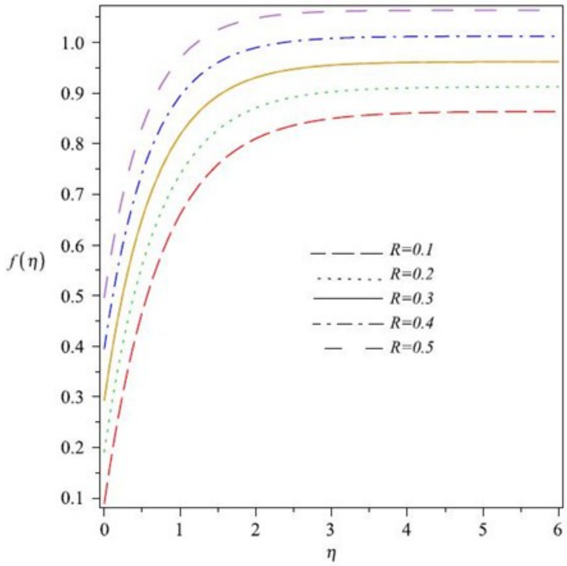

Fig. 3: Effects of suction velocity R on FADM and FHPM of f() with =0.3 and M = 0.5.

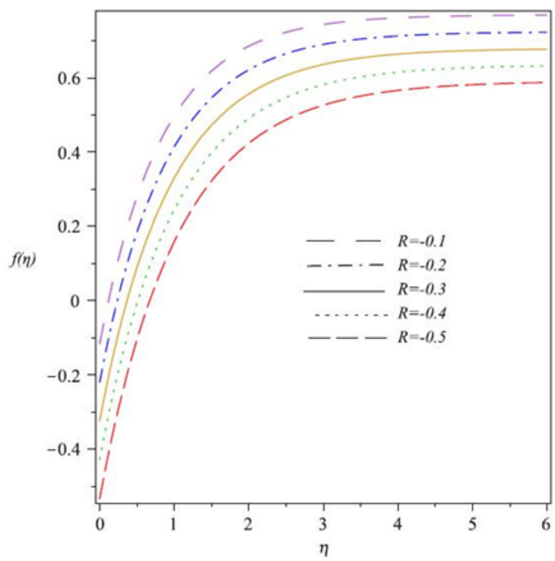

Fig. 4: Effects of injection velocity R on FADM and FHPM of$f$() with =0.3 and M = 0.5.

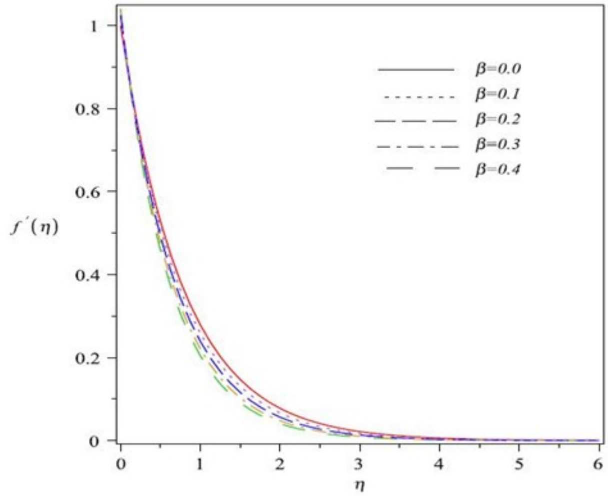

Fig. 5: Effects of relaxation time β on FHPM and FADM approximation of f' is drawn in case of suction at R = 0.3 and M =0.5.

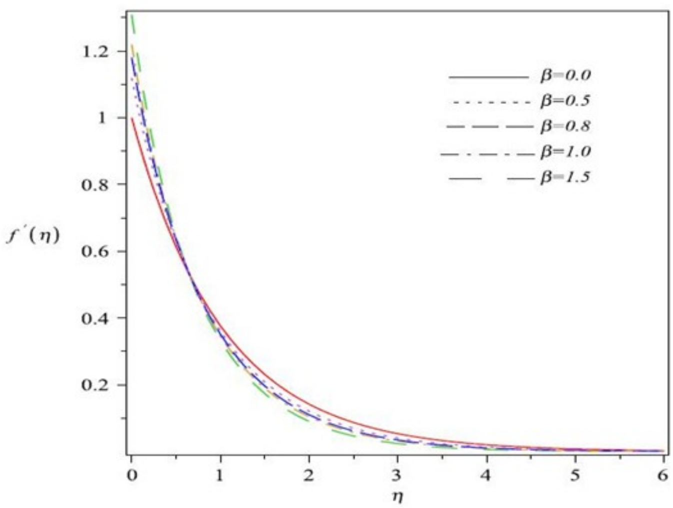

Fig. 6: Effects of relaxation time β on FHPM and FADM approximation of f' is drawn in case of injectionat R = -0.3 and M =0.5.

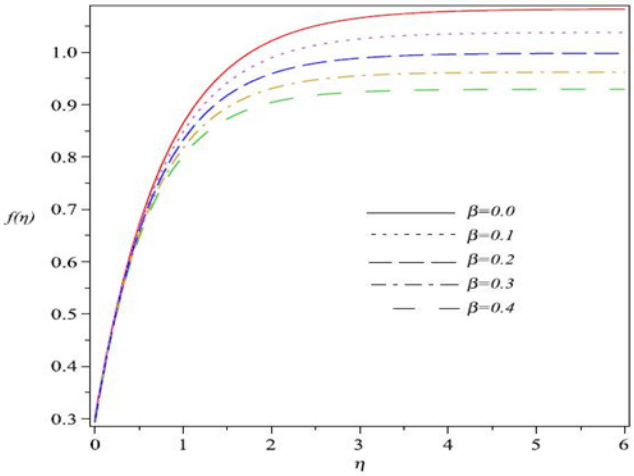

Fig. 7: Effects of relaxation time on FADM and FHPM approximation of f is drawn in case of suction with R = 0.3 and M =0.5

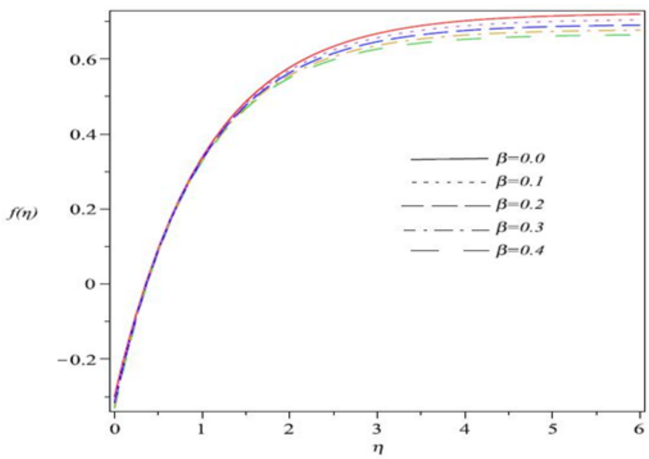

Fig. 8: Effects of relaxation time on FADM and FHPM approximation of f is drawn in case of injection with R = -0.3 and M =0.5.

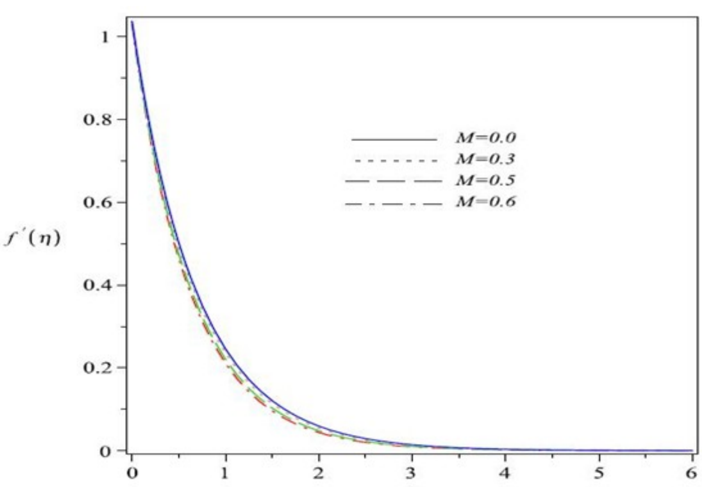

Fig. 9: Effects of MHD parameter M on FHPM and FADM approximation of f 0 is drawn in case of suction at R =0.3 and ⍰ = 0).

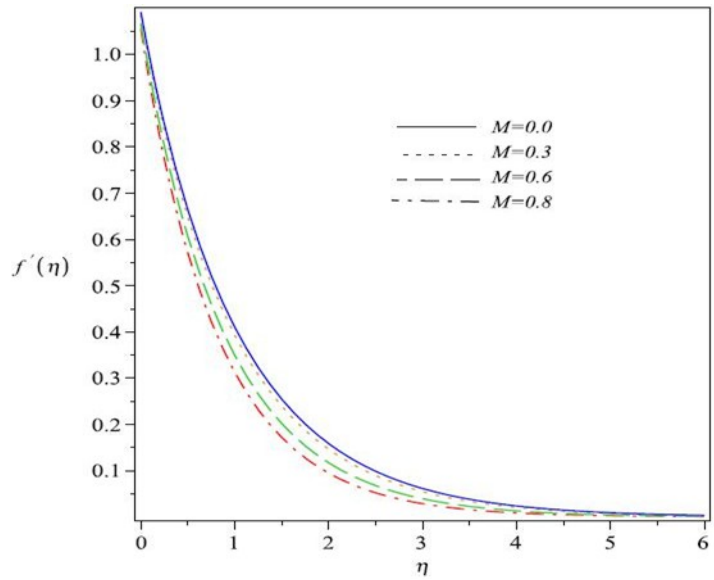

Fig. 10: Effects of MHD parameter M on FHPM and FADM approximation of f 0 is drawn in case of injection with R =0.3 and ⍰ = 0).

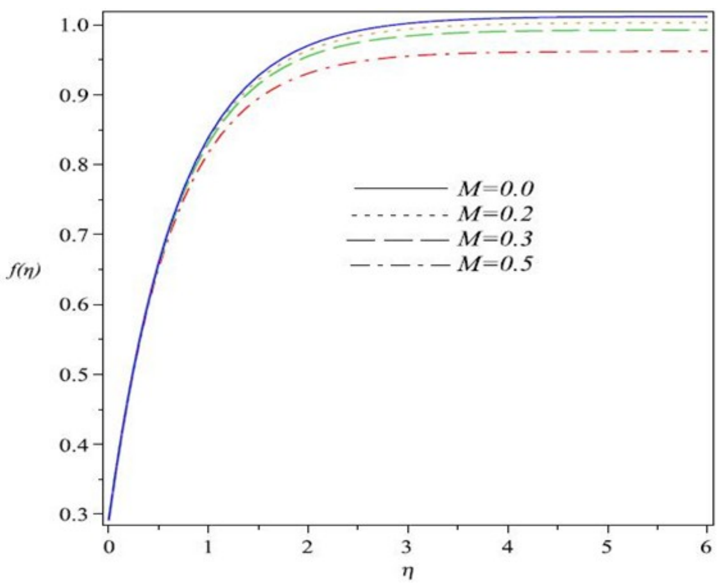

Fig. 11: Effects of MHD parameter M on FADM and FHPM approximation of f is drawn in case of suction at R = 0.3 and ⍰ = 0.

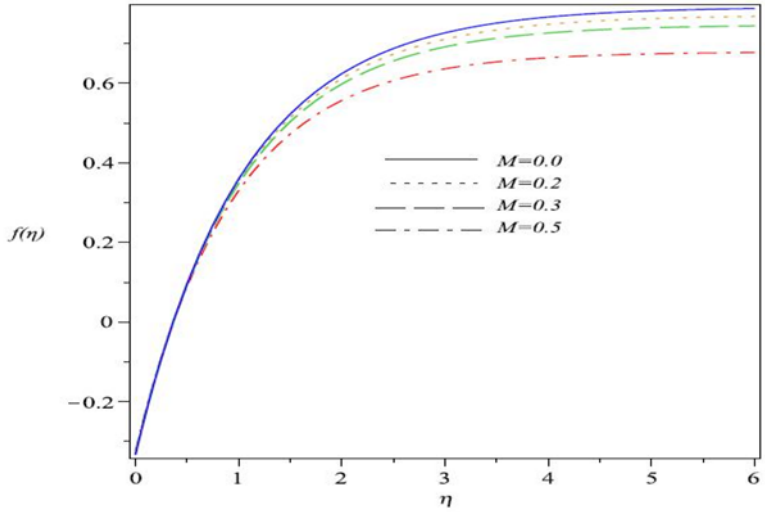

Fig.12: Effects of MHD parameter M on FADM and FHPM approximation of f is drawn in case of injection with R = -0.3 and ⍰ = 0.

Figs. 1-12 have been made in order to see the effects of the relaxation time ⍰ , suction orinjection parameter R and the MHD parameter M on the velocity field. Figs. 1-4 are made to see the effect of the suction and injection velocity R on the velocity components

f' and *f* respectively for the magneto hydrodynamic flow. Similar to the results obtained by HAM. It is found that for case of suction in Figs. 1 and 3, the velocity component *f'* decreases but *f* increases initially and then for the value of ⍰ between 1 and 2 it decreases with an increase in R. The boundary layer thickness decreases in both cases. For the case of injection in Figs. 2 and 4, we have the opposite effect. Figs. 5-8 are drawn for the effects of non-dimensional relaxation time ⍰ on the velocity components *f'* and *f*. In the case of suction,
Figs. 5 and 7 show that by increasing the velocity components *f'* and *f* decrease and the boundary layer thickness decreases. Figs. 9-12 are sketched in order to see the effects of MHD parameter M on the velocity components *f'* and *f*. These figures show that *f'* and *f* decreases by increasing M for the case of suction and injection. In Figs. 1-12 we find a good agreement between the results obtained by Hayat et al. (2006) and present method. We, therefore, can easily obtain the wall shear stress which reads $f''(0)$ ⍰ $u''(0)$.

V. Conclusion

In this study, we have applied Fuzzy Adomian Decomposition method in solving the MHD flows of UCM fluids above porous stretching sheets. Comparison between the Fuzzy Homotopy perturbation method (FHPM) and Fuzzy Adomian Decomposition methods (FADM) for the studied problem show a remarkable agreement and reveal that the FHPM need less work. It is important that we applied FHPM for the equation in unbounded domain without using Padé approximants.

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Fuzzy Adomian Decomposition Method For Viscous Flow and Heat Transfer Over A Nonlinearly Stretching Sheet

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ABSTRACT: *Fuzzy Adomian decomposition method is powerful methods which consider the approximate solution of a viscous flow and heat transfer over a nonlinearly stretching sheet as an infinite series usually converging to the accurate solution and also we discussed theoretical analysis of the method with numerical solution.*

Keywords: *Fuzzy A domain decomposition method (or) FADM , nonlinear ordinary differential equations (nonlinear ODE) , viscous flow, stretching sheet.*

1. Introduction

The study of two-dimensional boundary layer flow due to a stretching surface is important in variety of engineering applications such as cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of paper and plastic sheets. In all these cases, a study of flow field and heat transfer can be of significant importance since the quality of the final product depends on skin friction coefficient and surface heat transfer rate.

The problem of heat transfer from boundary layer flow driven by a continuous moving surface is of importance in a number of industrial manufacturing processes. Several authors have been analysed in various aspects of the pioneering work of Sakiadis (1961). Crane (1970) have investigated the steady boundary layer flow due to stretching with linear velocity. Vleggaar et al. (1977) have analysed the stretching problem with constant surface temperature and Soundalgekar et al. (1980) have analysed the constant surface velocity.

1.2 Formulation of The Problem

We consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane y = 0, the flow being confined to y > 0. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The basic boundary layer equations that govern momentum and energy respectively are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2} \tag{1.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{1.3}$$

Subject to the boundary conditions are

$$\begin{aligned} &u_w(x)=Cx^n, v=0\\ &u\to 0\ y\to\infty\\ &T=T_w\ at\ y=0;\ T\to T_\infty\ as\ y\to\infty \end{aligned} \tag{1.4}$$

where *(x,y)* denotes the Cartesian coordinates along the sheet and normal to it, *u* and *v* are the velocity components of the fluid in the *x* and *y* directions, respectively, and *v* is the kinematic viscosity. *C* and *n* are parameters related to the surface stretching speed. c_p and α are the specific heat of the fluid at constant pressure and the thermal diffusivity respectively.

The equation of continuity is satisfied if we choose a stream function ψ(x, y) such that

$u = \frac{\partial \psi}{\partial y}, \qquad v = \frac{-\partial \psi}{\partial x}.$

The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables

$$\eta = y\sqrt{\frac{C(n+1)}{2\nu}}\,x^{\frac{n-1}{2}}$$

u=C$x^n f'(\eta)$, (1.5)

v=-$\sqrt{\frac{C\nu(n+1)}{2}}\,x^{\frac{n-1}{2}}\left[f+\frac{n-1}{n+1}\eta f'\right]$

Substituting (1.4) into (1.2) and (1.3), we obtain the following set of ordinary differential equations:

$$f''' + ff'' - (f')^2\left(\frac{2n}{n+1}\right) = 0 \tag{1.6}$$

$$\theta'' + Prf\theta' + PrEc(f'')^2 = 0 \tag{1.7}$$

The boundary conditions (1.4) now become

η=0: f=0 $f' = 1, \quad \theta = 1$

η→ ∞: $f' = 0, \quad \theta = 0$ (1.8)

where the primes denote differentiation with respect to η

$E_c = \frac{u_w^2}{c_p(T_w - T_\infty)}$ is the Eckert number,Pr$\left(=\frac{\nu}{\alpha}\right)$ is the Prandtl number Further, the constants T_w ,T_∞ denote the temperature at the wall and at large distance from the wall, respectively.

1.3 Fuzzy Adomian Decomposition Method

To solve the system of coupled ODEs using Adomian decomposition method, rearranging (1.6) and (1.7) as follows

$$f''' = -ff'' - f'^2\frac{2n}{n+1} \tag{1.9}$$

$$\theta'' = -Pr[f\theta' + Ec(f'')^2] \tag{1.10}$$

While applying the standard procedure of FADM

Eqs (1.9) and (1.10) becomes

$$L_1 f=\left(-ff'' - f'^2\frac{2n}{n+1}\right) \tag{1.11}$$

$$L_2\theta=-Pr[f\theta' + Ec(f'')^2] \tag{1.12}$$

Where

$L_1 = \frac{d^3}{d\eta^3}$ and inverse operator $L_1^{-1}(.) = \int_0^\eta\int_0^\eta\int_0^\eta(.)d\eta d\eta d\eta$ and

$L_2 = \frac{d^2}{d\eta^2}$ and inverse operator $L_2^{-1}(.) = \int_0^\eta\int_0^\eta(.)d\eta d\eta$

Applying the inverse operator on both sides of (1.11) and (1.12)

$$L_1^{-1}L_1 f = L_1^{-1}\left(-ff'' - f'^2 \frac{2n}{n+1}\right) \tag{1.13}$$

$$L_2^{-1}L_2\theta = -Pr\, L_2^{-1}[f\theta' + Ec(f'')^2] \tag{1.14}$$

Simplify Eqs (1.13) and (1.14) we get

$$f(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \int_0^\eta \int_0^\eta \int_0^\eta \left[-N_1(f) - N_2(f)\frac{2n}{n+1}\right] d\,\eta d\,\eta d\,\eta \tag{1.15}$$

and

$$\theta(\eta) = \alpha_2 - \eta - \Pr \int_0^\eta \int_0^\eta N_3(f,\theta) + EcN_4(f)\, d\eta d\eta \tag{1.16}$$

Where $\alpha_1 = f''(0)$ and $\alpha_2 = \theta(0)$ are to be determined from the boundary conditions at infinity in (1.8). The nonlinear terms ff'', f'^2 and $f\theta'$ can be decomposed as Adomian polynomials $\sum_{n=0}^{\infty} B_n$, $\sum_{n=0}^{\infty} C_n$, $\sum_{n=0}^{\infty} D_n$ and $\sum_{n=0}^{\infty} E_n$ as follows

$$N_1(f) = \sum_{n=0}^{\infty} B_n = ff'' \tag{1.17}$$

$$N_2(f) = \sum_{n=0}^{\infty} C_n = (f')^2 \tag{1.18}$$

$$N_3(f,\theta) = \sum_{n=0}^{\infty} D_n = f\theta' \tag{1.19}$$

$$N_4(f,\theta) = \sum_{n=0}^{\infty} E_n = (f'')^2 \tag{1.20}$$

Where $B_n(f_0, f_1, ..., f_n)$ $C_n(f_0, f_1, ..., f_n)$ *and* $D_n(f_0, f_1, ..., f_n, \theta_0, \theta_1, \theta_2, ..., \theta_n)$ $E_n(f_0, f_1, ..., f_n)$ are the so called Adomian polynomials. In the Adomian decomposition method (1994) f and θ can be expanded as the infinite series

$$f(\eta) = \sum_{n=0}^{\infty} f_n = f_0 + f_1 + f_2 + ... + f_m + ...$$

$$\theta(\eta) = \sum_{n=0}^{\infty} \theta_n = \theta_0 + \theta_1 + \theta_2 + ... + \theta_m + ... \tag{1.21}$$

Substituting (1.17), (1.18), (1.19) and (1.20) into (1.14) and (1.16) gives

$$\sum_{n=0}^{\infty} f_n(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \int_0^\eta \int_0^\eta \int_0^\eta \left[-\sum_{n=0}^{\infty} B_n - \frac{2n}{n+1}\sum_{n=0}^{\infty} C_n\right] d\,\eta d\,\eta d\,\eta \tag{1.22}$$

and

$$\sum_{n=0}^{\infty} \theta_n(\eta) = \alpha_2 - \eta - \Pr \int_0^\eta \int_0^\eta \left[\sum_{n=0}^{\infty} D_n + Ec\sum_{n=0}^{\infty} E_n\right] d\eta d\eta \tag{1.23}$$

Hence, the individual terms of the Adomian series solution of the equation (1.6)–(1.8) are provided below by the simple recursive algorithm

$$f_0(\eta) = \eta + \frac{\alpha_1\eta^2}{2} \tag{1.24}$$

$$\theta_0(\eta) = 1 + \alpha_2\eta \tag{1.25}$$

$$f_{n+1}(\eta) = \int_0^\eta \int_0^\eta \int_0^\eta -B_n - C_n\, d\eta d\eta d\eta \tag{1.26}$$

$$\theta_{n+1}(\eta) = -Pr\int_0^{\eta}\int_0^{\eta} D_n + EcE_n\, d\eta d\eta \qquad (1.27)$$

For numerical calculation, we choice the m-term approximation of f(η) and $\theta(\eta)$ as
$\phi_m(\eta)=\sum_{n=0}^{m-1} f_n(\eta)$ and $\Omega_m(\eta) = \sum_{n=0}^{m-1}\theta_n(\eta)$

The recursive algorithms (1.24)–(1.27) are programmed in MATLAB. We have obtained upto 14th term of approximations to both f(η) and $\theta(\eta)$. We provided below only first few terms due to lack of space.

$$f_0 = \eta + \frac{\alpha_1\eta^2}{2}$$

$$f_1 = \left(\frac{1}{3} - \frac{1}{3n+3}\right)\eta^3 + \left(\frac{\alpha_1}{8} - \frac{\alpha_1}{6n+6}\right)\eta^4 + \left(\frac{\alpha_1^2}{40} - \frac{\alpha_1^2}{30n+30}\right)\eta^5$$

etc.,
and

$$\theta_0 = 1+\alpha_2\eta$$

$$\theta_1 = -Pr\left[\left(\frac{Ec\alpha_1^2}{2}\right)\eta^2 + \left(\frac{\alpha_2}{6}\right)\eta^3 + \left(\frac{\alpha_1\alpha_2}{24}\right)\eta^4\right]$$

etc.,

The velocity gradient (□$_2$ □ □'(0)) for various values of *Pr* at *n=3* and *Ec=1* using FHPM-Padé and FADM-Padé techniques

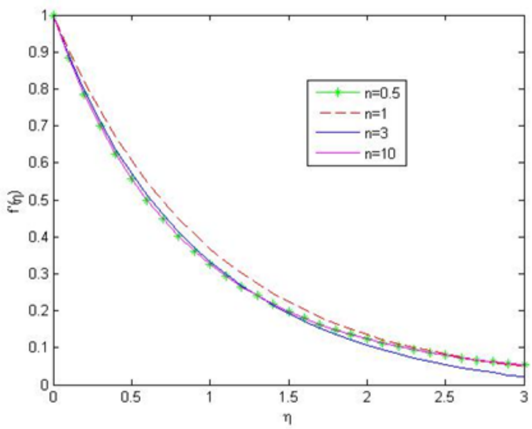

Fig. 1: Velocity profiles *f* '(□) for various values of *n* when *Pr*= 1 and *Ec*=1 Using □ '$_{15[7/7]}$.

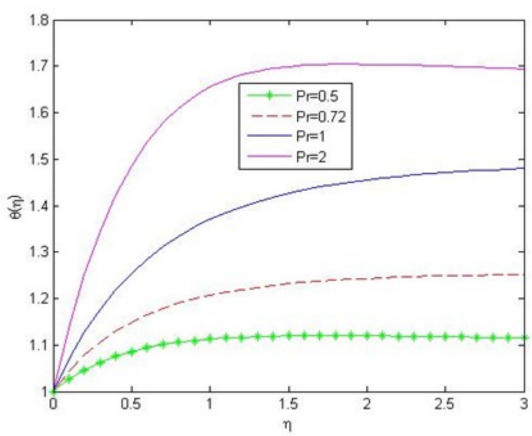

Fig. 2: Temperature profiles □ (□) for various values of Pr at n = 3 and Ec= 1 Using □$_{15[7/7]}$.

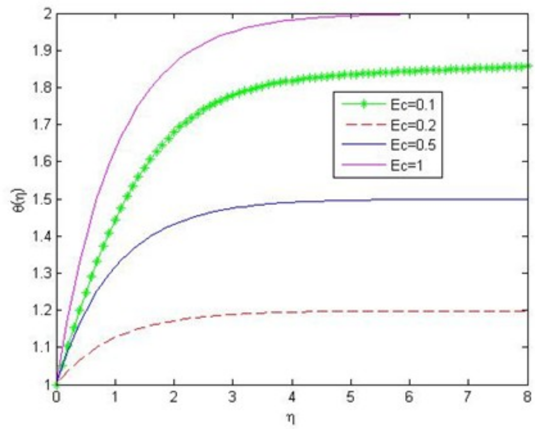

Fig. 3: Temperature profiles □ (□) for various values of Ec at n = 1 and Pr= 1 Using $□_{15[7/7]}$.

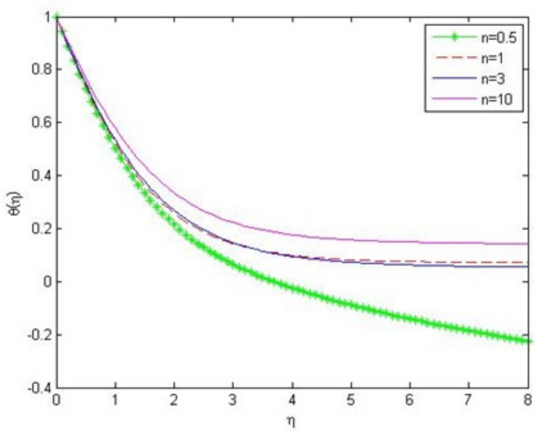

Fig. 4: Temperature profiles □ (□) for various values of n at Ec = 0 and Pr= 1 Using $□_{15[8/8]}$.

From Fig.1 we note that when unsteadiness parameter *n* increases, the velocity profiles decreases. In Figs. 2 and 3 we note that when Prandtl Number (Pr) increases that implies the temperature decreases within the boundary layer for all values of the Prandtl number. This is consistent with the well-known fact that the thermal boundary layer thickness decreases with increasing Prandtl number. In Fig 4 we note that when unsteadiness parameter *n* increases the temperature Profiles is decreases.

1.4 Conclusion

The Fuzzy Adomian decomposition method is applied to solve a system of two nonlinear ordinary differential equations with the specified boundary conditions that describes viscous flow and heat transfer over a nonlinearly stretching sheet. The obtained solutions have matched with the existing numerical result. The Fuzzy Adomian decomposition method techniques are very efficient alternative tools to solve nonlinear models with infinite boundary conditions.

Relative Study of Simplex Technique and Dynamic Programming Technique for Linear Programming Problem

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ABSTRACT: *In operational analysis contains so many problems. One amongst the chief problem is linear programming problem and it are often resolved by numerous ways specifically simplex technique, dual simplex technique ,big M technique etc., Dynamic programming is another one among the technique in operational analysis. Main thinking of this paper is the relative study of simplex technique and dynamic programming technique for lpp.*

Keywords: *Simplex technique, Dynamic programming, slack variable, best check*

1. Introduction

During this paper discuss regarding canonical type of applied mathematics issues shall be thought of. A general applied mathematics downside are often place within the kind maximize or Minimize $Z=C_1X_1+C_2X_2$. The higher than kind is clearly there if all original constraints are of ($\leq$) kind. Linear programming problem involving 2 variables are often simply resolved by the graphical approach .It conjointly provides associate insight into the ideas of Simplex Technique to be mentioned latter. Dynamic programming is worried with the idea of multi-stage call method. Associate best strategy has the property that regardless of the original state and original call.

Formula

$f_j(b_1,b_2,\ldots.b_m) = \max_{0 \leq X_j \leq b}[C_jX_j + f_{j-1}(b_1 - a_{1j}X_j) \ldots \ldots b_m - a_{mj}X_j]$

$b = \min\{b_1/a_{1j}, \ldots\ldots b_m/a_{mj}\}$

New Pivot equation=Old equation-Pivot element

Other equation=Old equation-Corresponding co-efficient$\times$new pivot element.

Definition

The condition expressing the relation between the variables are known as constraints, typically those constraints seem within the kind in equalities the variables that seem within the objective operate are known as call variable

Dynamic Programming for LPP

Dynamic programming approach is similar to divide and conquer in breaking down the problem into smaller and yet smaller possible sub-problems. ... An optimum solution can be achieved by using an optimum solution of smaller sub-problems.

Difference between Dynamic programming and linear programming:

(1) In DP, there's no set procedure as in platter to resolve any call downside. The DP technique permits to break the given downside into a sequence of easier and smaller sub-problems are then resolved in a very sequent orders.

(2) LP approach provides just the once amount answer to an issue. wherever as DP approach is beneficial for higher cognitive process over time and solves every sub downside optimality.

Dynamic Programming Approach

Algorithm:

The LPP general kind is Max $Z=C_1X_1+C_2X_2+\ldots\ldots\ldots+C_nX_n$

Subject to constraints,

$a_{11}x_1+\ldots\ldots+a_{1n}x_n \leq b_1$

$a_{21}x_2+\ldots\ldots+a_{2n}x_n \leq b_2$

$a_{m1}x_1+\ldots\ldots+a_{mn}x_n \leq b_m$

Each activity j (j=1,2,.....n) is taken into account as stage.
The constants $b_1,b_2,\ldots..b_m$ the amount accessible resources..
Let $f_n(b_1,b_2,\ldots..b_m)$ be best the rule relation is
$f_1(b_1,b_2,\ldots..b_m)=\max_{0\leq xj\leq b}[C_jX_j+f_{j-1}(b_1-a_{1j}x_j),(b_2-a_{2j}x_j)\ldots(b_m-a_{mj}x_j)]$

The maximum worth of b that xj assume is
$b=\min\left(\frac{b1}{a1j},\frac{b2}{a2j},\ldots\ldots,\frac{bm}{amj}\right)$

Example: 1

Use Dynamic programming to solve LPP Max $Z=3X_1+5X_2$
Subject to constraints, $X_1\leq 4$,
$X_2\leq 6$,
$3X_1+2X_2\leq 18$,
$X_1, X_2\geq 0$.

Solution

There are 2 variables: the problem are often treated as a 2 stage dynamic programming problem.
The three constraints b_1,b_2,b_3 are allocated to x_1 and x_2 at totally different stages.
Initial solution: $b_1=4$, $b_2=6$, $b_3=18$.

Stage: 1

Recursion relation
$f_j(b_1,b_2,\ldots\ldots b_m)= \max_{0\leq xj\leq b}[C_jX_j+f_{j-1}(b_1-a_{1j}x_j),\ldots.]$
most worth of b that xj will assume is
$b=\min\left(\frac{b1}{a1j},\frac{b2}{a2j},\ldots\right)$
$f_1(b_1,b_2,b_3)= \max_{0\leq x1\leq b}[3x_1]$
The possible worth x1 is satisfies all constraints most worth of b that x1 will assume is
$= \min\left(\frac{4}{1},\frac{6}{0},\frac{18}{3}\right)$
$= 4$
$f_1(b_1,b_2,b_3)=\max(3x_1)$
$f_1(b_1,b_2,b_3)=\max 3x_1$
$= 3\min\left(4,\frac{18-2X2}{3}\right)$
$X^*_1= \min\left(4,\frac{18-2X2}{3}\right)$

Stage: 2

$f_2(b_1,b_2,b_3) = \max_{0\leq x2\leq b}(5X2+3X1)$
$= \max\left[5X_2+3\min\left(4,\frac{18-2x2}{3}\right)\right]$
$=\begin{cases}4 & ,0\leq X2\leq 3\\ \frac{18-2X2}{3} & ,3\leq X2\leq 6\end{cases}$

Maximum worth of b that x2 will assume is
$= \min\left(\frac{4}{0},\frac{6}{1},\frac{18}{2}\right)$
$= 6$

$f_2(b_1,b_2,b_3)=\max\left[5x_2+3\min\left(4,\frac{18-2x2}{3}\right)\right]$

$$=\max\,(5x_2+3)\begin{cases}4 & ,0 \le X2 \le 3\\ \frac{18-2X2}{3} & ,3 \le X2 \le 6\end{cases}$$

$$=\max\,(5x_2+3)\begin{cases}4 & ,x2 = 3\\ \frac{18-2X2}{3} & ,x2 = 6\end{cases}$$

$$=\max\begin{pmatrix}5X2+12 & ,X2 = 3\\ 5X2+6 & ,X2 = 6\end{pmatrix}$$

$$=\max\begin{pmatrix}27 & ,X2 = 3\\ 36 & ,X2 = 6\end{pmatrix}$$

$f_2\,(b_1,b_2,b_3)=36$
$X_2=6$
$X^*_1=\min\,[4, 18-2x_2/3]$
$X_1=\min\,[4, 2]$
$X_1=2$
The Solution are: $X_1= 2$, $X_2=6$
Max Z=36.

Simplex Method
Step 1
Check whether or not the target operate of the given applied mathematics downside is to be maximized or minimize.

Step 2
If not all Zi - Cj≥0 then the add isn't best then we tend to visit next step.

Step 3
Getting into variable: The non-negative variable whose Zj-Cj is should negative it's the getting into variable. If getting into variable doesn't contain any positive worth then the given downside has on infinite answer otherwise visit next step.

Step 4
Going variable: just for a positive component of the getting into column. The fundamental variable that is that the answer variable that is that the going variable then we tend to visit next step.

Step 5
The intersection of component going row and getting into column is named pivot component.

Step 6
Visit Step a pair of and repeat the method and so get the possible answer..

Example: 2
Using Simplex method Max $Z=3X_1+5X_2$
Subject to constraints, $X_1\leq 4$,
$X_2\leq 6$,
$3X_1+2X_2\leq 18$,
$X_1, X_2\geq 0$.

Solution
To solve the problem by simplex method
Max $Z=3X_1+5X_2+0S_1+0S_2+0S_3$
Subject to, $X_1+S_1=4$
$X_2+S_2=6$
$3X_1+2X_2+S_3=18$

The basic solution $X_1, X_2=0$ and $S_1=4, S_2=6, S_3=18$

Table: 1

		C_j	3	5	0	0	0	
C_B	Y_B	X_B	X_1	X_2	S_1	S_2	S_3	θ
0	S_1	4	1	0	1	0	0	∞
0	S_2	6	0	1	0	1	0	6
0	S_3	18	3	2	0	0	1	9
	Z_j-C_j	0	-3	-5	0	0	0	

X_1-Entering variable, S_3-Leaving variable

First iteration

		C_j	3	5	0	0	0	
C_B	Y_B	X_B	X_1	X_2	S_1	S_2	S_3	θ
0	S_1	4	1	0	1	0	0	4
5	X_2	6	0	1	0	1	0	∞
0	S_3	6	3	0	0	-2	1	2
	Z_j-C_j	30	-3	0	0	5	0	

X_1-Entering variable, S_3-Leaving variable

Second iteration

		C_j	3	5	0	0	0
C_B	Y_B	X_B	X_1	X_2	S_1	S_2	S_3
0	S_1	2	0	0	1	2/3	-1/3
5	X_2	6	0	1	0	1	0
3	X_1	2	1	0	0	-2/3	1/3
	Z_j-C_j	36	0	0	0	3	1

All Z_j-C_j are all positive.
The optimal solution is,
$X_1=2$
$X_2=6$
Max Z=36

Conclusion

Linear Programming problem is the most important problem. Another important issue of the Operation Research problem is the dynamic problem associated with the decision problem. Solution available from both the simplex and dynamic methods that we used to solve the linear programming problem are identical. This paper we have learned is that using the dynamic approach we get the most correct solution because of this linear programming problem. The dynamic programming approach is an easy to solve LPP method.

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Mean Holomorphic Function Value Theorem

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ABSTRACT: *The article represents a general theorem of Myers and when the limit assumption ƒ '(a)=ƒ '(b) is removed and this result for the holomorpic functions of a complex variable is demonstrated. After that, the equivalence of Rolle's and Mean theorems of value in the complex plane demonstrated.*

Keywords: *Holomorphic, Mean Value Theorem , Rolle's Theorem, Flett's Theorem, Complex Rolle's Theorem, Complex Mean Value Theorem .*

1. Introduction

Let ƒ function continuously at a closed interval, [a, b]. The difference between the value of ƒ at the end points of [a, b], if ƒ '(a) is derivative, can be estimated using ƒ '(a)

ƒ (b) ƒ(a) $\approx$ ƒ ' (a)(b a),

Where the approach is good when b a is small. Actually, that is only the tangent line approximation

ƒ(a $+ \Delta x$) $\approx$ ƒ(a) $+$ ƒ '(a)Δx

where Δx replaced by b a. Actually, the approach can be replaced by the exact formula

ƒ(b) ƒ(a) = ƒ '(c)(b a)

If the derivatives ƒ ' is assessed between a and b at the point c instead of the end point a. Here we assume that ƒ can be distinguised at each point a and b, and the choice of c depends on the specific function ƒ.

Definition [Holomorphic]
Holomorphic function is a complex-valued function of one or more complex vaeiable that is, at every point of its domain, complex differntiable in a neighbourhood of the point. The existance of a complex derivative in a very strong condition, it implies that any holomorphic function.

Theorem –A [Mean Value Theorem]
If ƒ(x) is a function which fulfills both,
1. At the closed inteval [a,b] ƒ (x) is continuouss
2. At the open interval (a,b) ƒ (x) is differentiable

Then there is a number c such that $a < c < b$ and

ƒ (b) $-$ ƒ (a) = ƒ ' (c) $-$ (b$-$a)

Theorem – B [Rolle's Theorem]
If ƒ of x is continuous on the closed interval (a,b), and differentiable on the open interval from a to b, and ƒ of a equal ƒ of b, then the least number c exist and the open interval from a to b such that ƒ prime of c equals zero.
The equality between Rolle's and mean value theorem for real- valued functions has been proved. In particular, in 1958, Flett proved the following variation of mean value theorem.

Theorem – C [Flett's Theorem]

Let ƒ : [a,b] $\rightarrow$ R be differentiable on closed interval [a,b] and ƒ '(a) = ƒ ' (b). Then the point c exist in open interval (a,b) such that

$$f(b) - f(a) = f'(c) - (b - a)$$

In 1977, Myers gave the results.

Main Results

Our first objective in thispaper is to extend the theorem of Myers for real valued functions to a result that does not depends on the hypothesis ƒ '(a) = ƒ ' (b) , but it reduces to the theorem of Myers when this happens.

Theorem – 1

If ƒ : [a,b] $\rightarrow$ R is a differentiable function , then there exist a point point c belongs to open interval (a,b)such that

$$f(b)\ \ f(c) = f'(c)(b\ \ c) + \frac{f'(b) f'(a)}{2\ b\ a}(b\ \ c)^2.$$

Proof

Consider the auxilary feature h : [a,b] $\rightarrow$ R defined by

$$h(x) = f(x)\ \ \frac{f'(b) f'(a)}{2\ b\ a}(x\ \ a)^2$$

then h is distinguishable on closed interval [a,b], and

$$h'(x) = f'(x)\ \ \frac{f'(a) f'(b)}{2\ b\ a}(x\ \ a).$$

It follows that h'(a) = h'(b) = ƒ '(a). Applying theorem of Myers to h gives h(b) h(c) = h'(c) (b c) for some point c belons to the open interval (a,b). The rewriting of h and h' in ƒ gives the result.

Remark

That 's easyto see if ƒ '(a) = ƒ ' (b) , this result then shrinks to theorem. Moreover, theorem -1 remains valid if the h function id replaced by

$$h(x) = f(x)\ \ \frac{f'(b) f'(a)}{2\ b\ a}(x\ \ b)^2$$

This demonstrates that the h function is not unique . so, by using an auxilary function h, we can find the same result.

The secon objective of this paper is to demonstrate a version theorem of Myers for holomorphic functions in the spirit if Evar and Jafari.

Theorem –B

Let f be a holomorphic function defined on an convex subset open D_f of C . Let a and b be two separate points in D_f Then there exist , z_2 belongs to the open interval (a,b)

$$\mathrm{Re}\,(f'(z_1)) = \frac{[b\ a, f(b)\ f(z))}{(b\ a, b\ z)}\ \mathrm{Re}\frac{f'(a) f'(b)}{2\ b\ a}(b\ \ z)$$

and

$$\mathrm{Im}\,(f'(z_1)) = \frac{[b\ a, f(b)\ f(z))}{(b\ a, b\ z)}\ \mathrm{Im}\frac{f'(a) f'(b)}{2\ b\ a}(b\ \ z)$$

Proof

Let u(z) = Re(f(z)) and v(z) = Im(f(z)) for z belong to D_f . We define the auxiliary function : [0,1] $\rightarrow$ R by

φ (t)= (b – a, f(a + t(b – a)),

Which is
φ (t) = Re[(b – a) u(a + t(b – a))] + Im[(b – a) v(a + t(b – a))]

For every t belong to closed interval [0,1].
Therefore ,use of the Cauchy-Riemann equations, we obtain
φ' (t) = (b – a, (b – a)f0(a + t(b – a))
= Re($(b\ \ a)^2 \frac{\partial u(z)}{\partial x}$
= $(b\ \ a)^2$ Re (f '(z))

Applying theorem 1 we obtain the result.
It is easy to see that if it reduces this result to the next complex version of Myers theorem.

Theorem -3 (Complex Rolle's Theorem)
Let f be a holomorphic function defined on an convex subset open D_f of C . Let a and b belongs to D_f be such a way f (a) = f (b) = 0 and a ≠ b. Then there are $, z_1, , z_2$ belongs to the open interval (a,b) such that Re(f(z)) = 0 and Im(f(z)) = 0.

Theorem -4 (Complex Mean Value Theorem)
Let f be a holomorphic function defined on an convex subset open D_f of C . Let a and b be two separate points in D_f .Then there exist $z_1, , z_2$ belongs to the open interval (a,b) such that Re(f(z)) = Re $\frac{f\ (a) f\ (b)}{b\ \ a}$ and Im(f(z)) = Im $\frac{f\ (a) f\ (b)}{b\ \ a}$.

Proof of the equivalance

It is clear that Theorem 3 most hold if theorem 4 does. To show the conversation, assume that the condition of the theorem 4 is met .Then

$$g\ (z) = \frac{1}{a\ \ b} \begin{matrix} f(z) & f(a) & f(b) \\ z & a & b \\ 1 & 1 & 1 \end{matrix}$$

$$= f\ (z)\ f\ (a) \frac{z\ \ b}{a\ \ b} + f\ (b) \frac{z\ \ b}{a\ \ b}$$

Is also a holomorphic function for every z belonds to D_f . It is easy to see that the g function meets the requirement g (a) = g (b) = 0. Hence by theorem 3 , there exist $, z_1, , z_2$ belongs to the open interval (a,b) such that Re (g '(z_1)) = 0 and Im(g' (z_2)) = 0.

We obtain,
g'(z) = f ' (z) $\frac{f\ (b) f\ (a)}{b\ \ a}$

for every z belongs to D_f. Hence ,
0 = Re (g'(z)) = Re (f ' (z)) Re $\frac{f\ (b) f\ (a)}{b\ \ a}$,
0 = Im (g'(z)) = Im (f ' (z)) Im $\frac{f\ (b) f\ (a)}{b\ \ a}$,
Which is prove that theorem 3 this implies theorem 4. Therefore theorem 3 and 4 are equivalent.

Conclusion

This article we explained a general theorem of Myers and when the limit assumption $f'(a)=f'(b)$ is removed and this result for the holomorpic functions of a complex variable is demonstrated. After that, the equivalence of Rolle's and Mean theorems of value in the complex plane demonstrated. It is easy way to understand for learners.

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Linear Programming Problem With Simplex Method For Bounded Variables

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ABSTRACT: *Linear programming problem drawback with edge variables may be resolved victimization the finite simplex methodology while not the express thought of edge constraints. The edge constraints are thought-about implicitly during this methodology that reduced the scale of the idea matrix considerably. We developed bounded simplex variable algorithm and solve problem.*

Keywords: *LPP, Bounded simplex method ,Bounded variable ,Slack variable.*

Merits of bounded simplex method
Non negative constraints by setting x'=x-1. This set if conditions is helpful in algorithms for finite variables. Basically, the set is employed to determine the optimality condition for an extended basic possible answer in an LPP finite variables.

1. Introduction

Additionally to the constraints in any applied math downside the worth of some or all variables is restricted with lower and higher limits. In such cases the quality kind of associate degree applied math downside
Optimize(max or mini)z=cx
Subject to,
Ax=b and L≤X≤U

Where $L=(l_1, l_2, \ldots\ldots, l_n)$ and $u=(u_1,u_2,\ldots\ldots,u_n)$ denote lower and higher bounds of x severally.
The boundary constraint will be handled directly by subbing. $L=x-s_i$ (or) $x=l+s_i$,
where $s_i \geq 0$.
For associate edge constraint of the sort x≤u, the substitution $x=u-s_j$, $s_j \geq 0$ does not guarantee that x can stay non-negative .
This issue is overcome by employing a special technique known as delimited variable straightforward methodology.

The simplex algorithm for bounded variables:
Step-1:
1) If the matter diminution then convert into maximization kind.
2) If R.H.S. of any constraint is negative, create it possible by multiplying the constraint by -1.
3) Convert the inequalities of the constraint into equations by adding appropriate slack\surplus variable.

Step-2:
Acquire associate initial basic possible solution. If any basic variable at a positive boundary, then substituted it out at its boundary.

Step-3:
Calculate z_j-c_j as was common for all non-basic variables. Examine price of z_j-c_j .
1) If all z_j-c_j≥0, then the present soiution is perfect .
2) If atleast one z_j-c_j<0 choose most negative of z_j-c_j to enter non-basic variable into new solution .

Step-4:
Let x_j a non-basic variable at zero level that is chosen to enter the solution .
Let x_B^* the present basic solution .
Then calculate

Θ_1=min $\frac{XB*}{aij}$, $a_{ij}>0$

Θ_2=min $\frac{uj-XB*}{-aij}$,$a_{ij}<0$ and

Θ=min $\{\Theta_1,\Theta_2,u_j\}$
Wherever u_j is that the bound for the variable

Step-5:
Let $(x_B)_r$ be the variable like $\Theta=\min\{\Theta_1,\Theta_2,u_j\}$.
Then,
1) If $\Theta=\Theta_1$, then $(x_B)_r$ leaves the solution and x_j enters by victimization regular simplex technique.
2) If $\Theta=\Theta_2$,$(x_B)_r$ leaves the solution and x_j enters; then $(x_B)_r$ being non-basic at its boundary should be substituted out by victimization, $(x_B)_r=u_r-(x_B)_r'$, $0\leq(x_B)_r'<u_r$.
3) If $\Theta=u_j$, then x_j is substituted at its boundary distinction u_j-x_j'. whereas remaining non-basic.

Step-6:
Visit step-3 and repeat the procedure until all z_j-c_j are either positive or zero.

Example:
Using bounded variable simplex method. Solve the linear programming problem
Max $z=3x_1+5x_2+2x_3$
Subject to,
1. $X_1+2x_2+2x_3\leq14$
2. $2x_1+4x_2+3x_3\leq23$

$0\leq x1\leq4$, $2\leq x2\leq5$, $0\leq x3\leq3$

Solution
Given x2 has positive lower bound.
We taking $x2=x2'+2$
Max $z=3x+5(x_2'+2)+2x_3$
Max=$3x_1+5x_2'+2x_3+10$

Subject to,
1. $X_1+2(2x_2'+2)+2x_3\leq14$
   $X_1+2x_2'+2x_3\leq10$
2. $2x_1+4(x_2'+2)+3x_3\leq23$

$2x_1+4x_2+3x_3\leq15$
$0\leq x_1\leq4$
$2\leq x_2'+2\leq5$
$0\leq x_2'\leq3$
$0\leq x_3\leq3$

Standard form:
Max $y=z-10=3x_1+5x_2'+2x_3+0s_1+0s_2$

Subject to
$x_1+2x_2'+2x_3+s_1=10$
$2x_1+4x_2'+3x_2+s_2=15$
And $x_1,x_2',s_1,s_2,x_3\geq0$

Table 1

		u_i	4	3	3	∞	∞		
		c_j	3	5	2	0	0		
CB	Y_B	X_B	X_1	X_2'	X_3	S_1	S_2	θ_1	θ_2
0	S_1	10	1	2	2	1	0	5	∞
0	S_2	15	2	4	3	0	1	$\frac{15}{4}$	∞
Z_j-c_j		-	-3	-5	-2	0	0		

$\theta_1=\min\{5,\frac{15}{4}\} = \frac{15}{4}$

$\theta_2=\min\{\infty,\infty\}=\infty \quad ,u_2=3$

$\theta=\min\{\theta_1,\theta_2,u_2\}=\min\{\frac{15}{4},\infty,3\}=3$

Hence $\theta=u_2$
X_2' we substituted upper bound difference $x_2'=u_2-x_2''$ $0\le x_2''\le 3$
The problem becomes
Max $x=z-25=3x_1-5x_2''+2x_3$
Sub to , $x_1-2x_2''+2x_3+s_1=4$
$2x_1-4x_2''+3x_3+s_2=3$
$x_1,x_2'',x_3\ge 0$ $,s_1,s_2\ge 0$

Table 2

		Uj	4	3	3	∞	∞		
		Cj	3	-5	2	0	0		
CB	YB	XB	X1	X2”	X3	S1	S2	θ1	θ2
0	S1	4	1	-2	2	1	0	4	∞
0	S2	3	2	-4	3	0	1	$\frac{3}{2}$	∞
Zj-Cj			-3	5	-2	0	0		

θ1= min $\{4,\frac{3}{2}\} = \frac{3}{2}$

θ2=∞ ,u1=4
θ=θ1
Introduce x1 into basis and remove s2.

Table 3

		Uj	4	3	3	∞	∞		
		Cj	3	-5	2	0	0		
CB	YB	XB	X1	X2	X3	S1	S2	θ1	θ2
0	S1	$\frac{5}{2}$	0	0	$\frac{1}{2}$	1	$\frac{-1}{2}$	∞	∞
3	X1	$\frac{3}{2}$	1	-2	$\frac{3}{2}$	0	$\frac{1}{2}$	-VE	$\frac{5}{4}$
ZJ-CJ			0	-1	$\frac{5}{2}$	0	$\frac{3}{2}$		

θ1=∞ , θ2= $\frac{5}{4}$ θ3 =U3=3

θ=MIN$\{\infty,\frac{5}{4},3\}= \frac{5}{4}$

θ=θ2 ,X2”-entering ,x1-will leave the basis
Let x1=4-x1”

Table 4

		Ui	4	3	3	∞	∞
		Cj	3	-5	2	0	0
CB	YB	XB	X1	X2"	X3	S1	S2
0	S1	$\frac{5}{2}$	0	0	$\frac{1}{2}$	1	$\frac{-1}{2}$
-5	X2"	$\frac{-3}{4}$	$\frac{-1}{2}$	1	$\frac{-3}{4}$	0	$\frac{-1}{4}$
Zj-Cj		-	$\frac{-1}{2}$	0	$\frac{7}{4}$	0	$\frac{5}{4}$

Put x1=4-x1' , 0≤x1'≤4
Max z =-3x1'-5x2"+2x3

New table 5

		Uj	4	3	3	∞	∞
		Cj	-3	-5	2	0	0
CB	YB	XB	X1'	X2"	X3	S1	S2
0	S1	$\frac{5}{2}$	0	0	$\frac{1}{2}$	1	$\frac{-1}{2}$
-5	X2"	$\frac{5}{4}$	$\frac{1}{2}$	1	$\frac{-3}{4}$	0	$\frac{-1}{4}$
ZJ-CJ			$\frac{1}{2}$	0	$\frac{7}{4}$	0	$\frac{5}{4}$

All Zj-Cj≥0
Hence the optimal solution is
Max z= $\frac{123}{4}$
Hence the solution.

Conclusion

We conclude that the linear programming problem with finite simplex methodology and algorithms and realize the issues. This kind is extremely helpful in massive constraints and really straightforward to search out resolution.

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Some studies, on the other hand, found different positive dimensions within the relationship among colleagues focusing on the collectivity. Undoubtedly relationship among colleagues opens space for sharing values and norms and collective goal orientation. Hargreaves (1994) argued that in teacher culture there must be a shift from individualism to collaboration, from hierarchies to teams, from supervision to mentoring, from in-service training to professional development and from authority towards parents to a contract with parents(ibid).

Solving Difference Equations By Forward Difference Operator Method

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ABSTRACT: *In this paper a forward difference operator method was used to solve a set of difference equations. We also find the exacting solution of the non homogeneous difference equations with constant coefficients. In this case, a new operator term the forward difference operator Δr,s, defined as* $\Delta_{r,s} y_n = r\, y_{n+1} - s\, y_n$*, was introduce. Some of the properties of this original operator be also investigate.*

Keywords: *Forward Difference, Backward Difference, Finite Difference*

1. Introduction

In Numerical Analysis, we use some linear operators such as shift exponential operator E, with $Efj = fj+1$, forward difference operator Δ, with $\Delta fj = (fj+1) - fj$, and backward difference∇, with $\nabla fj = fj - (fj-1)$. These operator is used in some special numerical analysis

Under the forward difference operator Δ, the linear difference equations are written in one of the following forms

$$P(\Delta y_n) = 0, \text{ (homogeneous)} \tag{1}$$

$$P(\Delta y_n) = f_n \text{ (nonhomogeneous)} \tag{2}$$

Where *P* is a polynomial.

The results towards the development of the basic theory of the global behaviour of solutions of nonlinear difference equations of order greater than one.The techniques and results are also extremely used in analyzing the equations in the mathematical models of various biological systems and other applications

Finite differences

Given a function of f(x), with an argument x arranged at equal intervals, we could generate a table of logarithms or trigonometric functions or a Table of special computation for certain calculation (Odior, 2003). Given a certain function of x, say, with values f (a), f (a+h), f (a+2h),….., the difference between the consecutive values of x, denoted by h is called the interval differencing.

If $f(x_1) = f(x)$
$f(x_2) = f(x + h)$ and
$f(x_2) - f(x_1) = \Delta f(x)$

then $$\Delta f(x) = f(x + h) - f(x) \tag{3}$$

Where Δ is called "difference operator" and Δf(x) is called the first difference of f(x). The second difference of f(x) is given thus:

$$\Delta[\Delta f(x)] = \Delta^2 f(x) = \Delta f(x+h) - \Delta f(x). \tag{4}$$

Forward Difference table

X	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
X_0 X_1 X_2 X_3 X_4	$f(x_0)$ $f(x_1)$ $f(x_2)$ $f(x_3)$ $f(x_4)$	$\Delta f(x_0)$ $\Delta f(x_1)$ $\Delta f(x_2)$ $\Delta f(x_3)$	$\Delta^2 f(x_0)$ $\Delta^2 f(x_1)$ $\Delta^2 f(x_2)$	$\Delta^3 f(x_0)$ $\Delta^3 f(x_1)$	$\Delta^4 f(x_0)$

By Forward Difference formula:

$$y = f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{3!}\Delta^4 y_0 + \ldots\ldots$$

Example

Forward Difference table for the following data;

year:	1941	1951	1961	1971	1981	1991
Y:	20	24	29	36	46	51

Proof

Forward Difference table:

X	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
2 4 6 8 10	0 0 1 0 0	0 1 -1 0	1 -2 -1	-3 3	6

Example

Build up a difference Table for the function, $f(x) = x^3 + 3x^2 - 2x + 5$, taking $x = 0$ as the initial value,and $h=2$

Solution

The Table as follows:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
0 2 4 6 8 10	5 21 109 317 698 1285	16 88 208 376 592	72 120 168 216	48 48 48	0 0	0

Results and Discussions

We define the forward difference operator $\Delta_{r,s}$ as follow
$\Delta_{r,s}y_n = ry_{n+1} - sy_n = (rE - s)y_n$

Where y_n is the approximate value function $y(x)$ at point x_{n0} ,m_{xx}∈, then two operators (Δr,s) and (rE - s) are equivalent.

Difference Operations in Vector Space of Operator Δr,s. Some basic different operations in the vector space of difference operator are defined as:

(i) $\Delta r_{1,s} + \Delta r_{2,s} \equiv \Delta r_{1,s} + r_{2,s}$

(ii) $\Delta r,s_1 + \Delta r,s_2 \equiv \Delta r,s_1 + s_2$

(iii) $\Delta r_1,s - \Delta r_2,s \equiv \Delta r_1 - r_2,s$

(iv) $\Delta r,s_1 - \Delta r,s_2 \equiv \Delta r,s_1 - s_2$

(V) $\Delta r_1,s_1 \times \Delta r_2 ,s_2 \equiv \Delta r_2,s_2 \times \Delta r_1$.

Each of the above identities is used for finding particular solution of non-homogeneous difference equations with constant co-effiients.

Conclusion

From this paper we finish the first difference, second difference, third difference, and forth difference of a given polynomial function.The non-homogeneous difference equations with stable coefficients is a new method which can be used to fault all types of nonhomogeneous difference equations with stable coefficients.

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Application of Generalized Parikh Vector in Pattern Matching

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ABSTRACT: *In the area of combinatorial studies of string languages under formal grammars, the concept of Generalized Parikh Vector (GPV) gives the positions of symbols in a linear string. It has been noted that when GPVs of the strings over three symbols are plotted they lie on a hyper plane. This study has been extended to array of symbols too. This paper discusses the use of GPV in identifying two-dimensional subarray of symbols in a given array.*

Keywords: *Generalized Parikh Vector, Pattern Matching.*

1. Introduction

In the area of combinatorial studies of string languages under formal grammars, the concept of Generalized Parikh Vector (GPV) introduced by Siromoney et al. gives the positions of symbols in linear strings. It has been proved that GPVs of the strings of the same length lie on a hyper plane. These studies on GPV of strings over two lettered alphabet emerging with the concept of line languages was introduced by K.Sasikala and Huldah Samuel respectively. The classical parikh mapping also called the parikh vector, is an important tool in the theory of formal languages. One of the important results concerning this mapping is that the image by the parikh mapping of a context free language is always a semilinear set. The parikh vector of a word counts the number of occurrences of each letter of the alphabet. Siromoney et al. introduced another measure which reflects the position of each letter of the alphabet. While parikh mapping is defined for finite words, this measure is defined for infinite words also. The measure of an infinite word has a nice form and reflects the distinction between different infinite words and is called the Generalized Parikh vector (GPV). The concept of Generalized Parikh Vector proves to be a more powerful tool than the classical Parikh vector introduced by Parikh as it gives the positions of the letters in a word w. It has been proved that the GPVs of the words of the same length lie on a hyper plane. In the case of binary alphabet, the GPVs of words of same length lie on the straight line.

In Computer Science, pattern matching is the process of identifying the presence of a specific sequence of symbols in a larger domain of sequence. The issue of pattern matching includes identifying the positions or locations of a pattern within a sequence, to output the identified portions of matched pattern, and to replace the matched pattern with any other sequence.

Sequence patterns (e.g., a text string) are often described using regular expressions and matched using techniques such as backtracking. Tree patterns are used in some programming languages as a general tool to process data based on its structure, e.g., Haskell, ML, Scala and the symbolic mathematics language. Mathematica has special syntax for expressing tree patterns and a language construct for conditional execution and value retrieval based on it.

The contents of this paper has been organized as follows: In section 1, the preliminaries including definitions of Parkh Vector, Generalized Parikh Vector are recalled. Section 2 discusses algorithmic procedures for string pattern matching using GPV over $\Sigma = \{a_1, a_2, \dots, a_n\}$. In section 3, algorithms for array pattern matching using GPV for array over $\Sigma = \{a_1, a_2, \dots, a_n\}$ are discussed.

1. Preliminaries:

1.1. Parikh Vector:

Let $\Sigma = \{a_1 a_2 \dots a_n\}$. then the Parikh Vector of a word u is given by $\pi(x) = \left(|u|_{a_1}, |u|_{a_2}, \dots, |u|_{a_n}\right)$ where $|u|_{a_n}$ represents the number of times a_n occur in u.

1.1.1. Example:
Let $\Sigma = \{a, b, c\}$, if $u = abbcabac$ then the Parikh Vector of a word u is given by $\pi(x) = (3,3,2)$

1.2 Generalized Parikh Vector:
Let $\Sigma = \{a_1 a_2, \dots, a_n\}$ and $x \in \Sigma^\infty$. The Generalized Parikh Vector (GPV) of x denoted by $p(x)$ is $(p_1, p_2, \dots, p_n) \in [0,1]^n$, where, $p_i = \sum_{j \in A_i} \frac{1}{2^j}, A_i \subset N$(set of natural numbers) and A_i contains the position of $a_i (i = 1,2, \dots, n)$ in the word x.

1.2.1. Example:
Let u$=$ $ababab \ \in \Sigma^\infty$, then $p(u) = \left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5}, \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}\right) = \left(\frac{21}{32}, \frac{21}{64}\right)$.

2. String Pattern Matching
2.1 Pattern matching over $\Sigma = \{a_1, a_2, \dots, a_n\}$:
Let T be a text and P be a pattern over Σ

Step: 1
Constructing GPV for the strings T and P
$p(T) = (p_1, p_2, \dots, p_n)$ where, $p_i = \sum_{j_i \in A_i} \frac{1}{2^{j_i}}, A_i \subset N$(set of natural numbers) and A_i contains the position of $a_i (i = 1,2, \dots, n)$ in T.
$p(P) = (q_1, q_2, \dots, q_n)$ where, $q_i = \sum_{j_i \in B_i} \frac{1}{2^{j_i}}, B_i \subset N$(set of natural numbers) and B_i contains the position of $a_i (i = 1,2, \dots, n)$ in P.

Step: 2
Constructing a new word Q from P
Let $r_i = \sum_{j_i \in C_i} \frac{1}{2^{j_i}}, C_i \subset N$(set of natural numbers) and C_i contains the position of $a_i (i = 1,2, \dots, n)$ in $P + (m - 1)$. Where m is a constant $\in N$ $p(Q) = (r_1, r_2, \dots, r_n)$

Step: 3
If $\forall\ m = \{1,2, \dots, |T| - |P| + 1\}, C_i \not\subseteq A_i$ for any $i = \{1,2, \dots, n\}$
Pattern not found in T
Otherwise for any $b \in m = \{1,2, \dots, |T| - |P| + 1\}$, if $C_i \subseteq A_i \forall\ i = \{1,2, \dots, n\}$
Pattern occurs in T at b position

2.1.1 Example:
If $T = abbbaab$ is a text and $P = baa$ is a pattern over $\Sigma = \{a, b\}$

Step: 1 constructing GPV for the word T and P
$p(T) = \left\{\frac{1}{2^1} + \frac{1}{2^5} + \frac{1}{2^6}, \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^7}\right\}, A_1 = \{1,5,6\}, A_2 = \{2,3,4,7\}$
$p(P) = \left\{\frac{1}{2^2} + \frac{1}{2^3}, \frac{1}{2^1}\right\}, B_1 = \{2,3\}, B_2 = \{1\}$

Step: 2 constructing a new word Q from p
$p(Q) = \left\{\frac{1}{2^{m+1}} + \frac{1}{2^{m+2}}, \frac{1}{2^m}\right\}, C_1 = \{m + 1, m + 2\}, C_2 = \{m\}$

Step: 3

For $m = 1, C_1 = \{2,3\} \nsubseteq A_1 = \{1,5,6\}$

For $m = 2, C_1 = \{3,4\} \nsubseteq A_1 = \{1,5,6\}$

For $m = 3, C_1 = \{4,5\} \nsubseteq A_1 = \{1,5,6\}$

For $m = 4, C_1 = \{5,6\} \subseteq A_1 = \{1,5,6\}, C_2 = \{4\} \subseteq A_2 = \{2,3,4,7\}$

Pattern occurs in T at 4^{th} position

For $m = 5, C_1 = \{6,7\} \nsubseteq A_1 = \{1,5,6\}$

2.1.2 Example:

If $T = abbcacabacbcaa$ is a text and $P = bca$ is a pattern over $\Sigma = \{a, b, c\}$

Step: 1 constructing GPV for the word T and P

$p(T) = \left\{\frac{1}{2^1} + \frac{1}{2^5} + \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}}, \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^8} + \frac{1}{2^{11}}, \frac{1}{2^4} + \frac{1}{2^6} + \frac{1}{2^{10}} + \frac{1}{2^{12}}\right\}, A_1 = \{1,5,7,9,13,14\}, A_2 = \{2,3,8,11\}, A_3 = \{4,6,10,12\}$

$p(P) = \left\{\frac{1}{2^3}, \frac{1}{2^1}, \frac{1}{2^2}\right\}, B_1 = \{3\}, B_2 = \{1\}, B_3 = \{2\}$

Step: 2 constructing a new word Q from p

$p(Q) = \left\{\frac{1}{2^{m+2}}, \frac{1}{2^m} + \frac{1}{2^{m+1}}\right\}, C_1 = \{m+2\}, C_2 = \{m\}, C_3 = \{m+1\}$

Step: 3

For $m = 1, C_1 = \{3\} \nsubseteq A_1 = \{1,5,7,9,13,14\}$

For $m = 2, C_1 = \{4\} \nsubseteq A_1 = \{1,5,7,9,13,14\}$

For $m = 3, C_1 = \{5\} \subseteq A_1 = \{1,5,7,9,13,14\}, C_2 = \{3\} \subseteq A_2 = \{2,3,8,11\}, C_3 = \{4\} \subseteq A_3 = \{4,6,10,12\}$ Pattern occurs in T at 3^{rd} position

For $m = 4, C_1 = \{6\} \nsubseteq A_1 = \{1,5,7,9,13,14\}$

For $m = 5, C_1 = \{7\} \subseteq A_1 = \{1,5,7,9,13,14\}, C_2 = \{5\} \nsubseteq A_2 = \{2,3,8,11\}$

For $m = 6, C_1 = \{8\} \nsubseteq A_1 = \{1,5,7,9,13,14\}$

For $m = 7, C_1 = \{9\} \subseteq A_1 = \{1,5,7,9,13,14\}, C_2 = \{7\} \nsubseteq A_2 = \{2,3,8,11\}$

For $m = 8, C_1 = \{10\} \nsubseteq A_1 = \{1,5,7,9,13,14\}$

For $m = 9, C_1 = \{11\} \nsubseteq A_1 = \{1,5,7,9,13,14\}$

For $m = 10, C_1 = \{12\} \nsubseteq A_1 = \{1,5,7,9,13,14\}$

For $m = 11, C_1 = \{13\} \subseteq A_1 = \{1,5,7,9,13,14\}, C_2 = \{11\} \subseteq A_2 = \{2,3,8,11\}, C_3 = \{12\} \subseteq A_3 = \{4,6,10,12\}$

Pattern occurs in T at 11^{th} position

For $m = 12, C_1 = \{14\} \subseteq A_1 = \{1,5,7,9,13,14\}, C_2 = \{12\} \nsubseteq A_2 = \{2,3,8,11\}$.

Therefore P occurs at 3rd and 11th position in T

3. Array Pattern Matching

3.1 Algorithm for pattern matching of array over $\Sigma = \{a_1, a_2, \dots, a_n\}$:

Let T be a $i \times j$ text array and P be a $p \times q$ array pattern over Σ

Step: 1

Constructing GPV for the words of each rows of T and P

$p(T_{u_m}) = (u_{m_1}, u_{m_2}, \dots, u_{m_n}), m = 1,2, \dots, i$ where $u_{m_k} = \sum_{l_k \in A_k} \frac{1}{2^{l_k}}, A_k \subset N$(set of natural numbers) and A_k contains the position of $a_k (k = 1,2, \dots, n)$ in T_{u_m}.

$p(P_{v_r}) = (v_{r_1}, v_{r_2}, \dots, v_{r_n}), r = 1,2, \dots, p$ where $v_{r_k} = \sum_{l_k \in B_k} \frac{1}{2^{l_k}}, B_k \subset N$(set of natural numbers) and B_k contains the position of $a_k (k = 1,2, \dots, n)$ in P_{v_r}.

Step: 2
Constructing a new word Q from P
$w_{r_k} = \sum_{l_k \in B_k} \frac{1}{2^{l_k}}, C_k \subset N$ (Set of natural numbers) and C_k contains the position of a_k's $(k = 1,2, \dots, n)$ in $P_{v_r} + (s-1)$. Where s is a constant $\in N$. $p(Q_{w_r}) = (w_{r_1}, w_{r_2}, \dots, w_{r_n})$

Step: 3
For $p(T_{u_d}), p(Q_{w_r})$ where $d = \{1,2, \dots, i-p+1\} \subset m$
For $d = 1, r = 1$
If $\forall\ s = \{1,2, \dots, j-p+1\}$, $C_k \not\subseteq A_k$ for any $k = \{1,2, \dots, n\}$
Increment d by 1 and r=1
Repeat *
Otherwise for any $b \in s = \{1,2, \dots, j-p+1\}$, if $C_k \subseteq A_k \forall\ k = \{1,2, \dots, n\}$ *
$p(P_{v_r})$ occurs in $p(T_{u_d})$ at b position
Increment d and r by 1
Repeat * for s=b

3.1 Example:

If $T = \begin{bmatrix} a & b & a & b & a & a \\ b & a & b & a & b & b \\ a & a & b & b & a & a \\ b & a & a & b & b & b \\ a & b & a & b & b & a \\ b & b & b & b & b & b \end{bmatrix}$ is a 6×6 text matrix and $P = \begin{bmatrix} a & b & b \\ a & a & b \\ b & a & b \end{bmatrix}$ is a 3×3 pattern matrix

$P(T_{u_1}) = \left\{\frac{1}{2^1} + \frac{1}{2^3} + \frac{1}{2^5} + \frac{1}{2^6}, \frac{1}{2^2} + \frac{1}{2^4}\right\}, A_1 = \{1,3,5,6\}, A_2 = \{2,4\}$
$P(T_{u_2}) = \left\{\frac{1}{2^2} + \frac{1}{2^4}, \frac{1}{2^1} + \frac{1}{2^3} + \frac{1}{2^5} + \frac{1}{2^6}\right\}$ $A_1 = \{2,4\}, A_2 = \{1,3,5,6\}$
$P(T_{u_3}) = \left\{\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^6}, \frac{1}{2^3} + \frac{1}{2^4}\right\}$ $A_1 = \{1,2,5,6\}, A_2 = \{3,4\}$
$P(T_{u_4}) = \left\{\frac{1}{2^2} + \frac{1}{2^3}, \frac{1}{2^1} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6}\right\}$ $A_1 = \{2,3\}, A_2 = \{1,4,5,6\}$
$P(T_{u_5}) = \left\{\frac{1}{2^1} + \frac{1}{2^3} + \frac{1}{2^6}, \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^5}\right\}$ $A_1 = \{1,3,6\}, A_2 = \{2,4,5\}$
$P(T_{u_6}) = \left\{0, \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6}\right\} A_1 = \emptyset, A_2 = \{1,2,3,4,5,6\}$
$P(P_{v_1}) = \left\{\frac{1}{2^1}, \frac{1}{2^2} + \frac{1}{2^3}\right\} B_1 = \{1\}, B_2 = \{2,3\}$
$P(P_{v_2}) = \left\{\frac{1}{2^1} + \frac{1}{2^2}, \frac{1}{2^3}\right\} B_1 = \{1,2\}, B_2 = \{3\}$
$P(P_{v_3}) = \left\{\frac{1}{2^2}, \frac{1}{2^1} + \frac{1}{2^3}\right\} B_1 = \{2\}, B_2 = \{1,3\}$
$P(P_{w_1}) = \left\{\frac{1}{2^s}, \frac{1}{2^{s+1}} + \frac{1}{2^{s+2}}\right\} C_1 = \{s\}, C_2 = \{s+1, s+2\}$
$P(P_{w_2}) = \left\{\frac{1}{2^s} + \frac{1}{2^{s+1}}, \frac{1}{2^{s+2}}\right\} C_1 = \{s, s+1\}, C_2 = \{s+2\}$
$P(P_{w_3}) = \left\{\frac{1}{2^{s+1}}, \frac{1}{2^s} + \frac{1}{2^{s+2}}\right\} C_1 = \{s+1\}, C_2 = \{s, s+2\}$

For $P(T_{u_1}), P(Q_{w_1})$
For s=1, $C_1 = \{1\} \subseteq A_1 = \{1,3,5,6\}, C_2 = \{2,3\} \not\subseteq A_2 = \{2,4\}$
For s=2, $C_1 = \{2\} \not\subseteq A_1 = \{1,3,5,6\}$
For s=3, $C_1 = \{3\} \subseteq A_1 = \{1,3,5,6\}, C_2 = \{4,5\} \not\subseteq A_2 = \{2,4\}$
For s=4, $C_1 = \{4\} \not\subseteq A_1 = \{1,3,5,6\}$

For $P(T_{u_2}), P(Q_{w_1})$
For s=1, $C_1 = \{1\} \nsubseteq A_1 = \{2,4\}$,
For s=2, $C_1 = \{2\} \subseteq A_1 = \{2,4\}, C_2 = \{2,3\} \nsubseteq A_2 = \{1,3,5,6\}$
For s=3, $C_1 = \{3\} \nsubseteq A_1 = \{2,4\}$
For s=4, $C_1 = \{4\} \subseteq A_1 = \{2,4\}, C_2 = \{5,6\} \subseteq A_2 = \{1,3,5,6\}$
$P(P_{v_1})$Found in $P(T_{u_2})$ at s=4th position

For $P(T_{u_3}), P(P_{w_2})$
For s=4, $C_1 = \{4,5\} \nsubseteq A_1 = \{2,4\}$

For $P(T_{u_3}), P(Q_{w_1})$
For s=1, $C_1 = \{1\} \subseteq A_1 = \{1,2,5,6\}, C_2 = \{2,3\} \nsubseteq A_2 = \{3,4\}$
For s=2, $C_1 = \{2\} \subseteq A_1 = \{1,2,5,6\}, C_2 = \{3,4\} \subseteq A_2 = \{3,4\}$
For s=3, $C_1 = \{3\} \nsubseteq A_1 = \{1,2,5,6\}$
For s=4, $C_1 = \{4\} \nsubseteq A_1 = \{1,2,5,6\}$
$P(P_{v_1})$Found in $P(T_{u_3})$ at s=2nd position

For $P(T_{u_4}), P(P_{w_2})$
For s=2, $C_1 = \{2,3\} \subseteq A_1 = \{2,3\}, C_1 = \{4\} \subseteq A_2 = \{1,4,5,6\}$
$P(P_{v_2})$Found in $P(T_{u_4})$ at s=2nd position

For $P(T_{u_5}), P(P_{w_3})$
For s=2, $C_1 = \{3\} \subseteq A_1 = \{1,3,6\}, C_1 = \{2,4\} \subseteq A_2 = \{2,4,5\}$
$P(P_{v_3})$Found in $P(T_{u_5})$ at s=2nd position

For $P(T_{u_4}), P(Q_{w_1})$
For s=1, $C_1 = \{1\} \nsubseteq A_1 = \{2,3\}$
For s=2, $C_1 = \{2\} \subseteq A_1 = \{2,3\}, C_2 = \{3,4\} \nsubseteq A_2 = \{1,4,5,6\}$
For s=3, $C_1 = \{3\} \subseteq A_1 = \{2,3\}, C_2 = \{4,5\} \subseteq A_2 = \{1,4,5,6\}$
For s=4, $C_1 = \{4\} \nsubseteq A_1 = \{2,3\}$
$P(P_{v_1})$Found in $P(T_{u_4})$ at s=3rd position

For $P(T_{u_5}), P(P_{w_2})$
For s=3, $C_1 = \{2,3\} \nsubseteq A_1 = \{1,3,6\}$

Conclusion

In this paper the application of Generalised Parikh Vector is used as a combinatorial tool to identify occurences of string and array patterns in larger texts of strings and arrays respectively. The study has been extended to identify occurences of overlapping patterns and multiple occurences in the given texts. The scope of application includes identifying specific images in a larger image that are coded using a finite alphabet of symbols.

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Recognizability of Oxide Pictures by Oxide Wang Automata

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ABSTRACT: *In this paper we introduce a model of automata for the recognition of Oxide picture languages which is based on tiles and is called Oxide Wang automaton, as it depends on Oxide Wang systems. Oxide Wang automata assigns states to each position of the Oxide picture and the head moves from one position to an adjacent one according to some scanning strategy. We compare results of Oxide Wang automata with that of Oxide Tiling system and prove that Oxide Wang automata that recognize Oxide picture languages are equivalent to the Oxide tiling systems.*

Keywords: *Oxide picture languages, recognizable Oxide picture languages, Oxide Tiling system, Oxide Wang systems, Oxide Wang automaton.*

1. Introduction

Picture languages generalize string languages to two dimensions. A picture is a two-dimensional array of elements from a finite alphabet. Tiling systems were introduced in [1] defining local and recognizable picture languages. In each picture of the language, a specified set of square tiles is required to define local picture language. On the other hand, Wang system which is a formalism to recognize picture languages had been introduced in [10]. Pictures recognized by Wang systems and by Tiling Systems have been proved to be equivalent. Wang automaton were introduced in [11] based on the notion of Wang system.

The Oxide pictures, local Oxide picture languages, recognizable Oxide picture languages and Oxide Wang tiles were introduced and defined in [6]. Oxide Wang systems (OWS), a formalism to generate Oxide pictures were also introduced in [6] .

In this paper we present Oxide Wang automaton, a model to recognize Oxide picture languages, directed by polite scanning strategies. Studies have been done on Oxide Wang automata that recognize Oxide pictures.

2. PRELIMINARIES

The silicates are the important class of minerals whose basic chemical unit is SiO_4(a tetrahedral shaped anion. A ring of SiO_4 tetrahedron that are linked by shared oxygen nodes to other rings, forms a silicate sheet [12].An interconnection of fixed parallel silicate sheets forms a silicate network (Fig. 1.a). By deleting all the silicon nodes from a silicate network an Oxide Network (Fig. 1.b) is obtained [12][13].

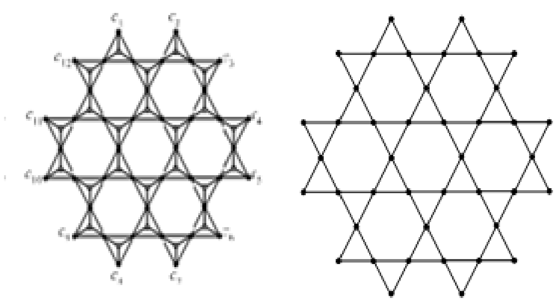

Fig. 1.Silicate and Oxide network

A coordinate system that is proposed for Oxide network is given in Fig.2.α = 0, β = 0,and γ = 0 are the three coordinate axes mutually at 120 degrees between any two of them and α-lines, β-lines, γ-lines are the lines parallel to the coordinate axes.

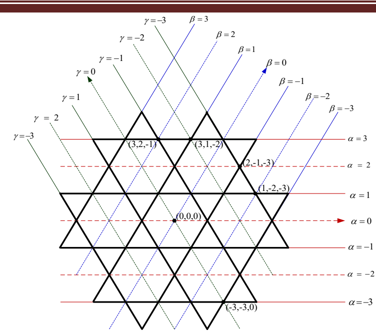

Fig.2. Coordinate System in Oxide Networks

Now we see some basic definitions of Oxide pictures, Oxide picture languages and Oxide Wang system from [6].

Definition 2.1: An Oxide picture OX_p is an oxide array of a finite element Σ. . An Oxide picture language over Σ is a subset of Σ^{**OX_p}. OX_p (n) denotes an Oxide picture of size n.

Example 2.1.
An Oxide picture over the alphabet {a, b, c} is shown in Fig. 4

a

c b c b

a a

b c b c

a

Fig. 3. An Oxide Picture

Definition 2.2: Consider a mapping $\pi : \Gamma \to \Sigma$, Γ and Σ are two set of finite alphabets. Let $OX_p \in \Gamma^{**OX_p}$ be an Oxide picture. The projection of OX_p is the Oxide picture $OX_p{}' \in \Sigma^{**OX_p}$ such that $\pi(OX_p(a,b,c)) = OX_p{}'(a,b,c)$

Definition 2.3: Consider an Oxide picture language $L \subseteq \Gamma^{**OX_p}$. The projection by $\pi : \Gamma \to \Sigma$ of L is the language $L' = \{OX_p{}' / OX_p{}' = \pi(OX_p), \forall OX_p \in L\} \subseteq \Sigma^{**OX_p}$

Definition 2.4: Let Γ be a finite alphabet and $L \subseteq \Gamma^{**OX_p}$ be its Oxide Picture language. An Oxide picture language L is local if there exists a finite set of Star of David tiles over $\Gamma \cup \{\#\}$

Definition 2.5: Let $OX_p \in \Sigma^{**OX_p}$,we get the boundary of an Oxide picture OX_p say $\hat{OX}_p$ when the special symbols $\# \notin \Sigma$ are added to the Oxide picture.

Definition 2.6: An Oxide picture language $L \subseteq \Sigma^{**OX_p}$ is called recognizable Oxide picture language if there exists a local Oxide picture language L' (given by a set of Star of David tiles) over an alphabet Γ and a projection $\pi : \Gamma \to \Sigma$ such that $L = \pi(L')$

Definition 2.7: An oxide tiling system $OXTS$ is a 4 tuple $(\Sigma, \Gamma, \pi, \theta)$ where Γ and Σ are two set of finite element, θ is a set of Star of David tiles over the alphabet $\Gamma \cup \{\#\}$ and the mapping $\pi : \Gamma \to \Sigma$ is a projection.

Definition 2.8: An oxide picture Language L is tiling recognizable if there exists an oxide tiling system such that $L = \pi(L(\theta))$

Definition 2.9: A labeled Oxide Wang tile consists of labeled triangular Wang tile(A tile and V tile) of three colors, a labeled hexagonal Wang tile of 6 colors and a label from a finite element set Σ. The colors are placed at right (R), left (L), and horizontal (H) edges of the labeled triangular Wang tile and at the lower horizontal (LH), upper horizontal (UH), upper right (UR), upper left (UL), lower right (LR), lower left (LL) edges of the hexagonal Wang tile (fig. 4)

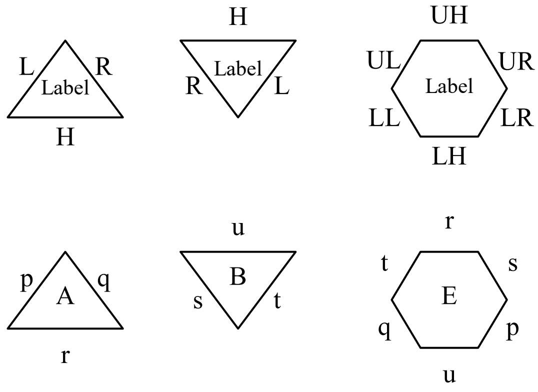

Fig. 4. Triangular and Hexagonal Wang tiles

A triangular Wang tile and a hexagonal Wang tile may be adjacent if and only if the adjacent positions are of the same color and no two triangular tiles(A tile and V tile) are adjacent.

Definition 2.10:

An Oxide Wang system is a triplet (Σ, Q, T_θ), where Σ is a finite symbol, T_θ is a set of labeled Oxide Wang tiles and Q is a finite set of colors. The language generated by $OXWS$ is the language $L(OXWS) \subseteq \Sigma^{**OX(n)}$.

3 Oxide Scanning Strategy

Two-dimensional scanning strategies for rectangular pictures[10], triangular pictures and hexagonal pictures [9] were already discussed. The scanning strategies do not depend on the size or content of the input picture. It is completely based on some pre-established and fixed kinds of scanning strategy. In this section we explore the scanning strategies for Oxide pictures.

Definition 3.1: An Oxide scanning strategy is a family $\mu = \{\mu_{\alpha\times\beta\times\gamma} : \{1,2...\} \to domOXP\}$, $\mu_{\alpha\times\beta\times\gamma}$ is a partial function. $\mu_{\alpha\times\beta\times\gamma}$ is called the Oxide scanning function over the Oxide picture domain $\alpha \times \beta \times \gamma$.

An Oxide scanning strategy provides a method to visit positions in any Oxide picture domain: $\mu_{\alpha\times\beta\times\gamma}(i)$ is the position visited in a domain $\alpha\times\beta\times\gamma$ at time i.

Definition 3.2: An Oxide scanning strategy is said to be continuous if $\mu_{\alpha\times\beta\times\gamma}(i+1)$ is adjacent to $\mu_{\alpha\times\beta\times\gamma}(i)$ provided both are defined.

Definition 3.3: An Oxide scanning strategy is one-pass if each position in each domain is visited exactly once.

Definition 3.4: An Oxide scanning strategy is said to be blind if the strategy proceeds locally, by scanning adjacent positions and it cannot "see" neither the size of the picture nor its picture content,: it can only "feel" an already considered position and a border, when it reaches it.

Definition 3.5: An Oxide scanning strategy is called polite if it is blind and one-pass.

Given a position h, we use (h) to denote the set of 6 edges of shared triangular tiles adjacent to horizontal tile of h. Dirs is the set of directions { (the lower horizontal and lower right shared triangular tiles), (upper horizontal and upper right shared triangular tiles) , (upper horizontal and upper left shared triangular tiles), $\leftarrow$ (upper left and lower left shared triangular tiles), $\rightarrow$ (upper right and lower right shared triangular tiles)}. For every position h and d ∈ Dirs, the edge of h in direction d is given by h_d and the position adjacent to h in direction d is denoted by $h*d$.

Definition 3.6: The next-position function η is a partial function that chooses where to go next, i.e. the direction towards the position to visit next. For a given position h , there may be set of already considered edges of shared triangular tiles given by the set D of directions and d the "last considered" edge.

4 Oxide Wang Automata

We now formally introduce Oxide Wang automata and show that they are equivalent to Oxide tiling systems.

Definition 4.1: A μ -directed Oxide Wang automaton (μ -OXWA) is a tuple $(\Sigma, \tau, Q, \mu, F)$

Σ is a finite input symbol, Q is a finite set of colors over Colrs, $F \subseteq \Sigma_{6Q}$ is a set of full colorings over Colrs, $\tau : \Sigma \times Q \times Dirs \rightarrow 2^{\Sigma_{6Q}}$ is a partial function such that each star of David tile $\tau(A,d)$ extends A, μ is a blind Oxide scanning strategy such that $\tau(A,d) \neq 0$ implies $\eta(A,d)$ have not reached the border.

An Oxide Wang automaton can be seen as a one having a head that visits hexagonal position of the star of David tile of an Oxide picture, moves from a position to an adjacent one and colors at each step the edges of the star of David tile it is visiting (in a sense, the element of $Q\times Dirs$ are the states of the automaton). The head movements are lead by the Oxide scanning strategy μ, whereas the coloring operations performed by the automaton are determined by a finite control function τ. Since the Oxide scanning strategy μ is blind, the automaton visits the Oxide picture positions independently and the choice of colors to assign to the edges is nondeterministic. If no move is possible, the automaton halts. The input Oxide picture is accepted if there is a computation in τ such that the coloring of the final position is in F. If the computation is accepted, the automaton produces an Oxide Wang-tiled picture whose label is equal to the input Oxide picture.

Theorem 4.1:

For every polite Oxide scanning strategy μ , we have $L(\mu - OXWA) = OXTREC$.

Proof

We prove that for every polite Oxide scanning strategy μ , μ -directed Oxide Wang automata are equivalent to Oxide Wang systems.

Let $W = (\Sigma, \tau, Q\ ,\ \mu, F\ )$ be a μ -directed Oxide Wang automata that recognize an Oxide picture language L. Consider the labeled Oxide Wang tile as defined in 2.9. The color assigned by automaton for the borders and the path followed by the head of the automaton, corresponds to the Oxide scanning strategy. It can be easily verified that each Oxide Wang picture over θ corresponds to an accepting computation of OXWA. Hence L is the language generated by the Oxide Wang system OXWS.

Conversely let OXWS be an Oxide Wang system recognizing an Oxide picture language L. Then take any polite Oxide scanning strategy μ and define $\mu - OXWA$. It can be proved that the language recognized by OXTWA is L.

Conclusion

Oxide Wang automata is a natural model to define OXREC. Since polite scanning strategies are used in OXTWA, the content and the size of the Oxide picture is not needed and is an eye opener for image analysis.

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General OSSP With Release Dates to Minimize the Total Completion Time

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ABSTRACT: *Scheduling problem exists almost everywhere in real industrial world situations. In this paper, we are considered the open shop scheduling problem with release dates, when pre emption is not allowed. The aim of this paper is to test the algorithm namely Dynamic longest processing time-Dense schedule(DLPT – DS) developed by Jayakumar S and Meganathan R [15] for the objective of total completion time (TCT). Numerical experiment shows that our DLPT – DS algorithm performs better than the algorithm available in the literature.*

Keywords: *Open shop scheduling problem, heuristic, total completion time.*

1. Introduction
Scheduling is the allocation of resources over a period of time to perform a collection of tasks. If the jobs will have different process sequence, the problem is known as Job-shop scheduling problem. In Open shop scheduling problem (OSSP), jobs have no predetermined processing sequence. i.e., jobs can be processed in any conceivable order. The OSSP is similar to the job shop scheduling problem with the exemption that there are no precedence relations between the operations of each job. The OSSP has considerably larger solution space than the other scheduling problems (Flow- shop & Job –shop) and seems to receive less attention in the literature, although it is an important and universal problem.
Open shop scheduling problems can arise in many practical applications, such as automobile repair, in a network of diagnostic testing facilities in a hospital, satellite communications and quality control centres. The most common example for open shop scheduling problems that we can came across in our day to day life are the teacher-classes assignments, examination scheduling and railway reservation, etc.,
A schedule is pre-emptive, if the execution of any operation may arbitrarily often be interrupted and resumed at a later time. Otherwise it is known as non pre-emptive schedule. i.e., each operation is executed continuously from start to completion without interrupted. In this paper, we consider non-pre-emptive open shop scheduling problem with release dates for the objective of minimizing the total completion time. The rest of the paper is organised as follows: In section 2, we gave the statement of the problem; section 3 is devoted for literature review relevant to our objective; section 4 details the scope of the objective; in section 5, we gave our proposed algorithm followed by examples in section 6; results and discussion was given in section 7.

2. Statement of the Problem
In an OSSP, a set of n jobs $J_1, J_2, \dots, J_n$ has to be processed on a set of m machines $M_1, M_2, \dots, M_m$. Every job consists of m operations each of which must be processed on a different machine for a given processing time. The operations of each job can be processed in any order. At any time one operation can be processed on each machine, and almost one operation of each job can be processed. There is no precedence relation between the operations. All jobs are independent and continuously available for their process with respect to their release times. All machines are continuously available. The process of a job cannot be interrupted. There are infinite buffer between machines (i.e., a job needs a machine that is occupied it can wait indefinitely until the machine becomes idle again). There are no transportation times between machines. It is assumed that the processing times of all operations are assumed to be given in advance. $O(i,j)$ denotes operation of job j on machine i . The processing time of job j on machine i, $i = 1, 2, \dots m$ is denoted by $p(i,j)$. It is assumed that the processing times are bounded by P_{max} and $i.i.d.$ (independently and identically distributed) random variables. $R(i,j)$ is the starting time of operation $O(i,j)$ and the completion time of job j

on machine i is denoted by $C(i,j)$. For each job j_i, there may be given a release date $r_j \geq 0$ which is the earliest possible time when the first operation of this job may start. The objective is to find a sequence of jobs with the given processing times on each machine to minimize the total completion time.

3. Litreature Review

In the literature of OSSP, most of the attention has been paid to the minimization of makespan without considering release dates or due dates. But, only in few papers, total completion time objective is considered. Most of the researchers focus on the problem with the assumption that all jobs are available at time zero, whereas we consider the release dates of the jobs also for our problem.

Achugbue and Chin [1] proved that the two machine case of scheduling open shop to minimize mean flow time is strongly NP – hard which means that the optimal solution cannot be found in polynomial time. If all operations have unit processing times, the problem can be transformed to a special pre-emptive identical parallel machine scheduling problem which can be solved in polynomial time (Brucker et al. [8]. Lann et al. [17] developed a simple heuristic which is asymptotically optimal, based on ordering the jobs by the average processing time of the m operations required for each job, for the case of job overlaps. Liaw et al. [18] presented the heuristic and branch – and – bound algorithms to deal with the problem when the sequence of jobs is given on one machine. Brasel and Hennes [6] presented a new lower bound and heuristic for the pre-emptive OSSP to minimize the average completion time. For the single machine sequencing problem with tool changes to minimize TCT, Akturk, Ghosh and Gunes [2] focused on the performance of the SPT list scheduling heuristic and provided theoretical worst –case bounds on it. The problem of scheduling multi-operation jobs on a single machine to minimize TCT was studied by Cheng, Ng and Yuan [9]. They showed that the problem is NP – hard and showed that some special case of the problems can be solved in polynomial time.

Scheduling tool changes to minimize TCT under controllable processing time was first considered by Akturk, Ghosh and Kayan [3]. They establish an important result which helps to solve the problem exactly for upto 30 jobs using a MIP formulation. Some constructive algorithms, matching algorithms priority dispatching rules and insertion and appending procedures combined with beam search, were developed and discussed by Brasel et al. [7]. Mastrolilli et al. [19] studied the sum of weighted completion times in a concurrent open shop. They gave a primal – dual 2 – approximation algorithm for this problem and lower bounds for in approximable problem. Tang L and Bai D [23] considered the OSSP to minimize TCT. They developed a Shortest Processing Time Block (SPTB) heuristic for the special case of the problem i.e., the job number is the multiple of the machine number and extend the heuristic to solve the general problem. They proved that the heuristic is asymptotically optimal when the job number goes to infinity. Scheduling open shops with parallel machines to minimize TCT was considered by Naderi et al. [20]. They developed an effective MILP model and apply memetic algorithm to solve the problem. Zang Z H and Bai D [24] studied OSSP for minimizing the sum of quadratic completion time. For small scale problems, they presented a solution based on Lagrangian relaxation method. A feasible schedule for the open shop problem is called dense when any machine is idle if and only if there is no job which currently could be processed on that machine. This concept was introduced by Racsmany (cf Barany I, & Fiala T [5]) and it has been shown that the makespan of any dense schedule is almost twice the optimum makespan. Sotskov Y N & Egorova N G [22] considered a single machine scheduling problem with uncertain durations of the given jobs with the objective of minimizing the sum of the job completion times. They applied the stability approach to the considered uncertain scheduling problem using a relative perimeter of the optimality box as a stability measure of the optimal job permutation. Also they investigated properties of the optimality box and developed algorithms for constructing job permutations that have the largest relative perimeters of the optimality box. In this paper we considered the dense schedule with release dates to minimize TCT.

4. Objective and Scope of the Problem

Even though many researchers focused on makespan problems, from the practical point of view the TCT objective is more important than makespan criterion. In 2013, Bai D and Tang L [4] had developed the DSPT – DS (Dynamic Shortest processing time-Dense schedule) algorithm to the open shop scheduling problems with release dates to minimize makespan. In [16], we gave our attention to the general OSSP and hypothetical case problems and developed DLPT – DS algorithm which gives better makespan value than DSPT – DS algorithm. In this paper, we test the effectiveness of DLPT-DS heuristic algorithm for the TCT objective by comparing its TCT value with those value obtained by DSPT-DS heuristic algorithm.

5. Proposed Algorithm

In this section, we described our heuristic algorithm. Let $B = O(i, j)_{m \times j}, 1 \le j \le n$, denote the operations that are available at time $t, t \ge 0$ and $R(i, j)$ be the starting time of operation $O(i, j)$.

5.1 DLPT-DS heuristic [15]

Step 1. At time $t, t \ge 0$ process the operation with the longest processing time, say $O(i_1, j_1)$ among all the available ones in matrix B. If some operations simultaneously satisfy the condition, give preference to the operation with the smallest $O(i_1, j_1)$ index. Update the starting times of the operations, which are at the same column and row with $O(i_1, j_1)$, to $t + p(i_1, j_1)$in matrix B. Remove operation from matrix B.

Step 2. If some jobs arrive, go to step3; if matrix B becomes empty, go to step4.

Step 3. Sort the operations of the arrivals into matrix B, and update the starting time of each new operation to the longest starting time of its row in matrix B. Then go to Step1.

Step 4. Let the machines remain idle until a job arrives, and go to step 3 .If the scheduling is completed, terminate the program.

6. Examples

In this section we illustrate our algorithm by considering the 4 jobs 4 machines for the general OSSP with release dates.

Consider the problem of scheduling four jobs on four machines. The processing times of job $J_j, j = 1, 2, 3, 4$ on machine $M_i, i = 1, 2, 3, 4$ and the release dates r_j are given below.

	J_1	J_2	J_3	J_4
M_1	3	5	2	6
M_2	5	7	8	4
M_3	7	5	3	4
M_4	3	2	2	4
r_j	5	3	2	8

If we schedule the operations according to DSPT-DS, we got that the completion times of J_1, J_2, J_3, J_4 are 29, 24, 24, 26 which gives the TCT value as 103 units of time (See figure 6.1.1). Meanwhile if we use DLPT – DS algorithm, the completion times of J_1, J_2, J_3, J_4 are 23, 24, 20, 27 which gives the TCT value as 94 units of time (See figure 6.1.2).

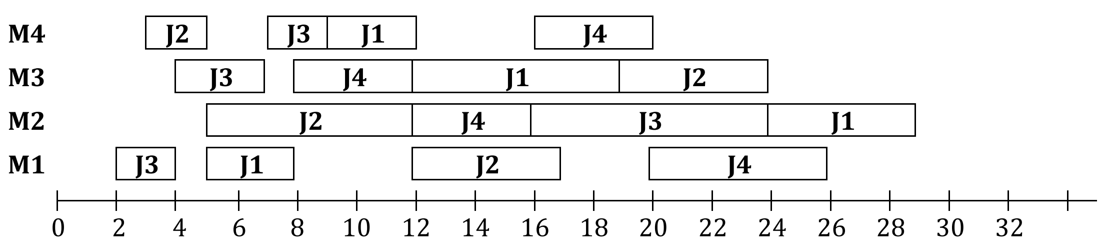

Fig. 6. 1.1. DSPT-DS Schedule for general OSSP

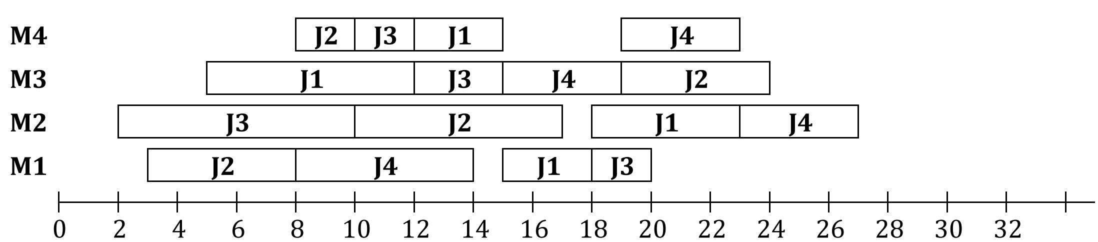

Fig. 6. 1. 2. DLPT-DS Schedule for general OSSP

7. Result and Discussion

To test the effectiveness of our DLPT- DS heuristic for the objective of TCT, comparison has been made between DLPT- DS and DSPT – DS. 20 problem instances were generated and tested for both the algorithms. The processing times of jobs on machines and release dates of jobs are randomly chosen from the numbers 1 to 9. At the outset, out of 20 problem instances, DLPT – DS algorithm performs better than DSPT – DS algorithm for about 13 problems and gives equal value for 2 problems. The results were shown in the following table 7.1. Computational result indicates that the DLPT-DS algorithm is better than DSPT-DS algorithm.

Table 7.1. TCT obtained for the OSSP with Number of Jobs equal to Number of Machines

Serial Number	Number of Jobs	Number of Machines	TCT obtained using DSPT – DS algorithm	TCT obtained using DLPT – DS algorithm
1	3	3	77	75
2	3	3	37	35
3	3	3	63	60
4	3	3	81	74
5	4	4	103	94
6	4	4	77	75
7	4	4	122	118
8	4	4	92	98
9	5	5	176	170
10	5	5	152	146
11	5	5	162	159
12	5	5	138	148
13	6	6	170	170
14	6	6	138	131
15	6	6	170	172
16	6	6	218	207
17	7	7	261	261
18	7	7	290	289
19	7	7	275	282
20	7	7	281	294

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Extensions of Liouville's Theorem

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ABSTRACT: *In this paper we first generalize the theorem of Liouville's into general forms based on the representation of power series for analytic functions. Secondly, harmonic functions can be identified as real parts of analytical functions in simply connected domains when we look at the relationship between analytical and harmonic functions we extend the theorem of Liouville's to harmonic functions by the Harnack's inequality.*

1. Introduction

If f (z) is analytic across entire complex plane, it is said to be a whole function. There is a lovely theorem known as Liouville's theorem for complete functions. It gives many important have been widely applied to complex analysis. And it allows us to prove algebra's basic theorem. In this paper, we first derive two extension form of the theorem of Liouville's by simplifying certain conditions in the classical theorem of Liouville's. In the meantime, we have two analytic results secondly, harmonic functions can be identified as real parts of analytical functions in simply connected domains and, for analytical functions there are many consequences some of these are the infinite differentiate of analytical functions, the theorem of Liouville's and the maximum theorem of modulus, consequently we believe these results have analog for harmonic functions. We extend the theorem of Liouville's to harmonic functions as far as ideas are concerned. The theorems of Liouville's generalized provide us with a theoretical basis for further studying the properties of all functions and harmonic functions.

Theorem: (Harnack's inequality)
If u(z) be a positive harmonic function defined in the disc $|z| < \rho$ *and* if $|z| = r < \rho$ then the Harnack's inequality is

$$\frac{\rho-r}{\rho+r}\, u(0) \leq u(z) \leq \frac{\rho+r}{\rho-r} u(o)$$

Proof
By using poisson's formula

$$U(z) = \frac{1}{2\pi}\int_0^{2\pi} \frac{\rho^2-r^2}{|\rho e^{i\theta}-z|^2} u\left(\rho e^{i\theta}\right) d\theta \qquad \text{----------------- (1)}$$

Consider the inequality

$$\rho - r \leq |\rho e^{i\theta} - z| \leq \rho + r$$

$$\frac{1}{\rho-r} \geq \frac{1}{|\rho e^{i\theta}-z|} \geq \frac{1}{\rho+r}$$

Taking square on both sides

$$\frac{1}{(\rho-r)^2} \geq \frac{1}{|\rho e^{i\theta}-z|^2} \geq \frac{1}{(\rho+r)^2}$$

Multiply by $\rho^2 - r^2$

$$\frac{\rho^2-r^2}{(\rho-r)^2} \geq \frac{\rho^2-r^2}{|\rho e^{i\theta}-z|^2} \geq \frac{\rho^2-r^2}{(\rho+r)^2}$$

$$\frac{\rho+r}{\rho-r} \geq \frac{\rho^2-r^2}{|\rho e^{i\theta}-z|^2} \geq \frac{\rho-r}{\rho+r}$$

$$\frac{\rho-r}{\rho+r} \leq \frac{\rho^2-r^2}{|\rho e^{i\theta}-z|^2} \leq \frac{\rho+r}{\rho-r} \qquad \text{----------------- (2)}$$

From (1) substitute second- half of equation (2)
U(z)$\leq \frac{1}{2\pi}\frac{\rho+r}{\rho-r}\int_0^{2\pi} u(\rho e^{i\theta})d\theta$ ------------------ (3)

Using the first-half in equation (2)
U(z)$\geq \frac{1}{2\pi}\frac{\rho-r}{\rho+r}\int_o^{2\pi} u(\rho e^{i\theta})d\theta$

$\frac{1}{2\pi}\frac{\rho-r}{\rho+r}\int_0^{2\pi} u(\rho e^{i\theta})d\theta \leq u(z)$ ------------------ (4)

From (3) and (4)
$\frac{1}{2\pi}\frac{\rho-r}{\rho+r}\int_0^{2\pi} u(\rho e^{i\theta})d\theta \leq u(z) \leq \frac{1}{2\pi}\frac{\rho+r}{\rho-r}\int_0^{2\pi} u(\rho e^{i\theta})d\theta$ -------------------- (5)

By mean value property,
U(0) = $\frac{1}{2\pi}\int_0^{2\pi} u(\rho e^{i\theta})d\theta$ --------------------- (6)

(5) ====> $\frac{\rho-r}{\rho+r}$u(0)$\leq u(z) \leq \frac{\rho+r}{\rho-r}u(0)$

Hence the proof is completed

The Generalized Liouville's Theorems

In this section, we consider the extension problem of classical Liouville's theorem. This is the main work of this paper. Firstly, we obtain two extension forms of Liouville's theorem by reducing the condition of classical Liouville's theorem. These theorems are stated below.

Theorem: (Liouville's theorem)

A function which is analytic and bounded in the whole plane must reduce to a constant. (i.e.), Bounded entire function is constant.

Proof

Let c be the circle, |z-a|$< r$
Let f(z) be the bounded entire function
∴ |f(z)|$\leq M$

By cauchy's inequality
$| f^n(a) | \leq \frac{M.n!}{r^n}$

Put n=1,
$|f'(a)| \leq \frac{M}{r}$

Since in this equality is true for all circle in the plane.
Taking limit on both sides, r $\to \infty$
$\lim_r \infty \mid f'$ (a)|$\leq \lim_r \infty \frac{M}{r} = 0$
$|f'(a)| = 0$
$f'(a) = 0$
$f'(z) = 0$, for all z in the whole plane

Taking integration,
f(z) = a is a constant

Hence the bounded entire function is constant.

Theorem: (Liouville's theorem for harmonic function)

Let ϕ be harmonic in the whole plane R^2 and bounded from above or below, then ϕ is constant.

Proof

Let us assume that ϕ is bounded from above, then there exists a constant M. such that ϕ$\leq M$ for any z $\in R^2$

Clearly, U = M - ϕ is harmonic and non-negative in the whole real plane.

Using the mean value theorem for harmonic function. For 0$\leq r < R < +\infty$

We have,

U(0)$\frac{\rho-r}{\rho+r} \leq U(\rho e^{i\theta}) \leq U(0)\frac{\rho+r}{\rho-r}$, $\rho \to \infty$ where r $\in [0, +\infty)$

Let us see that r is an arbitrary non-negative real number.

Hence U = M - ϕ is constant.

∴ ϕ is constant.

Hence the proof is completed.

Conclusion

Summing up, we have concluded the Liouville's theorem by simplifying some conditions. We have a tendency to extend Liouville's theorem to harmonic functions by Harnock's difference. In this topic we all come to know that the generalized Liouville's theorem. Meanwhile, we will get some vital conclusion for harmonic function.

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The Mapping of Bernhard Riemann Theorem in Complicated Analysis

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ABSTRACT: *During this technique we are going to prove the Bernhard Riemann mapping theorem by a limiting procedure. We are going to then ought to understand that the sequence of mapping made, or a minimum of a subsequence of it's a limit. To envision this, the sequence has to have a compactness property, analogues to the 'Bolzano-weierstrass' theorem for sequence of numbers. The acceptable conception is given by following definition.*

Keywords: *The Bernhard Riemann mapping theorem in complicated analysis, behavior use of the reflection principle, Schwartz's and cristoffel formula.*

1. Introduction

A mapping ϕ: Ω—› ∑ is named biholomorphic onto ∑, or conformal, if ϕ€O (Ω) and ϕ is invertible with ϕ-1€O(∑). If there's a conformal mapping between Ω and ∑, then they're a fore said to be conformally equivalent domains. Biholomorphic mappings square measure vital as a result of they'll be accustomed "plant" complicated analysis of 1 domain onto another domain: within the setting of the continuing definition, if f€O (∑) then f o ϕ €O (Ω), and if g€O (Ω) then goϕ-1€O (∑).
Conformal equivalence is associate equivalence relation on the area of all domains then in theory it suffices to check one representative from every of these equivalence categories is named conformal categories. Therefore, distinguishing conformal categories could be a elementary draw back in complicated analysis. The Bernhard Riemann mapping theorem characterizes the conformal category of the unit disk d= Z, that seems to be the gathering of all merely connected domains. With in the following, we have a tendency to discuss the Bernhard Riemann mapping theorem and a few alternative topics associated with it.

Definition

A family F of analytic perform outlined on a district Ω named is traditional if each sequence of functions in F contains a subsequence of domestically uniformly focused in Ω.

Theorem:1

A family F of functions analytic on a district Ω is traditional if and providing it's domestically equibounded.

Definition

A family F of complicated valued functions outlined in an exceedingly complicated regio Ω is named domestically equicontinuous i f for each ∈>0 and compact set E of Ω there's a δ>0 such |f(z)-f(w)| for each f∈ F and every one z, we satisfying |z-w|<δ.

Theorem:(Arzela-Ascoli)

Suppose $f_1,f_2,\ldots$ could be a sequence of complex-valued functions outlined on a district ΩcC, and assume the sequence is domestically equibounded and equicontinuous in Ω. Then there's domestically uniformly focused subsequence.

Proof

he set of points in Ω with rational real and fanciful elements is enumerable and dense in Ω.
That the set is enumerable implies that there's a sequence z_1, z_2,... consisting exactly of those points, which it's dense implies that any neighborhood of any purpose in Ω contains some extent from the sequence z_1,z_2,.... take into account currently the sequence $f_1(z_1)$, $f_2(z_2)$, $f_3(z_3)$,... of complicated numbers. This is often a finite sequence since the set is compact.
We know that ,"In arithmetic, specifically in real analysis, could be a elementary result regarding convergence in an exceedingly finite-dimensional Euclidean space R^n.

The theory states that every finite sequence in R^n contains a focused subsequence"
Therefore by the 'Bolzano-Weierstrass' theorem it's a focused subsequence, given by evaluating a sub-sequence $f_{11},f_{12},f_{13},...$ of $f_1,f_2,...$ at z_1; decision the limit $f(z_1)$.
The sequence $f_{11}(z_2),f_{12}(z_2),f_{13}(z_2),...$ is once more finite, therefore we are able to find a subsequence $f_{11},f_{22},f_{23},...$ of $f_{11},f_{12},f_{13},...$ that converges once evaluated at z_2;decision the limit $f(z_1)$.
Since a subsequence of a focused sequence converges to a similar issue because the sequence itself, we have a tendency to still have $f_{2n}(z_1) = f(z_1)$.
Continued during this fashion we have a tendency to get a sequence of sequences $f_{k1},f_{k2},f_{k3},...$, $k = 1,2,...$ such every sequence could be a subsequence of those returning before it,
and such that
$\lim_{n\to\infty} f_{kn}(z_j) = f(z_j)$ for $j \le k$.
Currently take into account the 'diagonal sequence' $f_{11},f_{22},f_{33},....$ this is often a subsequence of the sequence $f_{j1}, f_{j2}, f_{j3},...$ from its j:th part forward,
Therefore $\lim_{k\to\infty} f_{kk}(z_j) = f(z_j)$ for any j.
We have a tendency to shall finish the proof by showing that actually $f_{11},f_{22},f_{33},...$ converges domestically uniformly on Ω.
Let a compact set E of Ω and variety ε > zero incline.
By native equicontinuity we are able to then find δ > zero
so $|f_{nn}(z) - f_{nn}(w)| < \varepsilon/3$ for $z, w \in E$ and $|z - w| < \delta$.
Currently take into account the open cowl of E given by the balls of radius δ and focused at $z_{j,j} = 1, 2,....$ this is often a canopy since $z_1,z_2,...$ is dense in Ω.
We all know that,"If a collection of real numbers is closed and finite, then the set is compact.
That is, if a collection of real numbers is closed and finite, then each open cowl of the set contains a finite subcover"
By the Heine-Borel theorem there's a finite range of balls, say focused at $z_1, z_2,..., z_k$ that already cowl E.
Given $z \in E$ we are able to so find z_j with $j \le k$ such $|z - z_j| < \delta$ and
thus get

$$|f_{nn}(z) - f_{mm}(z)|$$
$$\le |f_{nn}(z) - f_{nn}(z_j)| + |f_{nn}(z_j) - f_{mm}(z_j)| + |f_{mm}(zj) - f_{mm}(z)| < \varepsilon/3 + |f_{nn}(z_j) - f_{mm}(z_j)|+\varepsilon/3.$$

By Cauchy's convergence principle (for complicated numbers) and our construction it follows that for each j there's variety Garden State
Such that $|f_{nn}(z_j) - f_{mm}(z_j)| < \varepsilon/3$ if $n, m > N_j$.
If we elect N because the largest of $N_1,...,N_k$ it follows that

$$|f_{nn}(z) - f_{mm}(z)| < \varepsilon \text{ if n and } m > N.$$

Using the opposite direction of Cauchy's convergence principle it follows that
$f(z) = \lim_{n\to\infty} f_{nn}(z)$ exists for each $z \in \Omega$, and belongings $m \to \infty$

within the expression on top of we have a tendency to
get $|f_{nn}(z) - f(z)| \le \varepsilon$ for each $z \in E$ if $n > N$.
This shows that $f_{nn} \to f$ domestically uniformly in Ω.

Definition: (Riemann mapping theorem)
In complicated analysis, the Bernhard Riemann mapping theorem mapping theorem states that if U could be anon-empty merely connected open set of the imaginary number plane C that isn't all of C, then there exists a biholomorphic mapping f from U onto the open unit disk this mapping is understood as a Bernhard Riemann mapping.

THEOREM: (Riemann mapping theorem).
Given a merely connected region Ω that isn't the whole complicated plane C and some extent $z_0 \in \Omega$ there's exactly one univalent conformal map f of Ω onto the unit disk such $f(z_0)$ = zero and $f(z_0)$ > zero. Note that

Liouville's theorem shows that it's out of the question to map the whole plane C conformally onto the unit disk; the sole finite entire functions square measure the constants.

Proof:
we've got already tested the distinctiveness in associate when Schwarz's lemma.

We know that,
"If f(z) is analytic for $|z|<1$ and satisfies the condition $|f(z)|$, $f(0)=0$.

Then $|f(z)|\le|z|$ and $|f^1(0)|\le1$.
Then $f(z)=cz$ with a continuing c for definite quantity 1".
To envision the way to get existence, note that if g solves the matter and f could be a map of Ω into the unit disk mapping z_0 onto zero and with positive by-product at z_0,
then $f\circ g-1$ satisfies the conditions of Schwarz's lemma
therefore $|(f\circ g-1)0(0)|\le$ one.
Scheming the by-product we have a tendency to see that this
implies that $f0(z_0)\le g0(z_0)$.
If we've got equality it follows from Schwarz's lemma that $f\equiv g$.
currently let F be the family of univalent functions f analytic in Ω
such $f(z_0)$ = zero, $|f(z)|\le$ one for $z\in\Omega$ and $f0(z_0)$ > zero.
We have a tendency to simply saw that if our drawback contains a answer it's the part of F that maximizes the by-product at z_0.

To finish the proof on these lines we'd like to:
(1) Show that F isn't empty,
(2)See that F has part f increasing the by-product at z_0 and, finally,
(3) Show that this f truly solves the mapping drawback.
(1) Since Ω isn't all of C there's some extent $a\notin\Omega$.
Since Ω is just connected we are able to define a single-valued branch h of $\sqrt{z-a}$ in Ω.
Clearly h cannot take the worth w if it somewhere takes the worth w.
however by the open mapping theorem there's a disk$|w-h(z_0)|<\rho$ contained within the image h(Ω).
It follows that $|h(z)+h(z_0)|\ge\rho$ for all $z\in\Omega$; above all $2|h(z_0)|\ge\rho$.

The perform
$\frac{h(z)-h(z_0)}{h(z)+h(z_0)} = h(z_0)\left(\frac{1}{h(z_0)}-\frac{2}{h(z)+h(z_0)}\right)$
maps z_0 to zero and is finite by $4|h(z0)|/\rho$.
Its by-product at z_0 is $\frac{h^1}{2h(z_0)}$.
If we have a tendency to currently place
$g(z) = \frac{\rho}{4}\frac{|h^1(z_0)|}{|h(z_0)|^2}\frac{h(z_0)}{h^1(z_0)}\frac{h(z)-h(z_0)}{h(z)+h(z_0)}$

It follows that g is univalent, $g(z_0)=0$, $|g(z)|\le1$, and $g0(z_0)$ > zero so $g\in F$.
therefore F6= ∅.
(2) Since all components of F have their values within the unit disk it follows that F is associate equibounded family.
Currently let $B=\sup_{f\in F} f0(z_0)$ so zero $< B\le\infty$.
We are able to then find a sequence $f_1,f_2,\ldots$ in F so $f0\,j(z_0)\to B$ as $j\to\infty$.
Since F is traditional we are able to find a domestically uniformly focused subsequence; decision the limit perform f.
it's then clear that $f0(z_0)=B$ so truly $B<\infty$ and f isn't constant.
It's clear that $f(z_0)$ = zero and f has its values within the closed unit disk; however by the open mapping theorem the values square measure then within the open unit disk.
(3) We had like to prove that f(Ω) is that the unit disk.
Suppose to the contrary that w0 is within the unit disk however $w_0\notin f(\Omega)$.

Since Ω is just connected we have a tendency to could define a single-valued branch of G (z) = $\sqrt{\frac{f(z)-w_0}{1-\overline{w}_0 f(z)}}$

Since the Mobius remodel w → $\frac{w-w_0}{1-\overline{w_0}w}$ preserves the unit disk, the perform G maps Ω univalently into the unit disk.

To get a member of F we have a tendency to currently set

F(z) =$\frac{|G^1(z_0)|}{G^1(z_0)}\frac{G(z)-G(z_0)}{1-\overline{G(z_0)}G(z)}$

It's once more clear that F has its values within the unit disk and maps z0 to zero. The by-product at z_0 is definitely calculated to be F0(z_0) = B$\frac{|1+G(z_0)|^2}{2|G(z_0)|} > B$. so F ∈ F.

however that is that the contradiction of the definition B.

Note that it's no accident that we have a tendency to get F0(z_0) > f0(z_0) ;

this simply expresses the actual fact that the inverse of the map f → F takes the unit disk into itself with zero fixed so Schwarz's lemma shows that the by-product at zero is less then one.

Conclusion

Some vital applications, corollaries and uses of the Bernhard Riemann mapping theorem square measure as follows:

The uniformisation theorem states that if could be a merely connected open set of the Bernhard Riemann surface, than is biholomorphic to the Bernhard Riemann sphere, the complicated plane, or the unit disk.

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THE New Technic of Simplex Method of Linear Programming Problems

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ABSTRACT: *In this paper discuss the Simplex method is associate approach to solving linear programming models by victimization slack and surplus variables, tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem. The Simplex method could be a technique for determination linear programs. This paper in the main focuses to solve a Simplex method for solving an linear programming problem to attain the optimal solution.*

Keywords: *Linear programming problems, Simplex method, Optimal, Feasible.*

1. Introduction

This method invited by George Dantzig, tests adjoint vertices of the feasible set in sequences that at every new vertex objective function improves or is unchanged. This method is used for calculating the optimal solution to the linear programming problem. A linear program could be a method of achieving the most effective outcome given a maximum or minimum equation with linear constraints.

Linear Programming Problem:

Linear programming deals with the optimization (maximization or minimization) of a function of variable known as objective function, subject to a set of linear quantities and (or inequalities known as constraints).

General Linear Programming Problems:

Max (or) Min $z = c_1x_1+c_2x_2+\ldots\ldots +c_nx_n$

Subject to $a_{11}x_1+a_{12}x_2+\ldots\ldots\ldots\ldots+a_{1n}x_n \le$ (or) = (or) $\ge b_1$

$a_{21}x_1+a_{22}x_2+\ldots\ldots\ldots\ldots+a_{2n}x_n \le$ (or) = (or) $\ge b_2$

$a_{31}x_1+a_{32}x_2+\ldots\ldots\ldots\ldots+a_{3n}x_n \le$ (or) = (or) $\ge b_3$

.

.

$a_{m1}x_1+a_{m2}x_2+\ldots\ldots\ldots+a_{mn}x_n \le$ (or) = (or) $\ge b_n$

Some Important Definitions

Solution

A set of variables $[x_1,x_2\ldots x_n]$ is called a solution to LPP is satisfies the constraints.

Feasible Solution

Any solution to a LPP which satisfies the non negative restriction is called feasible solution.

Optimum (Or) Optimal Solution

Any feasible solution which optimizes (maximizes (or) minimize) the objective function is called its optimum (or) optimal solution.

Slack Variable

Let the constraints of a LPP be $\sum_{j=1}^{n} a_{ij}x_j \ge b_i$, i = 1, 2, 3,……….,k then the non-negative variable s_i which are introduced to convert the inequalities into equalities $\sum_{j=1}^{n} a_{ij}x_j + s_i = b_i$ are called slack variable.

Surplus Variable

Let the constraints $\sum_{j=1}^{n} a_{ij}x_j \ge b_i$ then subtracted s_i from left hand side to convert inequalities to equalities. This variable is called surplus variable. (i.e.)., $\sum_{j=1}^{n} a_{ij}x_j - s_i = b_i$

Canonical Form

(1) Objective function is of maximization type.
(2) All constraints expressed all equations.
(3) Right hand side of each constraint is non-negative.

Simplex Method
Given the idea of the Simplex method of two elementary conditions.
(1)The feasibility condition : It ensures that if the begining solution is basic feasible, only basic feasible solutions will be obtained during computation.
(2)The optimality condition: It guarantees that solely higher solutions (as compared to this solution) will be encountered.

Simplex Method Algorithm:
Step 1
Convert the problem into maximization type.
(ie)., min z = -max(-z).

Step 2
Check whether all b_i's are negative.
If negative, multiply (-1) both sides.

Step 3
Introduce slack / surplus variable to convert the problem into standard form.

Step 4
Form the simplex table.

		C_j	C_1	C_2	C_3	………	0	0	0	…….
C_β	Y_β	X_β	X_1	X_2	X_3	………	s_1	s_2	s_3	………
C_{B1}	S_1	b_1	a_{11}	a_{12}	a_{13}	………	1	0	0	………
C_{B2}	S_2	b_2	a_{21}	a_{22}	a_{23}	………	0	1	1	………
C_{B3}	S_3	b_3	a_{31}	a_{32}	a_{33}	………	0	0	0	……
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
z_j-c_j			z_1-c_1	z_2-c_2	z_3-c_3	………	……	…..	….	……

C_j – co-

efficient of objective function.
C_β – co-efficient of basic variables. S in the objective function.
Y_β – Basic variables.
X_β – Values of basic variables.
$z_{j\text{-}Cj}$ – Net evaluation (or) Index for each column.

Step 5
Calculate z_j-c_j = $c_B a_j$-c
If all z_j-$c_j \geq 0$, the current solution is optimal, otherwise go to next step.

Step 6
To find entering variable:

Entering variable is non-basic corresponding to most negative values of z_j-c_j.

Step 7

To find leaving variable:

Find θ = min (x_{Bi} divided by a_{ij}, $a_{ij} > 0$.

(1) If all $a_{ir} \leq 0$ then there is an unbounded solution.

(2) If at least one $a_{ir} > 0$ then leaving variable is the basic variable corresponding to minimum ratioθ.

Key column: Entering column.
Key row: leaving row,
Pivot element: The element of intersection of pivot row and pivot column.

Step 8

New pivot equation = old equation – (corresponding column element) x New pivot row.

Step 9

Continue the process until all z_j-c_j are positive.

Example

Solve the LPP max $z = 12x_1+15x_2+14x_3$.
Subject to, $-x_1+x_2 \leq 0$,
$-x_2+2x_3 \leq 0$,
$x_1+x_2+x_3 \leq 100$ and $x_1,x_2,x_3 \geq 0$.

Solution

Max $z = 12x_1+15x_2+14x_3+0s_1+0s_2+0s_3$.
Subject to constraints,
$-x_1+x_2+s_1= 0$
$-x_1+2x_2+s_2 = 0$
$x_1+x_2+x_3+s_3 = 100$ and $x_1,x_2,x_3,s_1,s_2,s_3 \geq 0$
Basic solution: Put $x_1 = x_2 = x_3 = 0$
$s_1 = 0$, $s_2 =0$, $s_3 = 0$.

TABLE 1

			c_j	12	15	14	0	0	
C_β	Y_β	X_β	x_1	x_2	x_3	s_1	s_2	s_3	θ
0	s_1	0	-1	1	0	1	0	0	0→
0	s_2	0	-1	0	2	0	0	0	∞
0	s_3	100	1	1	1	0	1	1	100
Z_j-C_j		0	-12	-15 ↑	-14	0	0	0	

x_2 – entering variable.
s_1 – leaving variable.
1 – Pivot element.
New pivot equation = old equation ÷ pivot element.
New pivot equation = (0,-1,1,0,1,0,0).
New other equation = old equation – (corresponding column element) x new pivot equation.
New s_2 equation = (0,-1,0,2,0,0,0)-0(0,-1,1,0,1,0,0) = (0,-1,0,2,0,0,0).
New s_3 equation = (100,1,1,1,0,1,1)-1(0,-1,1,0,1,0,0) = (100,2,0,1,-1,0,1).

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FIRST ITERATION:

		c_j	12	15	14	0	0	0	
C_β	Y_β	X_β	x_1	x_2	x_3	s_1	s_2	s_3	θ
15	x_2	0	-1	1	0	1	0	0	-ve
0	s_2	0	-1	0	2	0	0	0	-ve
0	s_3	100	2	0	1	-1	0	1	50→
Z_j-C_j		0	-27 ↑	0	-14	15	0	0	

x_1 – entering variable.
s_3 – leaving variable.
2 – Pivot element

New pivot equation = (50, 1, 0, $\frac{1}{2}$, - $\frac{1}{2}$, 0, $\frac{1}{2}$).

New x_2 equation = (50, 0, 1, $\frac{1}{2}$, $\frac{1}{2}$, 0,1, $\frac{1}{2}$)

New s_2 equation = (50, 0, 1, $\frac{5}{2}$, $-\frac{1}{2}$, 1, $\frac{1}{2}$).

SECOND ITERATION

		c_j	12	15	14	0	0	0	
C_β	Y_β	X_β	x_1	x_2	x_3	s_1	s_2	s_3	θ
15	x_2	50	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	25
0	s_2	50	0	0	$\frac{5}{2}$	$\frac{-1}{2}$	1	$\frac{1}{2}$	$\frac{125}{2}$
12	x_1	50	1	0	$\frac{1}{2}$	$\frac{-1}{2}$	0	$\frac{1}{2}$	25
Z_j-C_j		1350	0	0	$\frac{-1}{2}$	$\frac{3}{2}$	0	$\frac{27}{2}$	

x_3 – entering variable.
x_2 – leaving variable.
Pivot element - $\frac{1}{2}$.

New pivot equation = (100, 0, 2, 1, 1, 0, 1).
New s_2 equation = (-200, 0, -5, 0, -3, 1, -2).
New x_1 equation = (0, 1, -1, 0, -1, 0, 0).

THIRD ITERATION:

		c_j	12	15	14	0	0	0
C_β	Y_β	X_β	x_1	x_2	x_3	s_1	s_2	s_3
14	x_3	100	0	2	1	1	0	1
0	s_2	-200	0	-5	0	-3	1	-2
12	x_1	0	1	-1	0	-1	0	0
Z_j-C_j		1400	0	1	0	2	0	14

All the values of Z_j-C_j are positive. The given solution is optimal.
Therefore the solutions are $x_1 = 0$, $x_2 = 0$, $x_3 = 100$.
The value max z = 1400.

Conclution

Simplex method is used to obtained optimal solution for an linear programming problems. The procedure and algorithm of simplex method with examples are discussed in this paper. So conclude as considerable number of methods has been so for presented for linear programming problem in which the Simplex method is more convenient. Therefore this paper attempts to propose a Simplex method for solving linear programming problem.

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A Study on Comparison of Simplex and Revised Simplex Method

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ABSTRACT: *The procedure for the revised simplex technique is printed by creating use of solely the rudiments of algebra . for a typical simplex iteration, a step by step comparison is created between the calculations for the revised and therefore the original technique. Finally , many reasons for recommending the revised technique are concisely mentioned.*

Keywords: *Linear programming , simplex method , basic feasible solution, optimum solution*

1. Introduction

The revised simplex methodology is mathematically corresponding to the quality simplex methodology however differs in implementation rather than maintaining a tableau that explicitly represents the constraints adjusted to a collection of basic variable ,it maintains a illustration of a basis of the matrix representing the constraints .The matrix homeward approach permits for bigger process potency by enabling thin matrix operations.

Relation between simplex and revised simplex method:

In the revised simplex method, there are m+1 rows and m+2 columns. So for moving from one iteration to another. We have to make $(m+1)^2$multiplication operations in order to get an improved solution in addition to m (n-m) operations for calculating $(z_j\text{-}c_j)$'s.

♦ In the revised simplex method we need to form (m+1)×(m+2) entries in every table, while within the simplex method there are (m+1)(m+1) entries in each table.

♦ The number of arithmetic operations within the revised simplex method could also be lesser then within the regular simplex method, depending on the size (m×n) of the problem.

♦ Revised simplex method reduces the accumulative round-off error whereas calculating $z_j\text{-}c_j$'s and updated column y_k due to use of original data.

♦ The inverse of the present basis matrix is obtained mechanically.

♦ The revised simplex method works with a reduced table because it stores only the essential variables. The basis inverse and also the RHS constants, hence less new data must be hold on within the memory of the computer from one iteration to the other.

♦ The theory of the revised simplex method ,especially the importance of the premise inverse and also the simplex multipliers is sort of useful in understood sensitively analysis and constant quantity programming.

The steps of simplex and revised simplex method

Consider an iteration of simplex method:

Step-1:Determine a beginning basic possible answer.

Step-2:choose associate degree getting into variable using the optimality condition. Stop if there's no getting into variable the last answer is perfect .Else, go to step 3.

Step-3:Select a feat variable exploitation the practicableness condition.

Step-4:Determine the new basic answer by exploitation the suitable Gauss-Jordan computations. Go to step 2.

Consider an iteration of Revised Simplex method:

Step-1:Define the initial basis matrix(B) for basic variables. Find B^{-1} using any one of the method

Step-2:Get $P_n' = B^{-1} \times P_n$ for the non-basis variables.X_B is the RHS of column vector.

Step-3;Get $Z_j\text{-}C_j = C_B P_n' \text{-} C_j$ value and choose entering variable for non-basic variables.

Step-4:Get $X_B' = B^{-1} X_B$,calculate the ratio and choose leaving variable.

Step-5:Update the initial table by introducing a non-basic variable into the basis and removing basic variable from the basic.

An example of simplex method:

Find the optimum integer solution to the following problem ,

max $z=3x_1+5x_2$ subject to the constraints,

$x_1 \le 4$

$x_2 \le 6$

$3x_1+2x_2 \le 18$ & $x_1,x_2 \ge 0$ and are integers.

Solution

To solve the problem by **simplex method,**

The LPP can be written as,

Max $z=3x_1+5x_2+0s_1+0s_2+0s_3$

s.t, $x_1+s_1=4$

$x_2+s_2=6$

$3x_1+2x_2+s_3= 18$ and $x_1 ,x_2 , s_1 ,s_2 ,s_3 \ge 0$.

The basic solutioins are,put $x_1 ,x_2=0$ then $s_1=4 ,s_2=6 ,s_3=18$

The initial table:

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	θ
0	s_1	4	1	0	1	0	0	∞
0	s_2	6	0	1	0	1	0	6
0	s_3	18	3	2	0	0	1	9
Z_j - C_j		0	-3	-5	0	0	0	

X_2–entering

S_2–leaving

New pivot equation=old equation ÷pivot element.

X_2 =6 , 0 ,1 , 0 ,1 ,0.

S_3-row:	Z_j - c_j row:
18-2(6)=6	0+5(6)=30
3-2(0) =3	-3+5(0)=-3
2-2(1)=0	-5+5(1)=0
0-2(0)=0	0+5(0)=0
0-2(1)=-2	0+5(1)=5
1-2(0)=1	0+5(0)=0

First Iteration

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	θ
0	s_1	4	1	0	1	0	0	4
5	x_2	6	0	1	0	1	0	∞
0	s_3	6	3	0	0	-2	1	2
Z_j-c_j	30	-3	0	0	5	0		

New pivot equation =2 , 1, 0 ,0, -⅔, ⅓.

S_1-row:	z_j-c_j-row:
4-1(2)=2	30+3(2)=36
1-1(1)=0	-3+3(1)=0
0-1(0)=0	0+3(0)=0
1-1(0)=1	0+3(0)=0

$0-1(-\frac{2}{3})=\frac{2}{3}$ $\quad 5+3(-\frac{2}{3})=3$
$0-1(\frac{1}{3})=-\frac{1}{3}$ $\quad 0+3(\frac{1}{3})=1$

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
0	s_1	2	0	0	1	⅔	-⅓
5	x_2	6	0	1	0	1	0
3	x_1	2	1	0	0	-⅔	⅓

All $z_j - c_j$ are the positive.
Hence Given solution is optimal and are integer
The solution is $x_1=2$
$x_2=6$
Max $z=36$

♦ Solve the **same** problem by using **Revised simplex method**

Solution
The LP problem can be rewritten as
Max $z=3x_1+5x_2+0s_1+0s_2+0s_3$
s.t, $x_1+s_1= 4$
$x_2+ s_2 = 6$
$3x_1+2x_2+s_3=18$ and $x_1, x_2, s_1, s_2, s_3 \geq 0$

Initial matrix

$$\begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & s_3 \end{array}$$
$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 1 \end{pmatrix}, \quad X_B=\begin{pmatrix} 4 \\ 6 \\ 18 \end{pmatrix}$$

$$Z=\begin{pmatrix} 3 & 5 & 0 & 0 & 0 \end{pmatrix}$$

Since s_1, s_2, s_3, are basic variables,

$$B=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B^{-1}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}=I$$

Calculate new p_n' for non- basic variables using $\mathbf{P_n'=B^{-1}\times P_n}$
The non-basic variable are (p_1, p_2)

$$P_1'=B^{-1}\times p_1=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

$$P_2'=B^{-1}\times p_2=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}=\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$X_B'=B^{-1}\times X_B=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \\ 18 \end{pmatrix}=\begin{pmatrix} 4 \\ 6 \\ 18 \end{pmatrix}$$

Calculate new coefficient for non-basic variables, using $\mathbf{z_j-c_j=c_B\times p_n'-c_n}$.
C_B for $s_1 \; s_2 \; s_3 =(0 \;\; 0 \;\; 0)$

$$Z_1 - c_1= c_B\times p_1' - c_1 = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} - 3= - 3$$

$Z_2 - c_2 = c_B \times p_2' - c_2 = = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} - 5 = -5$

The most negative is z_2-c_2=-5
$P_2'(x_2)$ is the entering variable.
Calculate the ratio min of x_B' / p_k'

Ratio= $\begin{pmatrix} 4/0 \\ 6/1 \\ 18/2 \end{pmatrix} = \begin{pmatrix} \infty \\ 6 \\ 9 \end{pmatrix} \rightarrow$ minimum positive

The variable s_2 is leaving.
Step 1:
The new basic variables ($s_1 \quad x_2 \quad s_3$)

B= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$ B^{-1}= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

Non-basic variables ($p_1 \quad p_4$)

$P_1' = B^{-1} \times p_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$

$P_4' = B^{-1} \times p_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$

$X_B' = B^{-1} \times X_B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 6 \\ 18 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 6 \end{pmatrix}$

Calculate the co-efficient for non-basic variables
C_B for ($s_1 \quad x_2 \quad s_3$) = $\begin{pmatrix} 0 & 5 & 0 \end{pmatrix}$

$Z_1 - c_1 = c_B \times p_1' - c_1 = \begin{pmatrix} 0 & 5 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} - 3 = -3$

$Z_4 - c_4 = c_B \times p_4' - c_4 = \begin{pmatrix} 0 & 5 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} - 0 = 5$

The most negartive is z_1-c_1=-3
P_1' (X_1) is entering variable.
Calculate the ratio = $\begin{pmatrix} 4/1 \\ 6/0 \\ 6/3 \end{pmatrix} = \begin{pmatrix} 4 \\ \infty \\ 2 \end{pmatrix} \longrightarrow$

S_3 is leaving variable.
Step 2:
The new basic variables ($s_1 \quad x_2 \quad x_1$)

$B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 3 \end{pmatrix}$ $B^{-1} = \begin{pmatrix} 1 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1/3 \end{pmatrix}$

Non-basic variables (p_4 p_5)

$$P_4' = B^{-1} \times p_4 = \begin{pmatrix} 1 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 1 \\ -2/3 \end{pmatrix}$$

$$P_5' = B^{-1} \times p_5 = \begin{pmatrix} 1 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/3 \\ 0 \\ 1/3 \end{pmatrix}$$

$$X_B' = B^{-1} \times X_B = = \begin{pmatrix} 1 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ 0 & -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} 4 \\ 6 \\ 18 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix}$$

Calculate new co-efficient for non-basic variables
C_B for (s_1 x_2 x_1) = (0 5 3)

$$Z_4 - C_4 = C_B \times p_4' - c_4 = \begin{pmatrix} 0 & 5 & 3 \end{pmatrix} \begin{pmatrix} 2/3 \\ 1 \\ -2/3 \end{pmatrix} - 0 = 5 - 2 = 3$$

$$Z_5 - C_5 = C_B \times P_5' - C_5 = \begin{pmatrix} 0 & 5 & 3 \end{pmatrix} \begin{pmatrix} -1/3 \\ 0 \\ 1/3 \end{pmatrix} - 0 = 1$$

All $z_j - c_j$ are positive.
The solution : $x_1 = 2$, $x_2 = 6$.
Max z =3(2) +5(6)
=6+30
Max z = 36
Hence the result.

Conclusion

Round off error tends to accumulate in each iteration of the simplex method. With revised simplex it also accumulates, but every once in a while we can go back to the original data, calculating the product form of the inverse from the original data and the current basis.

One more reason the revised simplex methodology is beneficial is that may be used even after you do not know all the variables- “column generation” can turn out them once they are required.

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Basic Concepts in Residue Theorem

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ABSTRACT: *The theorem of residue is powerful tool evaluated line integrals of analytic function over closed curves. Residue theorem can be used to evaluate different types of integrals of the real valued variable functions. The selected contour consists of straight lines and circular arcs.*

Keywords: *The Residues Theorem. Applications of the residue theorem to real integrals.*

1. Introduction

Residue theorem in complex analysis a discipline with in mathematics the residue theorem , sometimes, is called cauchy's residue theorem, is a powerful tool to evaluate line integrals of analytic functions over used to compute real integrals and infinite series as well.

The expansion coefficients a-1 is called residue of f(z) at the point z_0; res(f(z),z(0))=a-1. If the coefficients zero. Then the result of the integral is zero. The case where a functions is analytical, or has no poles is one of these cases.

Definition: Residue

The residue of f(z)at an isolated singularity 'a' is the unique complex number R which makes f(z)-R/z-a the derivative of a single valued analytic function in an annulus 0<|z-a|<δ the residue z=a of f(z) is denoted by R=$Res_{z=a} f(z)$.

Theorem: (Residue theorem):

Let f be defined on the simply connected domain D with a positive contour γ and with the poles $E_{1,\ldots,}E_n$ as the singularities. Then

$\int f(z)\,dz=2\pi i \sum_{m=1}^{N} res(f, E_n)$.

Therefore, the calculation of the integral is reduced to the calculation of the residues. How can they be obtained? Either by expanding f in a Laurent series or by doing the following derivatives for a pole of order m:

$a_{-1} = res(f, E_n)=\frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}((z- E_n)^m f(z)|_{z= En.}$

To calculate an integral enclosing poles, determine the poles and their order, o calculate the residuum of each pole and sum the results.

Examples:

$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ipx}\frac{P(x)}{Q(x)}dx$

Consider the Fourier integral

I= $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ipx}\frac{P(x)}{Q(x)}dx$, x$\epsilon R$,

With P,Q two polynomials in x, p>0, the degree of Q exceeding the degree of P by at least 1 an where the roots of Q are assumed to be not on the real axis, Using the residue theorem we writ

$\frac{1}{\sqrt{2\pi}}(\int_{-R}^{R} + \int \gamma R\, e^{ipz}\frac{P(z)}{Q(z)}$dxz)=$\sum_{upper\ half\ plane} res,$

Where γR is a semi circle with origin (0,0) and radius R such that all possible poles of Q are surrounded by the semi circle. Hence,

1. Write the integral over the reals as a complex integral
2. Introduce a closed contour in the complex plain.
3. Apply the residue theorem to the closed contour
4. Make sure that the part of the contour, which is not on the real axis, has zero contribution to the integral.

Prove that the semi circle integral has zero contribution follows from the Jordan Lemma :
Let f be an holomorphic function outside of a singularity at zero and assume that the condition

$\lim_{R\to\infty} f(Re^{i\emptyset}) = 0$

Holds uniformly for all 0 $\leq \emptyset \leq \pi$. Then $\lim_{R\to\infty} \oint \gamma R e^{iz} f(z)\, dz = 0$

Where γR the same semi circle as above

We apply these general fact to the Fourier integral

$F(\frac{1}{x^2+a^2})(p) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ipx}\frac{1}{x^2+a^2}dx = \frac{\sqrt{\pi}e^{-p|a|}}{\sqrt{2}|a|}$,a$\neq 0, P > 0.$

Clearly, the integer and $\frac{1}{x^2+a^2}$ satisfies the condition of Jordan's lemma, and the semi circle integral contribution around the pole has zero contribution. There are two poles: $\pm|a|i$. For the one is the upper half plane, the residue is

$a_{-1}=\frac{e^{-p|a|}}{2i|a|}$

Hence,
$F(\frac{1}{x^2+a^2})(p) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ipx}\frac{1}{x^2+a^2}dx = \frac{\sqrt{\pi}e^{-p|a|}}{\sqrt{2}|a|}$.

A comparable estimate can be carried out for p<0. This gives us then the result for p$\in R$.

Examples:

$$\oint f(z)dz, f(z) = \frac{z^2-2x}{(z+1)^2(z^2+4)}$$

Consider the integral,

$$\oint f(z)dz, z \in C,$$

Where f(z)=$\frac{4z-2}{z(z-1)}$

And γ is circle with radius 2 and center (0,0). The poles are z=-1 of order 2, z=$\pm 2i$ of order 1, that is circle encloses all poles. Calculating the residue
Res(f,-1)=-14/25, res(f,2i)=(7+i)/25, res(f,-2i)=(7-i)/25.

The residue theorem then implies

$$\oint f(z)dz = 2\pi i\sum_k res(f,k) = 0.$$

Example2:

$$\oint f(z)dz, f(z) = \frac{4x-2}{z(z-1)}$$

Consider the integral

$$\oint f(z)dz, z \in C,$$

Where f(z)=$\frac{4x-2}{z(z-1)}$

And γ is circle with radius and centre (0,0). The poles are z=0 of order 1, z=1 of order 1. The residues both have value 2 and therefore,

$$\oint f(z)dz, f(z) = 2\pi i4 = 8\pi i.$$

Example 3:

$$\oint f(z)dz, f(z) = \frac{e^z}{(z-1)^m}$$

Consider the integral

$$\oint f(z)dz, z \in C,$$

where

$f(z) = \frac{e^z}{(z-1)^m}$ for m$\in N,$

And gamma is circle with radius2 and center (0,0), hence, z=1 is a pole of order m. to calculate the residue, we use the Laurent series expansions:

$$\frac{e^z}{(z-1)^m} = e\frac{e^{z-1}}{(z-1)^m}$$

$$=e\frac{1}{(z-1)^m}(1 + (z-1) + \cdots + \frac{(z-1)^m}{m!}$$

Therefore,

Res (f,1)=e/(m-1)!

And

$$\oint f(z)dz = \frac{2\pi ie}{(m-1)!}$$

Conclusion

Powerful tool to evaluate line integrals of analytic functions over closed curves, it can often be used to compute real integrals as well.

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Alternative Move Toward To Revised Simplex Method

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ABSTRACT: *In this paper, an alternative towards to the revised simplex method of linear programming is suggested. The method sometimes involves not as much of or at the nearly all equal digit of iterations as compared to computational course of action for solving LPP on digital computer. We experimental that there is change in the regulation of selecting pivot vector at original stage and thereby for a few LPP it takes more number of iterations to achieve optimality. Here at the initial step we decide the pivot element on the basis of innovative rules of method described below. This powerful technique is better understood by resolving a cycling difficulty.*

Keywords: *Basic feasible solution, optimum solution conservative revised simplex method*

1. Introduction

The linear programming has its own importance in getting the answer of a controversy wherever 2 or a lot of activities complete for restricted resources. Allow us to take into account the linear programming drawback with n call variables and m constraints as:

Maximize Z $=\sum_{j=1}^{n} \gamma_j x_j$ (objective function)

Subject to constraints $\sum_{j=1}^{n} \beta_{ij} x_j \le c_i$, $x_j \ge 0$ (non negative restriction),

where $1 \le i \le m$, $1 \le j \le n$.

(i) $x_1, x_2, \ldots, x_n$ be call the decision variables.

(ii) The constant coefficients γ_j , j= 1, 2,... ,n represent the per unit contribution (profit or cost) of decision variables x_j to the value of objective function.

(iii) The coefficient β_{ij} , j = 1, 2,... ,n, i = 1, 2,... ,m are referred to as the technological or substitution coefficient. These represent the amount of variables c_i .

(iv) c_i (i = 1, 2,... ,m) is the stable in place of the requirement or availability of the i-th constant.

We can write as above in the matrix notation.

Maximize Z = γX;

Subject to constraint , $\beta X \le c$,

Where, $\gamma = [\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n]$, X, c are the column vectors

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{pmatrix}, \quad C = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_m \end{pmatrix}$$

and

$$\beta = \begin{pmatrix} \beta_{11} & \beta_{12} & \ldots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \ldots & \beta_{2n} \\ \vdots & & & \\ \vdots & & & \\ \beta_{m1} & \beta_{m2} & \ldots & \beta_{mn} \end{pmatrix}$$

introduce column variable of slack variables

$$X_s = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_m \end{pmatrix}$$

the constraint becomes

$$\begin{pmatrix} \beta, I \end{pmatrix} \begin{pmatrix} X \\ X_S \end{pmatrix} = C_1 \begin{pmatrix} X \\ X_s \end{pmatrix} \geq 0$$

where I am the matrix of m×m.
The initial basic feasible solution is Xρ = ρ −1 c, where ρ is the basis matrix

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \dots & \rho_{1n} \\ \rho_{21} & \rho_{22} \dots & \rho_{2n} \\ \vdots & & \\ \rho_{m1} & \rho_{m2} \dots & \rho_{mn} \end{pmatrix}$$

Imperative I. Determine the pivot basic vector by choosing the most negative of $^{\Phi}j$ given by
$^{\Phi}j = (Zj - Cj) / \sum R_i$ or $^{\Phi}j = (Zj - Cj) / \sum Xi$ for $ci \geq 0$.
Choose most negative of $^{\Phi}j$, corresponding vector will enter into the basis. Allow it be for varies j = k , hence y_k go through into the basis. If all $^{\Phi}j \geq 0$, the solution is optimum. choose the leaving vector by min(xBi/ xik) , allow it be for some i = r, hence xrk the pivot element.

Imperative II. Determine the web estimate for next iteration, before completing all the entries of next iteration by following above stated rule.
(i)It follows from the study of linear programming that for any fixed j, a set of feasible solutions can be construct such that $z < z_0$ for any member of the set where $zj - cj > 0$. If all $zj - cj \geq 0$, then the solution is optimum and so no need to conclude the entry of next iteration.
(ii) If there is too much of one negative zj – cj, choose most negative of them corresponding vector will enter the basis. Choose leaving variable and find out the corresponding line vector for all non basic j variables only.

Imperative III. Go to imperative II, repeat the process till optimum solution is obtained.

2. Statement of the Problem

In the following we will show the problem where the iterations are less than the other method.
Solve the following LPP:
Maximize z = 3x1 + 5x2 ,
subject to the constraints:
2x1 − 3x2 ≤ 4 ,
x1 − 2x2 ≤ 4 ,
2x1 + 2x2 ≤ 10 ,
x1 + 3x2 ≤ 20 ,
−2x1 +3 x2 ≤ 6
x1 ≤ 8 ,
x2 ≤ 6 .

3. Solution of the Problem

The above problem can be framed as
Maximize z = 3x1 + 5x2 + 0 (R1 + R2 + R3 + R4 + R5+R6+R7),
subject to the constraints
2x1-3x2+R1=4,

X1-2x2+R2=4,
2x1+2x2+R3=10,
X1+3X2+R4=20,
-2X1+3X2+R5=6,
X1+R6=8,
X2+R7=6,
X1,X2,R1,R2,R3,R4,R5,R6,R7 $\geq$ 0,
where R1, $\cdots$,R7 are slack variables.

Initial Step 1:

			Cj 3	5	0	0	0	0	0	0	0	
CB	YB	XB	X1	X2	R1	R2	R3	R4	R5	R6	R7	Ratio
0	R1	4	2	-3	1	0	0	0	0	0	0	-
0	R2	4	1	-2	0	1	0	0	0	0	0	-
0	R3	10	2	2	0	0	1	0	0	0	0	5
0	R4	20	1	3	0	0	0	1	0	0	0	2/3
0	R5	6	-2	3	0	0	0	0	1	0	0	2
0	R6	8	1	0	0	0	0	0	0	1	0	0
0	R7	6	0	1	0	0	0	0	0	0	1	6
Z_j-C_j		0	-3	-5	0	0	0	0	0	0	0	
ϕj		0	-3/5	-5/4	0	0	0	0	0	0	0	

TABLE - 1

3 is a pivot element
X2 – entering variable
R5 - leaving variable

Since the value ϕj = −5/4 is minimum(most negative), we make X2 as the entering variable in the basis and drop R5.

Step 2: Introduce X2 and drop R5

			Cj 3	5	0	0	0	0	0	0	0	
CB	YB	XB	X1	X2	R1	R2	R3	R4	R5	R6	R7	Ratio
0	R1	10	0	0	1	0	0	0	1	0	0	-
0	R2	8	-1/3	0	0	1	0	0	2/3	0	0	-
0	R3	6	10/3	0	0	0	1	0	-2/3	0	0	9/5
0	R4	14	3	0	0	0	0	1	-1	0	0	14/3
5	X2	2	-2/3	1	0	0	0	0	1/3	0	0	-
0	R6	8	1	0	0	0	0	0	0	1	0	8
0	R7	4	2/3	0	0	0	0	0	-1/3	0	1	6
zj-cj		10	-19/3	0	0	0	0	0	5/3	0	0	
ϕj			-19/21	0	0	0	0	0	0	0	0	

TABLE – 2

10/3 is a pivot element
X1 – entering variable
R3 – leaving variable

CB	YB	XB	X1	X2	R1	R2	R3	R4	R5	R6	R7
0	R1	10	0	0	1	0	0	0	1	0	0

0	R2	43/5	0	0	0	1	1/10	0	3/5	0	0
3	X1	9/5	1	0	0	0	3/10	0	-1/5	0	0
0	R4	43/5	0	0	0	0	-9/10	1	-2/5	0	0
5	X2	16/5	0	1	0	0	2/10	0	1/5	0	0
0	R6	31/5	0	0	0	0	-3/10	0	1/5	1	0
0	R7	14/5	0	0	0	0	-2/10	0	-1/5	0	1
zj-cj		107/5	0	0	0	0	19/10	0	6/5	0	0

All Zj-Cj $\geq$0 are positive
The given solution is optimal
X1=9/5 , X2=16/5
Maximum Z =3x1+5x2
=3(9/5)+5(16/5)
Maximum Z =107/5

Conclusion

It is known that the conventional revised simplex method is uncomfortable with degeneration and cycling problems, because the choice of the vectors plays an important role here. We observed that the optimum solution was obtained by our modified technique in less iteration or at the most equal iteration, where as usual, the simplex method took five iterations, whereas the LINDO package at step three and also the Bharamble method took three iterations to reach the optimum. Our technique therefore proves efficiency in resulting. Therefore our methodology reduces the number of iterations required.

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Factorization of Finite Abelian Groups

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ABSTRACT: *If G is a finite abelian group and n >1 is an integer, we say that G has the Hajós-n proper if from any factorization G = A_1……Ai......A_n of G into a direct product of subsets Such that every of the A_i's contains the identity element of G, it follows that a minimum of one amongst these A_i is periodic and we say that G has the Redei-n-property if from any factorization G = A_1...... A_i.......A_n of G, it follows that at least one of the A_i 's is such that (A_i) not equal G . We shall investigate some abelian groups with respect to these properties and will also study the relation between them*

Keywords: *factorization, abelian group*

1. Introduction

A famous conjecture of H. Minkowski was first proved by G . Hajos . He solved the conjecture after transforming the problem into a question about finite abelian groups. Below, we state two definition.

Definition of Periodic :
A group G is said to haven the Hajos -n- property if from any factorization
G = A_1..... A_i..... A_n of G , it follows that at least one of the subsets A_i is periodic. We say that G has the Redei -n-property if from any factorization G = A_1......A_i......A_n , it follows that at least one of the subsets A_i is such that (A_i) not equal G, where (A_i) denotes the subgroup generated by A_i .

Definition 1:
Let G be a finite abelian group. If a_i , a_2 ……,a_n are elements of G and r_1 , r_2 …… , r_n are positive integers such that each element of G is uniquely expressible in the form $a_1^{x1} a_2^{x2}$ ...... where $0 \le x_i \ge r_n - 1$ then ai =e for some i, $1 \le I \le n$.
A subset A of G is called cyclic if A={ e, a, a^2...... a^{r-1} }, where $r \le |a|$. Observe that, when r = a , A = (a) , the subgroup generated by a .

Definition 2:
Let G be a limited abelian group and let A_1..... A_i..... A n Subsets of G , each containing the identity element e of G. If G = A_1..... A_i..... A_n is factorization of G and each A_i is cyclic,
then at least one of the subsets A_i is a subgroup of G .

Factorization of finite Abelian groups
L. Redei proved the following theorem, which can be considered as a generalization of Definition 2 of Hajos theorem.
Let G be a finite abelian group and A_1..... A_i..... A n subsets of G each containing the identity element e of G, such that G = A_1..... A_i..... A_n is a factorization of G and each A_i has
of G , then for any h € H , hH=H.
This observation gave rise to the notion of periodic subsets of G . Namely, a subset A of G is called a periodic subset if there is an element a ≠ e in A such that a A =A

Hajoៈs Groups:
A Hajos group is a group G for which from any factorization of the form G = AB of G it follows that either A or B is periodic. The classification of Hajos finite Abelian groups was achieved by Sands and De Bruijn. These groups are listed below:

(p^α , q), (p^2, q^2),(p ,q ,r), (p ,q ,r ,s)
(p^3, 2, 2), (p^2 ,2,2,2), (p ,2^2 ,2), (p ,2,2, 2,2)

(p,q,2,2), (p,3,3), (3^2,3), (2^α,2) ,(2^2 ,2^2),(p,q)

Where p, q, r and s are primes and $\alpha > 1$ is an integer.
Any cyclic groups of order n is a Hajos group if n is of the form p^α , p^α q, p^α qr, pqr Where p , q, r and s are primes and $\alpha > 1$is an integers
If G is a finite abelian group and from each factorization $G = A_1 ..... A_i ..... A_n$ of G into subsets each containing the identity element e of G it follows that at least one of the subsets A_i is periodic, we say that G is n-good or has the Hajos -n-Property.
In [1], it is given away that the cyclic group G of order p^α , where p is a prime has the Hajos -n-property. In this document, we shall learn some non-cyclic p-groups with respect to Hajos -n-property.
We shall study the Hajos -n-Property for a general group G and show that there are many groups which do not posses the Hajos -n-property. We shall also study these groups with respect to the Redei-n-Property by which we mean. If G is a finite abelian group and from each factorization $G = A_1 ..... A_i .... A_n$ of G into subsets it follows that at least one of the subsets A_i is such that $(A_i) \neq G$, we say that G has the Redei-n-Property .

Lemma - 1
Suppose G has a subgroup H such that $|G \div H| = P$ for some prime
If H does not have the Hajós-n-Property. Then G itself does not have the Hajos -n-Property.

Proof
Let $H = A_1 A_2 ..... A_i ..... ,A_n$ be a factorization of H in which none of the A_i is periodic, and let $G= b_1 H+ b_2 H+....+ b_p H= BH$. where $b_1, b_2, ..., b_p$ is non-periodic . Then G has the factorization $G = (b_1 A + b_2 A + ...+ b_P A_1) A_2 ...... A_n$ Now, none of $A_2 ,..., A_n$ is periodic. Also,
$b_1 A_1 + b_2 A_1 + ... + b_p A_1$ is not periodic, for if it were periodic, then there must exist an element $g = b_{j(i)} a_1 \neq e$, $b_{j(i)}B$, $a_1 € A_1$ and j(i) a permutation of 1,2,…,p such that
$g (b_1 A_1 + b_2 A_1 + b_+ .... + b_p) = (b_1 A_1 + b_2 A_1 + ... + b_p A_1)$
It follows that $b_j^{(i)} (a_1 b_1 A_1 + b_2 A_1 + ... + b_p A) = (b_1 A_1 + b_2 A_1 + ... + b_p A_1)$
That is , $\sum^p_{i=1} b_{j(i)} a_1 A_1 = \sum^p_{i=1} b_i A_1$. Thus $b_{j(i)} a_1 A_1 = b_i A_1$

Hence $b_{j(i)} A_1 = b_i A_1$

Therefore $b_j^{-1} b_{j(i)} A_1 = A_1$ form which we get that A_1 is periodic with period $g=b_i^{-1} b_{j(i)} \neq e$ and this is a contradiction.
Hence the proof

Theorem
If G has the Hajos -n-Property and $G = A_1 ......A_i ......A_n$ is a factorization of G ,then at least one of the A_i is such that$(A_i) \neq G$.

Proof
Let G have the Hajos -n-Property and consider a factorization $G = A_1 ....A_i .....A_n$ of G. We proceed by induction on the number of distinct prime factors k of $|G|$. If k = 1, this is obvious since $|G| = |A_1| ......|Ai|.....|A_n|$ implies that one of A_i , say A_1 (since this is only a matter of reordering of A_i) has prime order p and the rest have order 1. So

$A_1 = G$, $A_2 = \{ e\}$,….. , $A_n = \{ e \}$ for all i . Thus, (A_i) snot equal G for every one $i \geq 2$.

Now, suppose $k \geq 2$. since G has the Hajos -n-Property, some A_i is periodic.
It follows that we get the factorization $A_i = HC$, where H is a proper subgroup of G . From the factorization $G = A_1 ....A_i .....A$, we get the factorization of the quotient group G/ H . Namely,

We now have $G/H= A_1 H/H…… A_i H/H….. A_n H/H$. Now, observe that $| G / H | < |G| / |H|$
also has the Hajos -n-Property. So by induction assumption, we get that some
$A_{j(i)} H/H \neq G/H$. where j(i) a permutation of {1,2,...,n}Consequently,$(A_{j(i)}) \neq G$.
Hence the proof

Conclusion

In this study for used in the finite abelian group , hajos-n-properties, subgroup, periodic for easy to the factorization of finite abelian groups

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Quadratic Programming Problem By Wolfe 'S Modified Simplex Method

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ABSTRACT: *In this paper, an alternate approach to the Wolfe's methodology for Quadratic programming is recommended. Here we have a tendency to planned a brand new approach supported the unvarying procedure for the answer of a Quadratic programming drawback by Wolfe's modified simple technique. The method typically involves less or at the most an equal range of iteration as compared to process procedure for finding NLLPP. We discovered that the rule of choosing pivot vector at initial stage and thereby for a few NLPP it takes a lot of variety iteration to attain optimality. Here at the initial step we decide the pivot vector on the premise of recent rules represented below. This powerful technique is best understood by breakdown a athletics drawback.*

Keywords: *Optimum solution, Wolfe's Methodology, Quadratic programming problem, modified simplex methodology.*

1. Introduction

Quadratic programming deals is concern with the Non-linear programming problem (NLPP) of maximizing (or minimizing) the quadratic problem objective operate subject group of linear difference constraints.

In this General Quadratic programming problem (GQPP) is form

Maximize M= f(x) $= \sum_{j=1}^{n} c_j x_j + 1/2 \sum_{k=1}^{n} c_{jk} x_j x_k$

Subject to constraints;

$\sum_{j=1}^{n} a_{ij} x_j \le b_i$, where i=1,2,………..,m.

$and\ x_j \ge 0$, j=1,2,…….n.

Where $C_{kj} = C_{jk}$ for all j and k, and also $b_i \ge 0$.

Let the Quadratic form

$\sum_{j=1}^{n} \sum_{k=1}^{n} c_{jk} x_j x_k$ be negative semi-definite.

The Wolfe's modified simplex algorithm:

Step 1

First convert the difference constraints into equation by introducing slack-variable q_1^2 with in the i^{th} constraint (i=1,2…….m) and the slack variable r_j^2 the j^{th} non-negative constraint (j=1,2…..n).

Step 2

Then construct the lagrangian function

$L(x,q,r,\lambda,\mu) = f(x) - \sum_{i=1}^{m} \lambda_i \left[\sum_{j=1}^{n} a_{ij} x_j - b_i + q_i^2\right] - \sum_{j=1}^{n} \mu_j [-x_j + r_j^2]$

Where $x= (x_1,x_2,\ldots x_n)$, $q=(q_1^2,q_2^2,\ldots q_m^2)$, $r=(r_1^2,r_2^2,\ldots r_n^2)$

And $\mu=(\mu_1,\mu_2\ldots\mu_n)$, $\lambda=(\lambda_1,\lambda_2\ldots \lambda_n)$

Differentiating the higher than perform 'L' partly with reference to the elements of x,q,r,λ,μ and equation the 1st partial derivatives to zero, we derive Kuhn-Tucker conditions from the resulting equations.

Step 3

Now introduce the non-negative artificial variable η_j (j=1,2….n) in the Kuhn-Tucker conditions

$C_j = \sum_{k=1}^{n} c_{jk} x_k - \sum_{i=1}^{m} \lambda_I a_{ij} + \mu_j = 0$ for j=1, 2….n, the construct an objective function

$M' = \eta_1 + \eta_2 + \ldots + \eta_n$

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Step 4

The feasible solution linear programming problem (LPP):

Minimize $M' = \eta_1+\eta_2+\ldots+\eta_n$

Subject to the constraints:

$\sum_{k=1}^{n} c_{jk}\, x_K - \sum_{i=1}^{m} \lambda_i\, a_{ij}+\mu_j+\eta_j = -\, c_j$ (j=1,2.....n)

$\sum_{j=1}^{n} a_{ij}\, x_j + q_i^2 = b_i$ (i=1,2.....m) $\eta_j, \lambda_j, \mu_j, x_j \geq 0$ (i=1,2......m; j=1,2.....n)

And satisfying the complementary slackness condition

$\sum_{j=1}^{n} \mu_j\, x_j + \sum_{i=1}^{m} \lambda_i s_i = 0$, [where $s_i = q_i^2$]

Step 5

Now apply 2-phase simplex method with in the usual manner to seek out associate optimum solution to the LP problem constructed in step four. The solution should satisfy the higher than complementary slack condition.

Step 6

The optimum solution obtained in above mentioned rule is associate degree optimum solution to given lpp.

Example 1

Maxiize $z=4x_1+6x_2-2x_1^2-2x_1x_{2\text{-}}2x_2^2$

Subject to constraints:

$X_1+2x_2 \leq 2$ and $x_1, x_2 \geq 0$

Solution

Maximize $z=4x_1+6x_2-2x_1^2-2x_1x_{2\text{-}}2x_2^2$

Subject to constraints:

$X_1+2x_2+s_1^2-2=0$

$-x_1+r_1^2=0$

$-x_2+r_2^2=0$

Lagrangian equation

$L[x,\lambda,\mu,s,r] = [4x_1+6x_2-2x_1^2-2x_1x_2-2x_2^2] - \lambda_1 [X_1+2x_2+s_1^2-2] - \mu_1 [-x_1+r_1^2] - \mu_2[-x_2+r_2^2]$

Necessary condition

$\partial L/\partial X_1=0$

$4X_1+2X_2+\lambda_1-\mu_1=4$ ....❶

$\partial L/\partial x_2=0$

$2x_1+4x_2+2\lambda_1-\mu_2=6$ ....❷

$\partial L/\partial \lambda_1=0$

$x_1+2x_2+s_1^2=2$ ....❸

$\partial L/\partial s_1=0$

$2\lambda_1 s_1=0$

$\partial L/\partial \mu_1=0$

$-x_1+r_1^2=0$

$\partial L/\partial \mu_2=0$

$-x_2+r_2^2=0$

Introducing artificial variable
Minimizing $z^{*}=A_1+A_2$
Subject constraints
$4x_1+2x_2+\lambda_1-\mu_1+A_1=4$
$2x_1+4x_2+2\lambda_1-\mu_2+A_2=6$
$X_1+2x_2+s_1^2=2$
$X_1, x_2, \lambda_1, s_1, \mu_1, \mu_2, A_1, A_2 \geq 0$
Using simplex method

Initial table

C_B	Y_B	X_B	X_1	X_2	S_1	λ_1	μ_1	μ_2	A_1	A_2	Θ
-1	A_1	4	4	2	0	1	-1	0	1	0	1
-1	-1	6	2	4	0	2	0	-1	0	1	3
0	S_1	2	1	2	1	0	0	0	0	0	2
Z_j-c_j		-10	-6	-6	0	-3	1	1	0	0	

The largest negative -6
Choose x_1 is entering variable and
A_1 is leaving variable.

First Iteration

C_B	Y_B	X_B	X_1	X_2	S_1	λ_1	μ_1	μ_2	A_1	A_2	Θ
0	X_1	1	1	1/2	0	1/4	-1/4	0	1/4	0	2
-1	A_2	4	0	3	0	3/2	1/2	-1	-1/2	1	4/3
0	S_1	1	0	3/2	1	-1/4	1/4	0	-1/4	0	2/3
Z_j-c_j		-4	0	-3	0	-3/2	-1/2	1	3/2	0	

X_2 is entering variable and S_1 is leaving variable.

Second Iteration

C_B	Y_B	X_B	X_1	X_2	S_1	λ_1	μ_1	μ_2	A_1	A_2	Θ
0	X_1	2/3	1	0	-1/6	1/3	-1/3	0	1/3	0	2
-1	A_2	2	0	0	-2	2	0	-1	0	1	1
0	X_2	2/3	0	1	2/3	-1/6	1/6	0	-1/6	0	_
Z_j-c_j		2	0	0	2	-2	0	1	1	0	

is entering variable and

S_1 is leaving variable

Third Iteration:

C_B	Y_B	X_B	X_1	X_2	S_1	λ_1	μ_1	μ_2	A_1	A_2
0	X_1	1/3	1	0	0	0	-1/3	1/6	1/3	-1/6
0	Λ_1	1	O	0	-1	1	0	-1/2	0	½
0	X_2	5/6	O	1	½	0	1/6	-1/12	-1/10	1/12
Z_j-c_j		0	O	0	0	0	0	0	0	0

Since all z_j-$c \geq 0$

The optimum solution is $x_1=1/3$, $x_2= 5/6$, $\lambda_1 =1$, $\mu_1=0$, $\mu_2=0$, $s_1=0$
Maximum z= 25/6.

Conclusion

In is seen that the prevailing methodology is lot of inconvenient in handing the degeneracy and sport downside as a result of here the selection of the vector, coming into and out going, play a crucial role. Here we have a tendency to discover that the optimum solution obtained in 3 iterations by our changed technique. We have a tendency to as Wolfe's simplex methodology took 5 iteration. Thence our technique provides potency in result as compared to different methodology in less iteration additionally we have a tendency to need less time change numerical problems.

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Introduction To The Convergence of Sequence

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ABSTRACT: *In this paper, we discuss the basic ideas involved in sequence and convergence, we start by defining sequence and follow by explaining subsequence, convergence and limit of a sequence with suitable examples and Relevant theorems will be given*

1. Introduction

A fundamental concept in mathematics is that of convergence. Let us distinguish sequences whose elements approach a single point as n increases (in this case we say that they converges) from those sequences whose elements do not. geometrically, it is clear that if the elements of the sequence (y_n) come finally inside every $\in -$ neighbourhood $(y_0 - \epsilon, y_0 + \epsilon)$ of y_0 then (y_n) approaches y_0.S

Definition

A sequence s = $\{s_i\}_{i=1}$ of real numbers is a function from I (the set of positive Integers) into R (the set of real numbers)

That is, $S:I \rightarrow R$

Notation

The real number s_i is s(i).

That is , $S=S(i)=\{Si\}_{i=1}^{\infty}$

Where s_i [i=1,2,3….] is called the i^{th} term of the sequence.

Example:1

(i) Let the sequence $X_n=1/n$

weget the following,

(1,1/2,1/3,1/4…..)

(ii) Consider the sequence is $x_n=2^n$

Weget,

(2,4,8,16,32…….)

These are the valid example of sequence because they are infinite list of real numbers.

Example:2

The followings are not an example of sequence

(i) (10,20,30,40)

(ii) (100,500,1000,100000)

We know that these are not an example of sequence because they are finite lists of real numbers.

Definition: Subsequence

Let $s_1,s_2,$.....be a sequence of real numbers and let $n_1 < n_2 <$…….be an increasing sequence of positive numbers. Then the sequence $(s_{n1},s_{n2},\ldots)$ is called the subsequence of (s_n) and is denoted by s_{nk}, where $k \in I$

Example:2

(i) The sequence of primes 2,3,5,7,11…..is a subsequence of {n}.

(ii) The set of positive integers is the subsequence of real numbers.

Definiton: Limit of a Sequence

Let sequence $\{s_n\}_{n=1}^{\infty}$ be the sequence of real numbers. We say that s_n

approaches the limit L (has $n\to\infty$) if for every $\in >0$ there is a positive integer N such that $|s_n-L|<\in$ $(n\geq N)$

That is, we write $\lim_{n\to\infty} s_n = L$
Or $s_n\to L$ as $n\to\infty$

Example: 1
Prove that sequence s= {1, 1/2, 1/3 …} has a limit.
Proof:

Given s= {1, 1/2,1/3,…..}

In general we write
$s_n = \frac{1}{n}$ n=1, 2,.....

$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{n}$
$=1/\infty$
$= 0$
$\{s_n\}$ has the limit 0

Example:2
Prove that $\{n/10^7\}_{n=1}^{\infty}$ does not have a limit

Proof
Given $\{n/10^7\}_{n=1}^{\infty}=\{\infty/10^7\}_{n=1}^{\infty}=\infty$
$\{n/10^7\}_{n=1}^{\infty}$ does not have a limit.

Theorem
If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers if $s_n\leq m$ $(n \in I)$ and if $\lim s_n=L$ prove $L\leq M$

Proof
Given $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers
Since $s_n\leq M$, $N \in I$
$M\geq s_n$
$M-s_n\geq 0$
$(M-s_n)\geq 0$ as $n\to\infty$
$M-\lim_{n\to\infty} s_n \geq 0$

Since $\lim_{n\to\infty} s_n = L$
$M-L\geq 0$
$M\geq L$
$L\leq M$
Hence proved.

Definiton: Convergent of Sequence
If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ has the limit L, then we say that $\{s_n\}_{n=1}^{\infty}$ is convergent to L.
If $\{s_n\}_{n=1}^{\infty}$ does not have a limit , we say that $\{s_n\}_{n=1}^{\infty}$ is divergent

Notation:
If (s_n) converges to L, then we write
$\lim_{n\to\infty} s_n = L$
(or) if for €>0, their exists a positive integer N such that $|s_n-L|<\in$, $\forall$ $n\geq N$
(or) $s_{n\to} L$ as $n\to\infty$

(or) simply written as $s_n \to L$
(or) $s_n \in (L-\in, L+\in)$ ∀, n≥N

Example

Fig. 1. The sequence {1,1,1...} converges to 1
Fig. 2. The sequence {1,1/2,1/3} are converges to 0
Fig. 3. It is already proved in the limit of the sequence

Example

Prove that $\{e^{1/n}\}_{n=1}^{\infty}$ converges to 1

Proof

Let $s_n=e^{1/n}$
$\lim_{n\to\infty} s_n = \lim_{n\to\infty} e^{1/n}$
$=e^{1/\infty}$
$=e^0$
Lim s_n= 1

Hence $\{e^{1/n}\}_{n=1}^{\infty}$ is convergent to 1
We now prove that a sequence cannot converge to more than one limt.

Theorem: Uniqueness Of Limit Theorem
Statement:
If the sequence of a real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L then $\{s_n\}_{n=1}^{\infty}$ cannot also to a limit distinct from L

That is if $\lim_{n\to\infty} s_n$ =L and $\lim_{n\to\infty} s_n$ =M then L=M

Proof:
Assume that the contrary
Let L≠M
|M-L|>0

Take |M-L|=2$\in$

By given hypothesis, $\lim_{n\to\infty} s_n$=L
Their exist N_1€I such that $|s_n-L|< \in, (n \geq N)(2)$
Similarly since $\lim_{n\to\infty} s_n$ =M
Their exist N_2€I such that $|s_n-M|<\in, (n \geq N)(3)$
let N=max(N_1,N_2)
$N \geq N1$ *and* $N \geq N2$
$|M-L|=|s_n-L+M-s_n|$
$=|(s_n-L)-(s_n-M)|$
$\leq|s_n-L+|s_n-M|$

<€+€ (from (2)&(3))
|M-L|<2$\in$
|M-L|< 2$\in$=|M-L|
|M-L|<|M-L|

Which is impossible
Hence our assumption is wrong
Hence L=M
Hence sequence s_n has a unique limit.

Theorem: Sandwich Theorem (Limit Theorem)

Suppose that (x_n),(y_n) and (z_n) are sequences such that $x_n \leq y_n \leq z_n$ for all n and that $x_n \to x_0$ and $z_n \to x_0$ then $y_n \to x_0$

Proof:

Let $\in > 0$ be given

Since $x_n \to x_0$ and $z_n \to x_0$ their exist N_1 and N_2

Such that x_n €$(x_0$-€$,x_0$+€$)$, $\forall$ $n \geq N_1$

z_n€$(x_0$-€$,x_0$+€$)$, $\forall n \geq N_2$

let $N=Max\{N_1,N_2\}$

then since $x_n \leq y_n \leq z_n$

we have y_n€$(x_0$-€$,x_0$+€$)$, $\forall$ $n \geq N_1$

Hence $y_n \to x_0$

Hence proved.

Conclusion

In the chapter we learn about the sequence, subsequence, convergence and its application.

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Problem Solved For N Jobs Through Three Machine in Operation Research

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ABSTRACT: *The first step in the process of operation research development in this problem environment observation, the second step is analyze and define the problem, the third step is develop a model, the fourth step is select appropriate data Input, the fifth step is provided a solution and test is reasonableness and implement the solution. This work is concerned about the minimization of the make span and also to establish the idle time for the machines in a flow shop scheduling evironmend by using Johnson's algorithm for three machines problems. The article also focuses on time-in and time-out strategy for two machines as well as for three machine problem.*

Keywords: *Number of machines, total elapsed time, processing time, optimal solution.*

1. Introduction

- A sequence is the order in which the jobs or processed. Sequence problems arise when we are concerned with situations where there is a choice in which a number of tasks can be performed. A sequencing problem could involve
- Jobs in a manufacturing plant.
- Aircraft waiting for landing and clearance.
- Maintenance scheduling in a factory.
- Programs to be run on a computer.
- Customers in a bank and so-on.

Definition: n jobs on 3 Machines:

The above procedure can be extended to special cases of 'n'jobs on three machines when one of the following conditions is satisfied.

(1).The smallest processing time for machine A is at least as great as the largest processing time for machine B. (i.e.) max ($_{Bi}$) ≤ min (Ai).

(2).The smallest processing time for machine C is at least as great as the largest processing time for machine B. (i.e.) max (Bi) ≤ min (Ci).

The method will reduce the problem of 'n' jobs with 3 machines to 'n' jobs with 2 machines

We introduce two fictitious machines G and H and determine their corresponding times as Gi and Hi

Where Gi=Ai+Bi and Hi = Bi+Ci.

Thus we have 'n' jobs with two machines G and H. We can obtain an optimal sequence for this problem. The resulting optimal solution also is the optimal solution to the original problem.

- **Processing Time:** Every operation requires certain time at each of machine. If the time is certain then the determination of schedule is easy. When the processing times are uncertain then the schedule is complex.
- **Total Elapsed Time:** It is the time between starting the first job and completing the last one.
- **Idle Time:** it is the time the machine remains idle during the total elapsed time.
- **Technological Order:** Different jobs may have different technological order. It refers to the order in which various machines are required for completing the jobs.

Procedure

(1).'n' jobs are to be processed on two machines A and B; all jobs to be processed and each job has to be processed in the given order AB.

(2).'n' jobs are to be processed on three machines A , B and C and each job has to be processed in the given order ABC.

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(3).'n' jobs are to be processed on 'm' machines, machine order being specified.
(4). 2 jobs are to be processed on 'n' machines each job being processed by the machines in a given order, but the given order is not necessarily the same for each job.

Example:-

Processing 'n' Jobs On 2 Machines:-

Suppose we have five jobs, each of which has to be processed on two machines A & B in the order AB. Processing time are given below you are required to determine the sequence for performing the jobs that would minimize the total elapsed time T, what is the value of T ?.

Job	Machine A	Machine B
1	6	3
2	2	7
3	10	8
4	4	9
5	11	5

Sol:

The minimum time in the above table is 2, which corresponds to job 2 on machine A.

2					

Now we eliminate job from further consideration. The reduced set of processing times are as follows.

Job	Machine A	Machine B
1	6	3
3	10	8
4	4	9
5	11	5

The minimum time is 3 for job 1 on machine B. Therefore, this job would be done in last the allocation of jobs fill this stage would be.

2				

After detection of job 1, the reduced set of processing times are as follows.

Job	Machine A	Machine B
3	10	8
4	4	9
5	11	5

The minimum time is 4 for job 4 on machine A.

2	4			1

After deleting job 4 from further consideration. The reduced set of processing times are as follows.

Job	Machine A	Machine B
3	10	8
5	11	5

The minimum time 5 for job 5. On machine B.

2	4		5	1

Now we eliminate job 5. Finally perform job 3 is the vacant plane.
Hence the optimal sequence is as follows.

2	4	3	5	1

Table:
Calculate of total elapsed time (T)

Job	MACHINE A		MACHINE B		IDLE TIME	
	Time in	Time out	Time in	Time out	A	B
2	0	2	2	9	-	2
4	2	6	9	18	-	-
3	6	16	18	26	-	-
5	16	27	27	32	-	1
1	27	33	33	36	3	1

The minimum total Elapsed time = 36
Idle Time for Machine A = 36 - 33 = 3Hours
Idle Time for Machine B = 4Hours.

Processing n Job On 3Machines:-

1. Find the sequence that minimize the total elapsed time required to complete the following job.

JOB	1	2	3	4	5
Machine A	3	8	7	5	2
Machine B	3	2	2	1	5
Machine C	5	8	10	1	6

Solution:
Minimum $_{Ai}$ = 2
Minimum Bi = 5
Minimum Ci = 5

(1) Minimum Ai ≥Maximum Bi
2 ≥ 5
(2) Minimum Ci ≥Maximum Bi
5 ≥ 5

The second of the conditions is satisfied we shall now determines Gi and Hi from then find optimal sequence
Here, Gi = Ai+Bi, Hi = Bi+Ci.

JOB	1	2	3	4	5
G	6	12	9	6	7
H	8	12	12	8	11

The optimal sequence in respect on n jobs processing on 2 machine.
The minimum time in the above table is 6 which corresponds to job 1 on G.

1				

Now we eliminate job 1 from further consideration .The reduced set of processing time are as follows.

JOB	2	3	4	5
G	12	9	6	7
H	12	12	8	11

The minimum time in 6 for job 4 for G machine.

1	4			

Now we delete job 4.

Job	2	3	5
G	12	9	7
H	12	12	11

The minimum time in 7 for job 5 on machine G.

1	4	5		

Delete job 5.

Job	2	3
G	12	9
H	12	12

The minimum time in 9 for job 3 on machine G.

1	4	5	3	

Delete job 3 about the remaining job 2 in the vacant place the sequence of all the jobs is,

1	4	5	3	2

Table
Minimum total elapse time.

JOB	MACHINE A		MACHINE B		MACHINE C		IDLE TIME		
	Time In	Time Out	Time In	Time Out	Time In	Time Out	A	B	C
1	0	3	3	6	6	11	0	3	6
4	3	8	8	9	11	18	-	2	-
5	8	10	10	15	18	24	-	1	-
3	10	17	17	19	24	34	-	2	-
2	17	25	25	29	34	42	17	6	-

Total elapsed time = 42
Idle time for machine A = 17hours
Idle time for machine B = 14hours
Idle time for machine C = 6hours.

Example:
Find the sequence that minimize the total elapsed time required to complete the following jobs.

JOBS	:	1	2	3	4	5
Processing time Machine A	:	5	7	6	9	5
(In hours) Machine B	:	2	1	4	5	3
Machine C	:	3	7	5	6	7

Solution:
This is a 5 jobs and 3 machines problem. We shall test whether it is possible to convert into 5 jobs and 2 machines problem. For this either minimum Ai ≥ maximum Bi or minimum Ci ≥ maximum Bi.
In this problem, minimum Ai = 5
Maximum Bi = 5
Maximum Ci = 3.
The condition minimum Ai ≤ maximum Bi is satisfied.
We can convert the problem into 5 jobs and 2 machines problem and adopt the earlier procedure to find the sequencing of jobs and minimum total elapsed time.
Let us create two machines G and H such that
Gi = Ai + Bi and Hi = Bi + Ci.
The problem can be treated as follows:

Job	Machine G	Machine H
1	7	5
2	8	8
3	10	9
4	14	11
5	8	10

Min (Gi, Hi) = 5 (job 1, machine 4)
Allot job 1 last.

				1

Delete job 1:

Job	2	3	4	5
G	8	10	14	8
H	8	9	11	10

Min (Gi, Hi) = 8 (job 2, G or Job 5, G)
Allot job 2 first and allot job 5 next to job 2.

2	5			1

Delete job 2 and job 5

Job	3	4
G	10	14
H	9	11

Min (Gi, Hi) = 9 (Job 3, H)
Allot job 3 just before 1.

2	5		3	1

Delete job 3:
Allot the remaining job 4 in the vacant place.
The sequencing of all the jobs is

2	5	4	3	1

To Find Minimum Total Elapsed Time:

Job	A		B		C		Idle Time		
	Time in	Time out	Time in	Time out	Time in	Time out	A	B	C
2	0	7	7	8	8	15	-	7	8
5	7	12	12	15	15	22	-	4	0
4	12	21	21	26	26	32	-	6	4
3	21	27	27	31	32	37	-	1	-
1	27	32	32	34	37	40	8	1+3	-
						TOTAL	8	22	12

The minimum total elapsed time = 40 hours
The idle time for machine A = 8 hours
The idle time for machine B = 22 hours
The idle time for machine C = 12 hours.

Conclusion

In this chapter it is clear that for two machine and more than two machine problems, Johnsons and extended Johnsons algorithm are used to find the make span and idle time for a particular factorial series. The prime critic observed in this algorithm is, it will take some time find to the optimal sequences. On practicing this strategy to three components and result are really good and we suggested to follow the system to all parts with continuous monitoring and improvement. The work can be also be extended to design a algorithm to minimize the idle time.

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Alternative Technique To Revised Simplex Method

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ABSTRACT: *In this paper, another algorithmic program to the revised simplex methodology of linear programming is usually recommended. The strategy generally involves less or at the foremost equal variety of iterations as compared to procedure for determination LPP on computer. We tend to discovered that there is modification within the rule of choosing pivot vector at initial stage and there by for a few LPP it takes additional variety of iterations to realize optimality. Here at the initial step we elect the pivot vector on the idea of recent rules of methodology represented below. In this paper we discussed a some basic definitions, some standard forms, Procedure of Revised Simplex Method and some problem and its solution.*

Keywords: *Linear Programming Problem , Objective Function, Feasible Solution, Optimal Solution, Leaving variable, Entering variable.*

1. Introduction

The Word Revised refers to the procedure of changing the simplex table. When size of the problems becomes, storing the entre simplex in memory will be computationally very expensive. In the Revised Simplex Method we only need to re compute values of B^{-1}, X_B, $C_B B^{-1}$ and Z value of all these new variable can be computed directly from their definition provided B^{-1} is known. At each iteration, B^{-1} is calculated from its previous value when only one y_j is changed at each iteration for which the non-basic variable is entered into the basis.

Linear programming problem

Linear programming deals with the Optimization (Maximization or Minimization) of a function of variable known as objective function, subject to a set of linear equalities or inequalities known as constraints.
General Linear programming problem form:

Max or Min $Z = c_1x_1+c_2x_2+\ldots\ldots\ldots\ldots+c_nx_n$
Subject to $a_{11}x_1+a_{12}x_2+\ldots\ldots\ldots\ldots+a_{1n}x_n \leq$ (or) $\geq b_1,$
$a_{21}x_1+a_{22}x_2+\ldots\ldots\ldots\ldots+a_{2n}x_n \leq$ (or) $\geq b_2,$
$\vdots$
$a_{m1}x_1+a_{m2}x_2+\ldots\ldots\ldots\ldots+a_{mn}x_n \leq$ (or) $\geq b_n$

Feasible solution

Any solution of a Linear programming problem satisfies the non-negative restrictions is called feasible solution.

Optimum (or) Optimal solution:

Any feasible solution optimizes (Maximizes or Minimizes) the objective function is called its optimum or optimal solution.

Revised Simplex Method in Standard Form:

Consider the Linear programming problem in standard form
Max $Z = c_1x_1+c_2x_2+\ldots\ldots\ldots\ldots+c_nx_n+0x_{n+1}+0x_{n+2}+\ldots\ldots\ldots\ldots+0x_{n+m}$
$a_{11}x_1+a_{12}x_2+\ldots\ldots\ldots\ldots+a_{1n}x_n+x_{n+1} = b_1,$
$a_{21}x_1+a_{22}x_2+\ldots\ldots\ldots\ldots+a_{2n}x_n+x_{n+2} = b_2,$
$\vdots$
$a_{m1}x_1+a_{m2}x_2+\ldots\ldots\ldots\ldots+a_{mn}x_n+ x_{n+m} = b_m,$
$x_1,x_2\ldots\ldots\ldots\ldots x_n , x_{n+m} \geq 0.$

In order to solve Linear programming problem using Revised simplex method, the objective function is also considered as one of the constraints equation.

Rewrite the objective function,
$Z - c_1x_1 - c_2x_2 - \ldots\ldots\ldots\ldots - c_nx_n - 0x_{n+1} - 0x_{n+2} - \ldots\ldots\ldots\ldots - 0x_{n+m}$

In matrix notation,
$Z - cx = 0$
$Ax = b$
$x \geq 0$.

Where c – Co-efficient of objective function,
A – Co-efficient of constraints equation,
b – Right hand side values.

Procedure of Revised Simplex Method:

Step 1

The given problem convert to the Revised simplex form with the objective functions as one of the constraints and adding slack variables. We convert the inequalities into the equalities.

Step 2

Identify the original column vectors(P_n),Co-efficient matrix(c_n)and the initial basis matrix (B) of basic variables. Calculate B^{-1} and using one of the method.

Step 3

Calculate the new column vectors P_n' using $P_n' = B^{-1}(P_n)$ of the non-basic variables and X_B is the RHS column vector.

Step 4

Find $Z_j - c_j = c_B P_n' - c_j$ value for non-basic variables.

1. All $Z_j - c_j \geq 0$, then the current $X_B' = B^{-1}X_B$ is the optimal solution.
2. If not, choose the most negative variable is entering variable.

Step 5

Calculate the ratio $\min\left(\frac{X_B'}{P_k'}\right)$.

Choose $\min\left(\frac{X_B'}{P_k'}\right)$ is leaving variable.

Step 6

Enter the initial table by introducing the non-basic variable into the basis variable and removing basic variable from the basis.

Step 7

Repeat steps 3, 4, 5, 6 until no more new basic variable can be identified.

Problem

Use the revised simplex method to solve the following linear programming problem
Maximize $Z = 2x_1 + x_2$,
Subject to the constraints

i. $3x_1 + 4x_2 \leq 6$,
ii. $6x_1 + x_2 \leq 3$,

$x_1, x_2 \geq 0$.

Solution

Step 1

The given problem can be written as,

Maximize $Z = 2x_1+x_2+0s_1+0s_2$,

$\Rightarrow Z - 2x_1-x_2-0s_1-0s_2=0$

Subject to the constraints

$3x_1+4x_2+s_1=6$,

$6x_1+x_2+s_2=3$,

$x_1 , x_2 ,s_1,s_2 \geq 0$.

Step 2

Initial matrix,

$\begin{pmatrix} 3 & 4 & 1 & 0 \\ 6 & 1 & 0 & 1 \end{pmatrix}$, $X_B=\begin{pmatrix} 6 \\ 3 \end{pmatrix}$, $Z=(\,2 \quad 1 \quad 0 \quad 0\,)$

s_1,s_2 are basic variables.

$B=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B^{-1}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=I$

Step 3

Calculate new P_n' for non-basic variables. Using $P_n'= B^{-1}(P_n)$.

Since, non-basic variables are (P_1,P_2)

$P_1'= B^{-1}(P_1)= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \end{pmatrix}=\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$P_2'= B^{-1}(P_2)= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix}=\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

and

$X_B'= B^{-1}(X_B)= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \end{pmatrix}=\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

Step 4

Calculate new co-efficient for non-basic variables. Using $Z_j-c_j=c_B(P_n')-c_n$

C_B for $s_1,s_2=(0 \;\; 0)$

$Z_1-c_1=c_B(P_1')-c_1=(0 \;\; 0)\begin{pmatrix} 3 \\ 6 \end{pmatrix}-2=-2 \quad \leftarrow$

$Z_2-c_2=c_B(P_2')-c_2=(0 \;\; 0)\begin{pmatrix} 4 \\ 1 \end{pmatrix}-1=-1$

The most negative is $Z_1-c_1=-2$.

$P_1'(x_1)$ is the entering variable.

Step 5

Calculate the ratio min$\left(\frac{X_B'}{P_k'}\right)$

$X_B'=\begin{pmatrix} 6 \\ 3 \end{pmatrix}$, $P_1'=\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

ratio $\frac{X_B'}{P_k'}=\begin{pmatrix} \frac{6}{3} \\ \frac{3}{6} \end{pmatrix}=\begin{pmatrix} 2 \\ \frac{1}{2} \end{pmatrix} \rightarrow$ smallest positive

Step 6

New basic variables (s_1,x_1)

$$B=\begin{pmatrix}1 & 3\\ 0 & 6\end{pmatrix},\ B^{-1}=\frac{1}{6}\begin{pmatrix}6 & -3\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & -\frac{1}{2}\\ 0 & \frac{1}{6}\end{pmatrix}$$

Non-basic variables (P_2,P_4)

$$P_2' = \begin{pmatrix}1 & -\frac{1}{2}\\ 0 & \frac{1}{6}\end{pmatrix}\begin{pmatrix}4\\ 1\end{pmatrix}=\begin{pmatrix}\frac{7}{2}\\ \frac{1}{6}\end{pmatrix}$$

$$P_4'=\begin{pmatrix}1 & -\frac{1}{2}\\ 0 & \frac{1}{6}\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}=\begin{pmatrix}-\frac{1}{2}\\ \frac{1}{6}\end{pmatrix}$$

$$X_B'=\begin{pmatrix}1 & -\frac{1}{2}\\ 0 & \frac{1}{6}\end{pmatrix}\begin{pmatrix}6\\ 3\end{pmatrix}=\begin{pmatrix}\frac{9}{2}\\ \frac{1}{2}\end{pmatrix}$$

Step 7

C_B for (s_1,x_1) is (0 2)

Calculate Z_j-c_j for non-basic variables.

$$Z_1-c_1=c_B(P_1')-c_1=(0\ \ 2)\begin{pmatrix}\frac{7}{2}\\ \frac{1}{6}\end{pmatrix}-1=-\frac{2}{3}\ \leftarrow$$

$$Z_2-c_2=c_B(P_2')-c_2=(0\ \ 2)\begin{pmatrix}-\frac{1}{2}\\ \frac{1}{6}\end{pmatrix}-0=\frac{1}{3}$$

The most negative Z_2-c_2= $-\frac{2}{3}$

P_2' (x_2) is the entering variable.

Step 8

Calculate ratio $\min\left(\frac{X_B'}{P_k'}\right)$

$$X_B'=\begin{pmatrix}\frac{9}{2}\\ \frac{1}{2}\end{pmatrix},\ P_2'=\begin{pmatrix}\frac{7}{2}\\ \frac{1}{6}\end{pmatrix},$$

$$\text{ratio}\ \frac{X_B'}{P_k'}=\begin{pmatrix}\frac{9}{2}/(\frac{7}{2})\\ \frac{1}{2}/(\frac{1}{6})\end{pmatrix}=\begin{pmatrix}\frac{9}{7}\\ 3\end{pmatrix}\rightarrow \text{smallest positive}$$

First variable(s_1) is leaving variable.

The new basic variables are (x_2 x_1).

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B= $\begin{pmatrix} 4 & 3 \\ 1 & 6 \end{pmatrix}$, B^{-1}= $\frac{1}{21}\begin{pmatrix} 6 & -3 \\ -1 & 4 \end{pmatrix}$= $\begin{pmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{1}{21} & \frac{4}{21} \end{pmatrix}$

Non-basic variables are (s_1, s_2).

P_3' = $\begin{pmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{1}{21} & \frac{4}{21} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$=$\begin{pmatrix} \frac{2}{7} \\ \frac{-1}{21} \end{pmatrix}$

P_4'=$\begin{pmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{1}{21} & \frac{4}{21} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$=$\begin{pmatrix} -\frac{1}{7} \\ \frac{4}{21} \end{pmatrix}$

X_B'=$\begin{pmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{1}{21} & \frac{4}{21} \end{pmatrix}\begin{pmatrix} 6 \\ 3 \end{pmatrix}$=$\begin{pmatrix} \frac{9}{7} \\ \frac{2}{7} \end{pmatrix}$

Calculate Z_j-c_j for non-basic variables.
C_B for (x_2 x_1)=(1 2)

Z_3-c_3=(1 2) $\begin{pmatrix} \frac{2}{7} \\ \frac{-1}{21} \end{pmatrix}$ – 0=$\frac{4}{21}$

Z_4-c_4=(1 2) $\begin{pmatrix} -\frac{1}{7} \\ \frac{4}{21} \end{pmatrix}$-0=$\frac{5}{21}$

All Z_j-c_j are positive.
The solution is optimal

X_B'=$\begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$ =$\begin{pmatrix} \frac{9}{7} \\ \frac{2}{7} \end{pmatrix}$

x_1=$\frac{2}{7}$, $x_2 = \frac{9}{7}$

Max Z = $\frac{13}{7}$.

Conclusion

It is far-famed that the traditional revised simplex technique is quite inconvenient in handling the degeneracy and cycling issues as a result of here the selection of the vectors, coming into and outgoing, play a vital role. We tend to discovered that the optimum answer obtained in less iteration or at the foremost equal iteration by our changed technique. Thus the amount of iterations needed is reduced by our methodology.

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Fermat's Little Theorem and Chineses Remainder Theorem in Number Theory

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ABSTRACT: *During this article, we'll prove the theorems Fermat's very little theorem and Chinese Remainder theorem. Fermat's very little theorem is useful within the learning of the integers and their property, that is a part of arithmetic referred to as variety theory. Gift be extra than a just some numerous sorts of proof to Fermat's theorem, however this quality theory provides a really elegant one. Conjointly Chinese remainder theorem of variety theorem, that states that if one is aware of the rest of the geometer division of an integers n by many ,then one will conclude severally the residue of the rending from n by the multiply of those integers, below condition that the divisors are combine wise co-prime.*

1. Introduction

Pierre de Fermat's theorem was initial planned by Fermat in 1640, however a confirmation wasn't magisterially revealed till 1736. Fermat's very little Theorem is wide proverbial throughout mathematical community. On the opposite hand , smallest exponent that may be utilized in the distinctive theorem. Fermat's very little theorem is an important possessions of integers to a main modulo. One case of the celebrated Chinese Hypothesis is solely a reduplication of Fermat's very little theorem mistreatment the quantity The Chinese remainder theorem is wide wont to computing with whole number, because it permits replace a calculation that one understand a sure on the dimension of the end result by many similar computations on tiny integers. .

Basic Definitions

(i) Number Theory

Number is the branch of the pure mathematics denoted primarily to the study of integer its divided in to several areas including elementary theory, number theory algebraic number theory.

(ii) Congruences

Given three integer a, b and m we say that "a is congruent to b modulo" and write
$a \equiv b \pmod m$ if the difference a-b is divisible by m, m is called the modulus of the congruences.

(iii) Prime Number

An integer as no positive divisor other than one and itself. A number is composite if it as at least non-trivial divisor. (eg: 2,3,5,7)

(iv) Remainder

Let m>0 be an integer . We say that integer a and b are congruent modulo n if n divides their difference. We can write $a \equiv b$ (mod m) ,where the number n is called the modulus and b is called the remainder.

(v) Properties Of Congruence

(1) $a \equiv a \pmod m$
(2) $a \equiv b \pmod m$ then $b \equiv a \pmod m$
(3) if $a \equiv b \pmod m$ and $b \equiv c \pmod m$ then $a \equiv c \pmod m$
(4) if $a \equiv b \pmod m$ and $c \equiv d \pmod m$ then $a+c \equiv b+d \pmod m$ and $ac \equiv bd \pmod m$

Fermat's Little Theorem

Statement

Let 'p' be a prime any integer 'a' satisfies $a^p \equiv a \pmod p$ and any intger a not divisible by p satisfies conditions $a^{p-1} \equiv 1 \pmod p$

Proof

We consider the integer a,2a,3a,.....(p-1)a complete residue system.
Now, Let us consider the equation

$$\left.\begin{array}{r} \gcd(1,p) = 1 \\ \gcd(2,p) = 1 \\ \vdots \\ \gcd(p-1,p) \end{array}\right\} \ldots\ldots\ldots\ldots\ldots 1$$

since gcd(a,p)=1

$$\left.\begin{array}{r} \gcd(1,p) = 1 \\ \gcd(a,p) = 1 \\ \gcd(1a,p) = 1 \\ \gcd(2,p) = 1 \\ \gcd(a,p) = 1 \\ \gcd(2a,p) = 1 \\ \vdots \\ \gcd(p-1,p) = 1 \\ \gcd((p-1)a,p) = 1 \end{array}\right\} \ldots\ldots\ldots\ldots\ldots 2$$

we know that
if gcd(u,α)=1 and gcd(v,α)=1
then gcd(uv,α)=1
consider the remainder left by a,2a,.....(p-1)a when divisible by p.
(2) $\Rightarrow$none of the remainder can be zero.if it is possible

Suppose 'ia' and 'ja' have the same remainder when divided by p. when $1 \leq i \leq p$,
$1 \leq j \leq p$, where $i \neq j$, without loss of generality ,
we can assume that i>j
ia=ja (modp)
Now,
$ia-ja \equiv o(modp)$
Since p/(i-j)a
(i-j)<p
Also(i-j)>0
Since p divides natural number smaller than itself.
Which is a contradiction.
Since when a,2a,....(p-1)a are divided by p,

No remainder is zero and two remainder are not equal.
Since these remainder are 1,2,3,.....p-1 in a certain order.
Suppose $a \equiv r_1 \pmod p$
$2a \equiv r_2 \pmod p$
.
.
.

$(p-1)a \equiv r_{p-1} \pmod p$

Then $r_1, r_2, \ldots r_{p-1}$ as a certain order.
Since $a.2a.3a \ldots (p-1) = r_1, r_2, \ldots r_{p-1} \pmod p$
$1.2.3 \ldots (p-1)\, a.a \ldots a \equiv 1.2 \ldots p-1 \pmod p$
$(p-1)!\, a^{p-1} \equiv (p-1)! \pmod p$
$p/(p-1)!(a^{p-1}-1)$
since p is prime , $p/(p-1)!$
But $p \nmid (p-1)!$
Since $p/(a^{p-1})$
ie)$a^{p-1}-1 \equiv o \pmod m$

$a^{p-1} \equiv \pmod p$
Hence The Theorem.

Chinese Remainder Theorem
Statement
Suppose that we want to solve a system of congruence to different modulo $x \equiv a_1 \pmod{m_1}$, $x \equiv a_2 \pmod{m_2} \ldots x \equiv a_r \pmod{m_r}$. Suppose that each pair of modulo is relatively prime $\gcd(m_i, m_j)=1$ for $i \neq j$.then there exists a simultaneous solution x to all of the congruence and any two congruent to one another modulo $M = m_1.m_2 \ldots m_r$.

Proof
Let us define $M_i = M / m_i$ for $i=1,2, \ldots k$
ie) M_i is the product of the moduli except m_i
since m_i & M_i have no common factor other than 1,where $i \neq j$.
We have $\gcd(M_i, M_j)=1$
M_i has a multiple inverse modulo m_i
Since it is possible to solve the linear congruence $M_i N_i \equiv 1 \pmod{m_i}$ has a unique solution.
$M_i N_i \equiv 1 \pmod{m_i}$ multiple a_i on both side, we get
Since $a_i M_i Ni \equiv a_i \pmod{m_i}$ for i= 1 to k.....(1)
Let construct $x = \sum a_i M_i N_i$ such that $x \equiv a_1 M_1 N_1 \pmod{m_1}$
$x \equiv a_2 M_2 N_2 \pmod{m_2} \ldots x \equiv a_k M_k N_k \pmod{m_k}$
$x = a_1 M_1 N_1 + a_2 M_2 N_2 + \ldots + a_k M_k N_k \equiv a_i \pmod{m_i}$
since m_i / M_j. $j \neq i$. then for each i, the sum of other than i^{th} trem are all divisible by m_i. hence the system has the a solution
since $x = a_1 M_1 N_1 + a_2 M_2 N_2 + \ldots + a_k M_k N_k$ is a simultaneous soln of all the congruences.

Uniqueness
Suppose x' and x'' are two solution then $x^1 \equiv x^{11} \pmod{m_i}$
$M_i/(x'-x'')$ for each i
Since $\gcd(m_i, m_j)=1$ for $i \neq j$
We have $m_1, m_2, m_3, \ldots m_k/(x'-x'')$
Since $x' = x'' \pmod{m_i}$
This proves uniqueness.

Conclusion
In this paper we have demonstrated the theorems of the Fermat's theorem and the Chinese remainder very easily. Those readers can easily understand these theorems. It has demonstrated the theorems with the existing proofs and has demonstrated a new way.

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Fuzzy Logic and Fuzzy Expert Systems

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ABSTRACT: *Fuzzy logic is a superset of classical logic introduced by Dr. Lotfi Zadeh. This paper discusses a mathematical basis of fuzzy logic along with the concepts of fuzzy sets, membership functions and rules of reasoning. Fuzzy expert systems accept inputs numbers and convert them into linguistic values which are then manipulated by if-then rules given by a human expert. The concept of a fuzzy expert system with its rule-base and set membership functions is discussed in detail. After defining the rules and membership functions, the application of this knowledge to the input variables to compute the values of the output variables is studied. A case study is presented along with an implementation in MATLAB.*

Keywords: *Fuzzy expert systems, Linguistic variables and linguistic values, Fuzzy propositions and truth assignments, Fuzzy logical operations, Fuzzy sets and membership functions.*

1. Introduction

In the classical two-valued logic, propositions can take only two truth values from the set $\{0,1\}$. It has been shown using De Morgan's algebras that the propositional logic obtained when the set of truth assignments consists of all values in the unit interval with conjunction, disjunction and negation as defined in the table 1 (Appendix C) is the same as fuzzy logic [4]. De-Morgan algebra is a structure $\mathrm{A} = \{A, \vee, \wedge, 0, 1, \neg\}$ such that $\{A, \vee, \wedge, 0, 1, \neg\}$ is a bounded distributive lattice. ¬ is a De Morgan involution such that $\neg(x \wedge y) = \neg x \vee \neg y, \neg\neg x = x$. When the classical set of truth values is extended to include a third value called '*undecided*', the De Morgan algebra generated gives rise to three-valued logic. A structure $([0,1], \wedge, \vee, \neg, 0, 1)$ forms a De Morgan's algebra. Using this as the algebra of truth values, classical fuzzy logic is generated. It has been shown that the equational class of all De-Morgan's algebras can be generated by both three-valued logic and fuzzy logic, hence they are the same. Fuzzy logic extends propositional logic to all values in the interval [0,1]. In such a case, a statement may have a truth value that is neither completely true nor completely false. Basic fuzzy logic operations as defined by Dr. Zadeh are shown in the table 1 in appendix C.

2. Fuzzy sets and membership functions.

A fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. Some common examples are the sets of all young women or the set of small cars. In a fuzzy set an object may have a *grade of membership* intermediate between full membership, represented by 1, and non-membership, represented by 0. Mathematically, the transition from regular sets (crisp sets) to fuzzy sets can be explained by assuming a universal set X such that A is a crisp subset of X. Define a function $v: X \to \{0,1\}$ such that $v(x)$ is 1 if $x \in A$ and $v(x)$ is 0 when $x \notin A$. This function is the characteristic function of any crisp subset A of X. Fuzzy sets generalize the characteristic function by allowing all values in the unit interval. This is known as the membership function. Any fuzzy subset F of X is defined by its membership function $\mu: X \to [0,1]$. Then $\mu(x)$ where $x \in F$ denotes the grade of membership of *x* in the fuzzy set F. X is called the *universe of discourse*. A fuzzy subset (or fuzzy set as it is commonly referred to) is defined as a set of ordered pairs $F = \{(x, \mu(x)) \mid x \in F, F \subset X\}$. The *membership function* is denoted by μ_F for the fuzzy set *F*.

For example, let *X* be the set of people and *F* be the set of 'Tall' people. Each person is assigned a degree of membership in the fuzzy subset TALL. For this we define a membership function based on the person's height. The intervals of 5ft and 7ft are arbitrarily chosen for the sake of the example and can be replaced by any other suitable intervals.

$$Tall(x) = \begin{Bmatrix} 0,\ \text{if height}(x)\ <\ 5ft \\ \text{height}(x)\text{ - }5ft/2ft,\ \text{if } 5ft.\ \leq\ \text{height }(x)\ \geq\ 7ft \\ 1, \text{if height}(x)\ >\ 7ft \end{Bmatrix}$$

Based on the above definition if there is two individuals 3 ft and 6ft tall would have 0 and 0.5 membership grade respectively [2].

3. Fuzzy logical operations.

Some fuzzy set theorems are presented in table 2 in appendix C. The negation operation defined in table 1 is standard. The computation of AND and OR are however not standard. The AND and OR operators in fuzzy logic are generalizations from classical logic and are called t-norms and t-conorms respectively. T-norms and t-conorms are functions from $[0,1]\times[0,1]$ into $[0,1]$ [1]. If $z = T(x, y)$, then x, y, z all belong to the interval $[0,1]$. All t-norms and t-conorms have the properties of commutativity, monotonicity, boundary and associativity. Let *tv* be a truth assignment, then $tv(P\ AND\ Q) = T(tv(P), tv(Q))$ for any t-norm. The basic t-norms are:

1) $T_m(x, y) = \min(x, y)$ (Zahedian intersection)
2) $T_L(x, y) = \max(0, x + y - 1)$ (Bounded difference intersection)
3) $T_p = xy$ (Algebraic product)

As for t-norms, If *C* is a t-conorm then, $tv(P\ OR\ Q) = C(tv(P), tv(Q))$. The basic t-conorms are:

1) $C_m(x, y) = \max(x, y)$ (Standard Union)
2) $C_L(x, y) = \min(1, x + y)$ (Bounded sum)
3) $C_p = x + y - xy$ (Algebraic sum)

4. Fuzzy propositions and truth assignments.

A *fuzzy proposition* is a statement that takes a fuzzy truth value. It includes logical connectives like AND, OR, NOT and Implication. An *atomic* fuzzy proposition cannot be decomposed into simpler fuzzy propositions and are combined together to create compound propositions. For example, a fuzzy rule of the form *(x is fast)* is an *atomic fuzzy proposition* because it can not be decomposed into simpler propositions. Other examples are *x* and *size is small*. A *compound fuzzy proposition* consists of atomic propositions that are conjuncted, disjuncted, complemented or connected by implication. An example of a complex fuzzy proposition is ((x is A) AND (x is B) AND (y is C) AND (z is D)). The truth value of a proposition *x*, that simply asserts that *x* exists is the truth value of *x.* If, for example, x is assigned a value 10 with truth value 0.75, then truth value of the proposition is 0.75. Most fuzzy propositions involve only single valued data like integers or strings [1]. They are of the general form (A(comparison operator)B). For example, A :($x = y$) where value of $x = y = 3$ and the membership values are $\mu_x = 0.8, \mu_y = 0.7,$ comparison operator = 1. Therefore, truth value of A = min(0.8,1,0.7) = 0.7. For propositions of the form (Size is small), the truth value (tv) of the fuzzy set size, tv(size) = 1, of the comparison operator 'is' is 1. The truth value of SMALL in size is its grade of membership in size, for example 0.4. Then tv((Size is small)) = min(1,1,0.4) = 0.4. Truth values of complex propositions are obtained by evaluating the logical operators. For example [1], let P be a complex proposition, $P = A\ AND\ B\ OR\ NOT\ C$. The truth values of A, B and C are 0.6, 0.8 and 0.3 respectively.

Then, tv(A)= 0.6, tv(A AND B) = min(0.6,0.8) = 0.6, tv(NOT C) = 1 - 0.3 = 0.7. Therefore, tv((A AND B) OR (NOT C)) = max(0.6,0.7) = 0.7.

5. Linguistic variables and linguistic values.

A linguistic variable is a variable whose value is expressed in terms of spoken language. These terms are imprecise and are represented by fuzzy sets. Let *t* be a variable that denotes temperature over an interval [0, T]. Let X be the domain of real numbers. Let there be three fuzzy sets L, M and H that have the member ship functions μ_L, μ_M, μ_H respectively. Each of these fuzzy sets can be referred to as LOW, MEDIUM and HIGH. TEMPERATURE can be treated as a *linguistic variable*. LOW, MEDIUM and HIGH are *linguistic values* of the *linguistic variable* TEMPERATURE. Therefore *t* can be LOW, MEDIUM or HIGH.

6. Fuzzy rules and inference.

Inference involves the modification of the values or truth values of data using rules [1]. A *fuzzy rule* is a proposition in the if-then format. It contains a main implication connective and fuzzy propositions. These can be of the form,

((x is A) AND (x is B) AND (y is C) AND (z is D)) => (r is Q)

where x, y and z are input variables, r is an output variable, A,B and C are membership functions defined on x, y and z respectively and Q is a membership function defined on r.

The rule's premise (left hand side) is called the antecedent and it describes to what degree the rule applies, while the conclusion (right hand side) is called the consequent. This assigns a membership function to each of one or more output variables. The set of rules used for inferencing in a fuzzy expert system is known as the rule base or knowledge base.

The most common tool used for fuzzy logical inference is *approximate reasoning,* used to infer new logical propositions from old ones. Approximate reasoning employs the generalized modus ponens. Classical modus ponens is *if A then B.* The fuzzy version is formulated as: If X is A then Y is B, from X = A' infer that Y = B'. Here A and A' are fuzzy sets belonging to the same universe and B and B' are also defined on the same universe which maybe different from that of A and A'.

7. Fuzzy expert systems.

An expert system involves the collection and encoding of human knowledge about prediction and classification, together with an inference engine for evaluating the rule base for a given set of inputs [1].

The basic components of a fuzzy expert system are a set of fuzzy rules, a set of fuzzy set membership functions and a *fuzzy inference engine* [2]. They are used in several wide-ranging fields including, linear and nonlinear control, pattern recognition, and financial systems and data analysis. The fuzzy inference engine contains a *scheduler* that selects the rules in a sequence for processing. A scheduler is needed because certain rules must be checked before others. Some rules fire and make their consequents true. These consequents are the antecedents in the certain rules. Thus a firing order is needed implemented by a scheduler. It also contains a *rule processor* that examines the left hand side of a rule to check the truth values of the conditions. It uses these truth values to fire the rule. A *set membership builder* builds an approximate membership function for the output from the firing of the rules using approximate reasoning. A *defuzzifier* then converts the inferred fuzzy values of the consequents to a *non-fuzzy* value. The operation of the expert system proceeds as follows.

1. *Fuzzification*: This process involves finding grades of membership of the linguistic values of a linguistic variable corresponding to an input value given by the user or process. The degree of truth for each rule antecedent is calculated by applying the membership functions to the input values.
2. *Inference*: The if–then rules are implemented and the truth value of each rule is computed using the truth values of the linguistic values. Approximate reasoning as describe earlier is used. Commonly used inference rules are *min* and *product*. In *min* inferencing, the output membership function is determined by either the truth value of the implemented rule or the original value of the membership function, whichever is lower (fuzzy logic AND). In *product* inferencing, the output membership function is scaled by the rule premise's truth value.
3. *Composition*: All of the fuzzy subsets assigned to each output variable are combined together to form a single fuzzy subset for each output linguistic variable. The methods used are *max* and *sum*. In *max*

composition, the fuzzy sets for the output variable are ORed with each other so that the maximum value from all the fuzzy subsets is used (fuzzy logic OR). In *sum* composition, the fuzzy subsets associated with the output value are summed to generate a single fuzzy subset.
4. *Defuzzification*: This is the process of calculating a scalar value from the fuzzy output. From composition we obtain a single fuzzy set. Defuzzification aggregates the set into a single value. The common techniques used are the *centroid* and *maximum* methods. In the *centroid* method, the scalar value of the output variable is computed by finding the variable value of the center of gravity of the membership function for the fuzzy value. The general formula is

$$\bar{X}(centroid) = \frac{\int_b^a x\mu(x)dx}{\int_b^a \mu(x)dx}$$

where [a, b] is the interval of the aggregated membership function. In the *maximum* method, the value at which the fuzzy subset has its maximum truth value is chosen as the value for the output variable.

8. Implementation of a simple fuzzy expert system.
To demonstrate the use of fuzzy expert systems, a fuzzy washing machine is implemented. The details of the system were obtained from reference [5]. A detailed implementation of the expert system is provided in appendix A.

1. Developing membership functions: The washing machine receives input from the user in the form of scalar values on a scale of 1 to 10. These correspond to two linguistic variables, "degree of dirtiness" and "type of dirt". Each has 3 linguistic values, Degree of dirtiness (D_d) = {Low, Medium, High}= $\{\mu_{LOW}, \mu_{MED}, \mu_{HI}\}$ and Type of dirt (D_t) = {Not greasy, Medium greasy, Greasy}= $\{\mu_{NG}, \mu_{MG}, \mu_{G}\}$. The expected output from the expert system is the linguistic variable "duration of the wash cycle (W_d)" ranging from 0 to 60 minutes. The linguistic values for the output are W_d = {Very short, Short, Medium, Long, Very long} = $\{\mu_{VL}, \mu_{L}, \mu_{M}, \mu_{S}, \mu_{VS}\}$. The linguistic variables are the fuzzy sets and each of the 11 linguistic values (6 for the inputs and 5 for the outputs) are fuzzy membership functions. The function definitions are listed in appendix A.

2. Developing the rule-base: We have 2 inputs comprising of 3 membership functions each. Thus we will have 3x3 = 9 rules of inference comprising the rule base.
If degree of dirtiness is High and type of dirt is Greasy, then duration of wash cycle is Very long;
If degree of dirtiness is Medium and type of dirt is Greasy, then duration of wash cycle is Long;
If degree of dirtiness is Low and type of dirt is Greasy, then duration of wash cycle is Long;
If degree of dirtiness is High and type of dirt is Medium, then duration of wash cycle is Long;
If degree of dirtiness is Medium and type of dirt is Medium, then duration of wash cycle is Medium;
If degree of dirtiness is Low and type of dirt is Medium, then duration of wash cycle is Medium;
If degree of dirtiness is High and type of dirt is not greasy, then duration of wash cycle is Medium;
If degree of dirtiness is Medium and type of dirt is not greasy, then duration of wash cycle is Short;
If degree of dirtiness is Low and type of dirt is not greasy, then duration of wash cycle is Very short

Since none of the above rules depend on the firing on any of the other rules, a scheduler for the firing of rules is not needed.

3. The expert system algorithm:
1. Accept user inputs from an interface. These are values on a scale of 1 to 10.
2. Fuzzify the inputs, i.e. determine the membership values in each of the 6 membership functions for both inputs.
3. Apply the membership values of the inputs to the rule-base to generate the truth values of the consequents of the 9 rules.

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4. Use MIN inferencing to generate new output membership functions. The new membership values are compared point wise to the original membership functions to get the modified output membership functions
5. Combine the new membership functions to form a single membership function using MAX composition method. In this all the newly generated membership functions are combined with each other using the fuzzy OR and the maximum possible membership value is chosen for each output value.
6. Perform defuzzification using the centroid method to get a scalar output value for the actual duration of the wash cycle.

9. Conclusion

Fuzzy expert systems provide a convenient way of handling uncertain values. They are used in several wide-ranging fields including, linear and nonlinear control, pattern recognition, and financial systems and data analysis. A table of the results obtained for the fuzzy washing machine expert system is attached in appendix A. The performance can be improved by using better defined rules of inference and membership functions.

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Trapezoidal Rule

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ABSTRACT: *In this paper we are writing a note on Trapezoidal Rule.In this Method we are using the general quadrature formula. Also we discuss a error in Trapezoidal Rule.*

Keywords: *Numerical Integration. Simpson's 1/3 rule, Simpson's 3/8 rule and Error in Trapezoidal Rule.*

1. Introduction

The Numerical Integration has been succcessful to study problems in Mathematics,Engineering,computes science and physical science.
It is helpful for the following cases:
(1) The integrand f(x) is not known explicity,but a set of data points is given for this integrand.
(2) The integrand f(x) may be known only at certain points,which are obtained by sampling.

1. Numerical Integration

The process of evaluating a definite integral from a set of tablated values of the integrand f(x) which is not known explicitly is called Numerical Integration.
Ie.,) Given a set of data point $(x_0,y_0),(x_1,y_1),\ldots\ldots,(x_n,y_n)$ of the function y=f(x) where f(x) is not explicity known it is required to compute the value of the definite integral.

I=$\int_a^b y\,dx$ (or)$\int_a^b f(x)dx$

Let be Interval a,b we divide into 'n' equal subintervals of with 'h' such that
$a=x_0< x_1 <x_2<_{\ldots}<x_n=b$.

Where $x_1=x_0+h, x_2=x_0+2h,\ldots\ldots\ldots, x_n=x_0+nh$.
Therefore, We have to evaluate the integral
I=$\int_{x0}^{x0+h} y\,dx$

Approroximating 'y' the Newton's formard difference formula we have,
I=$\int_{x_0}^{x_0+nh}[y_0 +p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0+\ldots\ldots..]dx$ ....(1)

Where $p=\frac{x-x_0}{h}$
$x=x_0+ph$
x_0 is a constant
dx=hdp

X	x_0	x_0+h
P	0	n

Sub (1) ⇒ I=$h\int_0^n[y_0 +p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0+\ldots\ldots..]dp$
$=h[py_0+\frac{p^2}{2}\Delta y_0+\frac{1}{2}\left(\frac{p^3}{3}-\frac{p^2}{2}\right)\Delta^2 y_0+\ldots\ldots\ldots\ldots..]_0^n$

$\int_{x_0}^{x_0+nh} f(x)\,dx$ =h[ny_0+$\frac{n2}{2}$ Δy_0+$\frac{1}{2}\left(\frac{n3}{3}-\frac{n2}{2}\right)$ $\Delta^2 y_0$+…………] ……………….(2)

This formula is called “Newton’s cotis quadrature formula”.

2.Trapezoidel Rule

Put n=1 in equation (2) we find that all the differences expect the first order difference Δy_0 becomes 0.

$\int_{x_0}^{x_0+h} f(x)\,dx$ =h[y_0+$\frac{1}{2}$ Δy_0]

$\int_{x_0}^{x_0+h} f(x)\,dx$ = h[y_0+$\frac{1}{2}$(y_1-y_0)]

Since y_0=y_1-y_0

where x_1=x_0+h

x_2=x_0+2h

$\int_{x_0}^{x_0+h} f(x)\,dx$ = h[$\frac{1}{2}$$y_1$+$\frac{1}{2}$$y_0$]

$\int_{x_0}^{x_0+h} f(x)\,dx$ = $\frac{1}{2}$h(y_1+y_0)

x	Y	Δ	Δ^2	Δ^3
x_0	y_0			
		(Δy_0)		
x_1	y_1		($\Delta^2 y_0$)	
		(Δy_1)		
x_2	y_2			($\Delta^3 y_0$)
		(Δy_2)	($\Delta^2 y_1$)	
x_3	y_3			

For the next interval [x_1,x_2] we deduce

$\int_{x_1}^{x_2} f(x)dx$ = $\frac{h}{2}$(y_1+y_2)

⋮

$\int_{x_{n-1}}^{x_n} f(x)dx$ = $\frac{h}{2}$ (y_{n-1}+y_n)

Adding this n integrals we obtained

$\int_{x_0}^{x_n} f(x)dx$ = $\frac{h}{2}$[y_0+y_n+2(y_1+y_2+…….+y_{n-1})]

This is called as “Trapezoidal Rule”.

3.Simpson’s 1/3 rule

Put n=2 in equation (2) that is by replacing the curve by n/2 arcs of 2^0 polynomial or parapolar.we have

$\int_{x_0}^{x_2} f(x)dx$ = h[2y_0+2Δy_0+$\frac{1}{2}\left(\frac{8}{3}-2\right)$ $\Delta^2 y_0$]

= h[2y_0+2(y_1-y_0)+$\frac{1}{3}$ $\Delta^2 y_0$]

= h[2y_0+2y_1-2y_0)+$\frac{1}{3}$(y_2-2y_1+y_0)]

= h[$\frac{1}{3}$$y_0$+$\frac{4}{3}$$y_1$+$\frac{1}{3}$$y_2$]

$\int_{x_0}^{x_2} f(x)dx$ = $\frac{h}{3}$(y_0+4y_1+y_2)

$\Delta^2 y_0$ = Δ(Δy_0)

$= \Delta(y_1 - y_0)$
$= \Delta y_1 - \Delta y_0$
$= y_2 - y_1 - y_1 - y_0$

$\Delta^2 y_0 = y_2 - 2y_1 - y_0$
$\Delta^3 y_0 = \Delta(\Delta^2 y_0)$
$= \Delta(y_2 - 2y_1 - y_0)$
$= \Delta y_2 - \Delta 2y_1 - \Delta y_0$
$= y_3 - y_2 - 2(y_2 - y_1) + y_1 - y_0$

$\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$
$\int_{x_2}^{x_4} f(x)dx = \frac{h}{3}(y_2 + 4y_3 + y_4)$
$\vdots$
$\int_{x_{n-1}}^{x_n} f(x)dx = \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n)$

Adding,
$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}[y_0 + y_n + 4(y_1 + y_3 + \dots\dots + y_{n-1}) + 2(y_2 + y_4 + \dots\dots + y_{n-2})]$

4.Simpson's 3/8 Rule
This rule is obtained by put n=3 in equation(2) and we observe that all difference higher then the 3 will become zero.

$\int_{x_0}^{x_n} f(x)dx = \frac{3h}{8}[y_0 + 3y_1 + 3y_2 + y_3]$
$\int_{x_0}^{x_3} f(x)dx = h[3y_0 + \frac{9}{2}\Delta y_0 + \frac{1}{2}\left(9 - \frac{9}{2}\right)\Delta^2 y_0 + \frac{1}{6}\left(\frac{81}{4} - 27 + 9\right)\Delta^3 y_0]$

Problem
Evaluate $I = \int_0^1 \frac{1}{1+x}dx$ correct to 3 decimal places by both the Trapezoidal and Simpson's Rules with h = 0.5 , 0.25 .

Solution
Given that,
$f(x) = \frac{1}{1+x}$

Case 1
When h=0.5

x	0	0.5	1
f(x)	1	0.667	0.5

By Trapezoidel rule
Area I $= \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + \dots\dots + y_{n-1})]$
$I = \frac{0.5}{2}[1 + 0.5 + 2(0.667)]$
$= 0.25(2.834)$
$I = 0.709$

Simpon's 1/3 Rule:
$I = \frac{h}{3}[y_0 + y_n + 4(y_1 + y_3 + \dots\dots + y_{n-1}) + 2(y_2 + y_4 + \dots\dots + y_{n-2})]$

$I = \frac{0.5}{3}[1+0.5+4(0.667)]$

$I = 0.695$

Case 2

When h = 0.25

X	0	0.25	0.5	0.75	1
f(x)	1	0.667	0.667	0.571	0.5

By Trapezoidal Rule

$I = \frac{0.25}{2}[1+0.5+2(0.8+0.667+0.667)]$

$I = 0.125(5.576)$

$I = 0.697$

Simpson's 3/8 Rule

$I = \frac{0.25}{3}[1+0.5+4(0.8+0.571)+2(0.667)]$

$= 0.083[1.5+5.484+2.334]$

$= 0.083(9.318)$

$I = 0.7733$

5. Error in Trapezoidal Rule

The Taylor's series expansion y = f(x) about x_0 is given by

$$f(x) = f(x_0) + \frac{x-x0}{1!}f'(x_0) + \frac{(x-x0)2}{2!}f''(x_0) + \ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\int_{x0}^{x1} f(x)\,dx = \int_{x_0}^{x_1}[f(x_0) + \frac{(x-x_0)}{1!}f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + \cdots]dx$$

$$= \left[xf(x_0) + \frac{(x-x_0)^2}{2!}f'(x_0) + \frac{(x-x_0)^3}{3!}f''(x_0) + \cdots\right]_{x_0}^{x_1}$$

$$= (x_1 - x_0)f(x_0) + \frac{(x_1-x_0)^2}{2!}f'(x_0) + \frac{(x_1-x_0)^3}{3!}f''(x_0) + \cdots$$

$$\int_{x_0}^{x_1} f(x)\,dx = hy_0 + \frac{h^2}{2!}y_0' + \frac{h^3}{3!}y_0'' + \cdots \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where $h = x_1 - x_0$

$f(x_0) = y_0$

$f'(x_0) = y_0'$

The area of the Trapesion in the interval is given by

$$A_0 = \int_{x_0}^{x_1} y\,dx$$

$$A_0 = \frac{h}{2}(y_0 + y_1) \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

Sub $x = x_0$ in (1) we get,

$f(x_1) = f(x_0) + \frac{(x_1-x_0)}{1!}f'(x_0) + \frac{(x_1-x_0)^2}{2!}f''(x_0)+\ldots\ldots.$

$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \ldots\ldots. \quad \ldots\ldots\ldots\ldots..(4)$

sub (4) in (3) we get.

$A_0 = \frac{h}{2}$ [$2y_0+\frac{h}{1!}y_0'+\frac{h^2}{2!}y_0''+$.......]

$= hy_0+\frac{h^2}{2}y_0'+\frac{h^3}{2.2!}y_0''+$....... ....(5)

(3) – (5) given that,

$\int_{x_0}^{x_1} f(x)\,dx$ $-A_0 = + h^3y_0''\left(\frac{1}{6}-\frac{1}{4}\right)$........

$= h^3y_0''\left(-\frac{1}{12}\right)+$............

The principal part of the error in the interval

$(x_0,x_1) = -\frac{h^3y_0''}{12}$

Similarly, the principal part of the error in the interval

$(x_1,x_2) = -\frac{h^3y_1''}{12}$

$\vdots$

and the principal part of the error in the interval

$(x_{n-1},x_n) = -\frac{h^3y_{n-1}''}{12}$

Therefore, the total error E is given by

$E \simeq -\frac{h^3}{12}[y_0''+y_1''+y_2''+\cdots+y_{n-1}'']$

$|E| < n\frac{h^3}{12}M$

Where $M = \max(|y_0''|, |y_1''|, \ldots\ldots |y_{n-1}''|)$

$|E| < (b-a)\frac{h^2}{12}M$

Where $h = \frac{(b-a)}{n}$

Hence the error in the trapezoidal rule is of order h^2.

Conclusion

As simpson's 1/3 rule required the devision of the hole range into an even number of sub-interval of with h.

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A Study on Kuhn-Tucker Conditions

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ABSTRACT: *In this paper, we discussed the constrained optimization with inequality constraints and state the Kuhn-Tucker necessary conditions for a solution; after an example, we state the Kuhn-Tucker sufficient conditions for a maximum.*

Keywords: *Kuhn-Tucker condition, Invex function*

1. Introduction

The Kuhn-Tucker conditions have been used to derive many significant results in economics, particularly in decision problems that occur in static situations, for instance, to show the existence of an equilibrium for a competitive economy, to carry out the first-order approach to principal-agent problems and to examine the need for land reform. Also, the Kuhn-Tucker conditions and/or the method of Lagrange multipliers are usually contained in standard microeconomics textbooks, for instance, Mas-Colell, Whinston and Green, where the Kuhn-Tucker conditions for the problem with both inequality and equality constraints are discussed.

The necessary conditions for optimality of solution points in mathematical programming problems will be studied. Because of the orientation of this book to present optimization theory as an instrument for qualitative economic analysis, the theory to be described is not immediately concerned with computational aspects of solution techniques, which can be found in many excellent books on mathematical programming, The discussion begins with the extension of the Lagrange theory by Kuhn and with the derivation of necessary optimality conditions for the optimization problems including inequality constraints.

Preliminaries

The problem to be addressed is as follows:

$$
\begin{array}{lll}
\textbf{(P)} & \textbf{maximize} & f(x) \\
& \textbf{subject to} & g_i(x) \geq 0, \quad i \in I, \\
& & h_j(x) = 0, \quad j \in J,
\end{array}
$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $g_i : \mathbb{R}^n \to \mathbb{R}, i \in I$, and $h_j : \mathbb{R}^n \to \mathbb{R}, j \in J$ are continuously Fréchet differentiable functions, and $|I| = m$, $|J| = \ell$ ($m, \ell \in \mathbb{Z}_+$). If there are no inequality (equality) constraints, we think that $m = 0$ ($\ell = 0$).

Here, we should pay attention to the fact that the problem (P) naturally includes the optimization problem with equality constraints considered by Lagrange in the 18th century.

We postulate the following Mangasarian-Fromovitz constraint qualification (MF) (Mangasarian and Fromovitz) in association with (P). We define $I(\bar{x}) = \{i \mid g_i(\bar{x}) = 0, i \in I\}$.

(MF) For $\bar{x} \in \mathbb{R}^n$, $\nabla h_j(\bar{x})$, $j \in J$ are linearly independent, and there exists an $\exists d \in \mathbb{R}^n$ s.t. $\langle \nabla g_i(\bar{x}), d \rangle > 0, i \in I(\bar{x})$ and $\langle \nabla h_j(\bar{x}), d \rangle = 0, j \in J$.

Remarks
The linearly independent constraint qualification, which is usually assumed in practice, implies (MF) is equal to the Cottle constraint qualification without the presence of equality constraints, and if the problem (P) is a concave program without equality constraints, the Slater constraint qualification implies the Cottle constraint qualification (Bazaraa and Shetty). Finally, we recall the following result to the linear system including equalities for the sake of convenience.

Lemma 1. Corollary 2 to Theorem 2.3.5) For $A \in \mathbb{R}^{n \times m}$, $B \in \mathbb{R}^{n \times \ell}$, and $c \in \mathbb{R}^n$, either
(a) $\exists y \geq 0$, $\exists z \in \mathbb{R}^\ell$, $c + Ay + Bz = 0$,
or
(b) $\exists x \in \mathbb{R}^n$, $\langle c, x \rangle > 0$ and $A^T x \geq 0$, $B^T x = 0$
but never both.

The Kuhn-Tucker Theorem
The basic mathematical programming problem (1.28), as described that of choosing values of *n* variables so as to minimize a function of those variables subject to *m* inequality constraints:
minimize f_0 (x)
subject to *ft**(x)*** ^ 0 *(i = 1, 2,...,m).*

Stationarity
∇**x***f*(**x**)+∑*mi*=1∇**x***λihi*(**x**)+∑*ni*=1*μi*∇**x***gi*(**x**)=0 (minimization)
∇**x***f*(**x**)+∑*mi*=1∇**x***λihi*(**x**)−∑*ni*=1*μi*∇**x***gi*(**x**)=0 (maximization)

Equality constraints
∇*λf*(**x**)+∑*mi*=1∇*λλihi*(**x**)+∑*ni*=1*μi*∇*λgi*(**x**)=0

Inequality constraints a.k.a. complementary slackness condition
μigi(**x**)=0,∀*i*=1,..,*n*
μi≥0,∀*i*=1,..,*n*
This problem is a generalization of the classical optimization problem (which uses constraints in equation form), since equality constraints are a special case of inequality

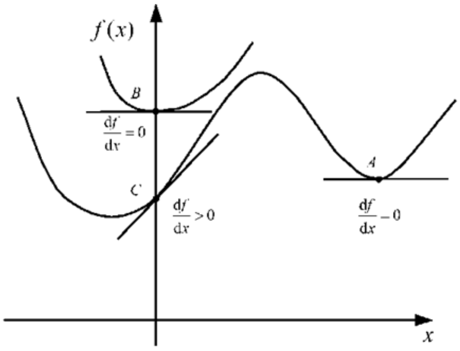
Rationale of the Kuhn-Tucker Conditions
The Kuhn-Tucker conditions are the natural generalization of the Lagrange multiplier approach, from classical differential calculus replacing equality constraints by inequality constraints, to take account of the possibility that the maximum or minimum in question can occur not only at a boundary point but also at an interior point. The calculus requirements are generally appropriate only if the extremum (i.e., the maximum or minimum) occurs at a point at which all of the variables (including the slack variables) take nonzero values.
Now we consider—for simplicity, but without loss of generality—the minimization of the function *f(x)* subject to *x* > 0. In this case, the matter can be illustrated graphically. Suppose first that we are at a point at which the value of *x* can either be increased or decreased. By the usual logic of marginal analysis, we must have ^{f} = 0, for otherwise either a rise or a fall in the value of *x* could increase the value of *f,* and *f* would not be at its minimum.
On the other hand, suppose we are testing for the possibility of a boundary minimum at which *x* = 0.Two possibilities for local minimum of the function *f(x)* subject to *x* > 0 can be observed. If *dx* = 0, the point

with $x = 0$ may be a minimum for the usual reasons, and if ${}^{d}df\!x > 0$, it may be a minimum point simply because it is impossible to reduce the value of x any further

Direct generalization for the function with n variables leads to the following conclusions. Given a differentiable function $f(x\backslash, x_2,...,x_n)$,

- for an interior minimum (maximum), it is necessary that $= 0$ $(j = 1, 2,...,n)$;
- for a boundary minimum, it is necessary that > 0 $(j = 1, 2,...,n)$.

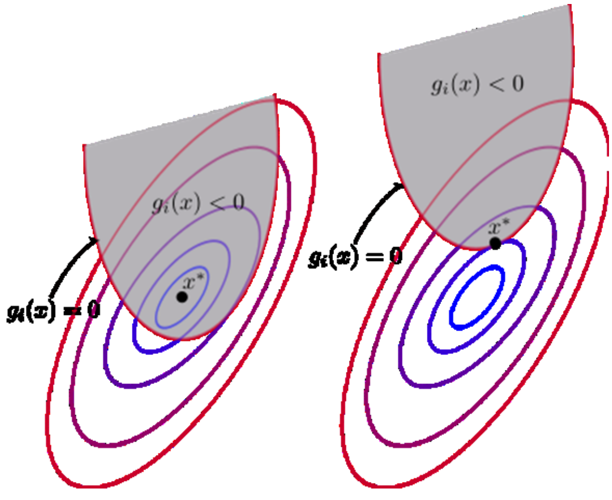
The reader may check that—by the same reasoning—for a boundary maximum it is necessary that *dxj*- ^ 0.



Similar to the interpretation of the complementary slackness conditions (2.3')or (2.24), the conditions in (2.21) serve to determine which solution case occurs; if the value of *xj* under consideration is positive (interior minimum case), then (2.21) requires Hj- = 0. If Hj- > 0, then we can only have a boundary minimum *(xj* = 0).

Necessary conditions

Suppose that the objective function and the constraint functions and are continuously differentiable at a point . If is a local optimum and the optimization problem satisfies some regularity conditions (see below), then there exist constants and , called KKT multipliers, such that



Sufficient conditions

In some cases, the necessary conditions are also sufficient for optimality. In general, the necessary conditions are not sufficient for optimality and additional information is necessary, such as the Second Order Sufficient Conditions (SOSC). For smooth functions, SOSC involve the second derivatives, which explains its name.

The necessary conditions are sufficient for optimality if the objective function of a maximization problem is a concave function, the inequality constraints are continuously differentiable convex functions and the equality constraints are affine functions.

It was shown by Martin in 1985 that the broader class of functions in which KKT conditions guarantees global optimality are the so-called Type 1 **invex functions**.

Second-order sufficient conditions

For smooth, non-linear optimization problems, a second order sufficient condition is given as follows.
The solution found in the above section is a constrained local minimum if for the Lagrangian,
then,where is a vector satisfying the following,where only those active inequality constraints corresponding to strict complementarity (i.e. where) are applied. The solution is a strict constrained local minimum in the case the inequality is also strict.

Non-negativity conditions

Many of the optimization problems in economic theory have non negativity constraints on the variables. For example, a consumer chooses a bundle x of goods to maximize her utility $u(x)$ subject to her budget constraint $p\cdot x \le w$ and the condition $x \ge 0$. The general form of such a problem is
$\max_x f(x)$ subject to $g_j(x) \le c_j$ for $j = 1, ..., m$ and $x_i \ge 0$ for $i = 1, ..., n$.
This problem is a special case of the general maximization problem with inequality constraints, studied previously: the non negativity constraint on each variable is simply an additional inequality constraint. Specifically, if we define the function g_{m+i} for $i = 1, ..., n$ by $g_{m+i}(x) = -x_i$ and let $c_{m+i} = 0$ for $i = 1, ..., n$, then we may write the problem as
$\max_x f(x)$ subject to $g_j(x) \le c_j$ for $j = 1, ..., m+n$
and solve it using the Kuhn-Tucker conditions

$L_i'(x) \qquad = 0$ for $i = 1 ,..., n$

$\lambda_j \ge 0$, $g_j(x) \le c_j$ and $\lambda_j[g_j(x) - c_j] = 0$ for $j = 1, ..., m + n$,

Where $L(x) = f(x) -\sum m+n\ j=1\lambda_j(g_j(x) - c_j)$.

Result

We now establish the main result,
Theorem 5.3.1 in that our result includes complementarity conditions and the boundedness of Lagrange multipliers under (MF).

Theorem 1. Suppose that $\bar{x} \in \mathbb{R}^n$ is a local solution for (P), and that the constraint qualification (MF) holds at $\bar{x}$. Then, it holds that for $\exists \bar{\lambda}_i \ge 0, i \in I$ and $\exists \bar{\mu}_j \in \mathbb{R}, j \in J$,

$$\nabla f(\bar{x}) + \sum_{i \in I} \bar{\lambda}_i \nabla g_i(\bar{x}) + \sum_{j \in J} \bar{\mu}_j \nabla h_j(\bar{x}) = 0, \qquad (1)$$

$$\bar{\lambda}_i g_i(\bar{x}) = 0, \quad \bar{\lambda}_i \ge 0, \quad i \in I,$$

and $\bar{\lambda}$, $\bar{\mu}$ are bounded.

Proof.

At a local solution $\bar{x} \in \mathbb{R}^n$, if we choose x^k in the feasible region such that $x^k - \bar{x} = t_k s + o(t_k)$,

$$\frac{g_i(x^k) - g_i(\bar{x})}{t_k} = \frac{1}{t_k}\left[t_k \langle \nabla g_i(\bar{x}), s\rangle + o(t_k)\right] = \langle \nabla g_i(\bar{x}), s\rangle + \frac{o(t_k)}{t_k} \ge 0 \quad \text{as } t_k \downarrow 0, \quad i \in I(\bar{x}),$$

$$\frac{h_j(x^k) - h_j(\bar{x})}{t_k} = \frac{1}{t_k}\left[t_k \langle \nabla h_j(\bar{x}), s\rangle + o(t_k)\right] = \langle \nabla h_j(\bar{x}), s\rangle + \frac{o(t_k)}{t_k} = 0 \quad \text{as } t_k \downarrow 0, \quad j \in J,$$

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which shows that $s \in \mathbb{R}^n$ satisfies $\langle \nabla g_i(\bar{x}), s \rangle \geq 0$, $i \in I(\bar{x})$ and $\langle \nabla h_j(\bar{x}), s \rangle = 0$, $j \in J$.

Then, for a local solution $\bar{x}$, it follows that

$\langle \nabla f(\bar{x}), s \rangle > 0$

does not hold, since, if so, $\frac{f(x^k) - f(\bar{x})}{t_k} = \frac{1}{t_k}\left[t_k \langle \nabla f(\bar{x}), s \rangle + o(t_k)\right] > 0$ as $t_k \downarrow 0$ for $x^k \to \bar{x}$, which con-tradicts the local optimality of f at $\bar{x}$.

Note that (MF) guarantees the existence of such $0 \neq s = \exists d \in \mathbb{R}^n$ from the implicit function theorem, Appendix D-3 with $h(x_I, x_{II}) = 0$ and $(x_I, x_{II}) \in \Lambda \subset \mathbb{R}^{n-\ell} \times \mathbb{R}^{\ell}$) if $|I(\bar{x})| + \ell \geq 1$; otherwise (2) does not hold for $0 \neq \forall s \in \mathbb{R}^n$. By applying Lemma 1 to (2) and (MF), even if the active constraints are empty, we obtain

$$\nabla f(\bar{x}) + \sum_{i \in I(\bar{x})} \bar{\lambda}_i \nabla g_i(\bar{x}) + \sum_{j \in J} \bar{\mu}_j \nabla h_j(\bar{x}) = 0, \ \ \bar{\lambda}_i g_i(\bar{x}) = 0, \bar{\lambda}_i \geq 0, i \in I(\bar{x}),$$

or, equivalently,

$$\nabla f(\bar{x}) + \sum_{i \in I} \bar{\lambda}_i \nabla g_i(\bar{x}) + \sum_{j \in J} \bar{\mu}_j \nabla h_j(\bar{x}) = 0, \quad \bar{\lambda}_i g_i(\bar{x}) = 0, \bar{\lambda}_i \geq 0, i \in I$$

for $\bar{\lambda}_i = 0$, $i \in I \setminus I(\bar{x})$.

The rest part of the proof is as follows. From (3) we obtain

$$\left\langle \nabla f(\bar{x}) + \sum_{i \in I(\bar{x})} \bar{\lambda}_i \nabla g_i(\bar{x}), d \right\rangle = 0, \quad \exists d \text{ in } (\mathrm{MF}).$$

So, if $|I(\bar{x})| \neq 0$,

$$-\frac{\langle \nabla f(\bar{x}), d \rangle}{|I(\bar{x})| \min_{i \in I(\bar{x})} \langle \nabla g(\bar{x}), d \rangle} \geq \bar{\lambda}_i \geq 0, \quad i \in I(\bar{x}),$$

and if $|I(\bar{x})| = 0$, $\bar{\lambda}$ vanishes. In any case, (3) reduced to

$$(\text{a bounded vector}) + \sum_{j \in J} \bar{\mu}_j \nabla h_j(\bar{x}) = 0.$$

Since $\nabla h_j(\bar{x})$, $j \in J$ are linearly independent by (MF), $\bar{\mu}$ is determined to a single bounded vector. □

Example 1.
Consider the problem

$$\begin{aligned} \text{maximize} \quad & -x_1^2-(x_2-1)^2 \\ \text{subject to} \quad & -(x_1+1)^2-(x_2-1)^2+2\geq 0, \\ & -(x_1-1)^2-(x_2-1)^2+2\geq 0, \\ & x_1=0, \end{aligned}$$

with an optimal solution $\bar{x}=(0,0)^T$. At $\bar{x}$, the linearly independent constraint qualification does not hold, whereas (MF) holds for $d=(0,1)^T$.
Indeed, (MF) is valid for problems with a number of inequality constraints and admits feasible directions around d in the orthogonal complementary space of $\nabla h_j(\bar{x})$, $j \in J$.

Conclusion
In this paper, we discussed the constrained optimization with inequality constraints and state the Kuhn-Tucker necessary conditions for a solution. After an example, we state the Kuhn-Tucker sufficient conditions for a maximum are easily explained. It is easy understand for readers.

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Vertex and Edge Colouring in Graph Theory

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ABSTRACT: *A Graph G is a mathematical structure consisting of two sets V (G) (Vertices of G) and E (G) (Edge of G). Proper Colouring of a graph is an assignment of colour either to vertex of the graph or to the edge, G such a way that adjacent vertices edge or coloured differently. And we consider many classes of graphs to colour with applications. And in this we recall some basis definition then we prove the theorem X (G) ≤ (G) + 1 for vertex colouring. And we discuss about vising's theorem existing results for vertex colouring and edge colouring.*

1. Introduction

This Chapter is targeted to introduce definitions & notations required for the advancement of the content. Normally the aim is to use the smallest number of colours, which is denoted by X' (G). This notion is one of the main theme of the theory of graph colouring and is studied extensively. And the concept of colouring has found applications in such diverse fields as the theory of automata and storage problems in operations research. And the we discuss the theory of colourings, which grew along wt attempts to settle the 4CC.

Definition

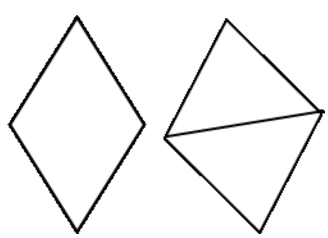
An assignment of colours to the graph so that no two adjacent vertices get the same colour is called a colouring of the graph". For each colour, the set of all points which gets that colour is independent and is called an colour class. G is an n colouring. The Chromatic number X(G) of a graph G is the minimum number of colours needed to colour G. Then a graph G is called, n – Colourable if X(G) ≤ n.

Examples:

Graph	Kp	Kp-x	Kp	Km, n	C_{2n}	C_{2n+1}
Chromatic Number	P	P-1	1	2	2	3

Graph Colouring Problem:

Graphs are made of vertices connected by edges. If you colour each vertex in a "Unit graph". Where every edge is the same length and require that connected vertices have different Colours.



Vertex Colouring

A K-vertex colouring of a graph G is an assignment of K colours to the vertices of G Such that no two adjacent vertices receive the same colour.

Normally colours are not always represented by positive integers. A K-Colourings of G is a function

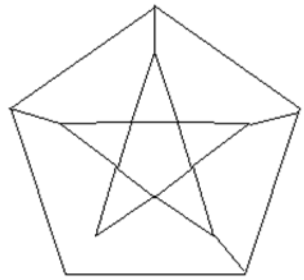
f:V(G) → (1, 2 ....K)

: f(u) = f(v), Whenever (u, v) E E(G).

Remarks:

Every graph with P vertices is p – Colourable.

If a graph is k – Colourbale, then it is t – Colourable for every integer t, t ≥ k.



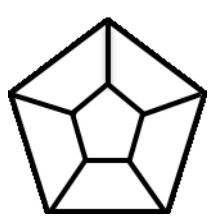
Theorem: 6.4
For any graph G, $X(G) \leq \Delta(G) + 1$

Proof
We Prove the theorem by induction on p (= |V(G) |).
If p=1, then the result is obvious.
Now we assume that the result for all graphs on p – 1 Vertices.
Let G be a graph on p ($\geq$ 2) vertices. Let v be a vertex in G of maximum degree and Let V_1, V_2............V_{Δ} be the adjacent to v.

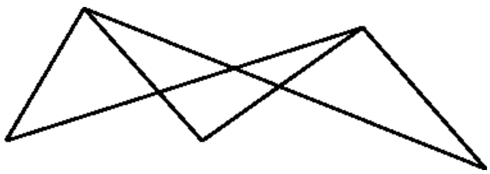
Now, consider G – v. Using induction hypothesis,
$X(G - v) \leq \Delta(G - v) + 1$
$\leq \Delta(G) + 1$

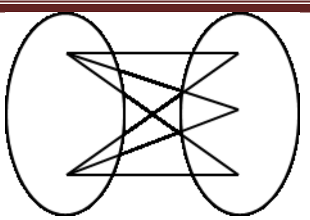
Hence (Δ(G) +1) – Colouring of G – v.
We can now easily extend this Colouring a (Δ(G) +1). Colouring of G.
Sice deg (v) = Δ, (Δ(G)+1 Colours atleast one colour.
We obtain a (Δ(G) +1) – Colouring of G.
$X(G) \leq \Delta(G) + 1$
$X(kp) = \Delta(kp) + 1$
$X(C_{2n+1}) = \Delta(C_{2n+1}) + 1$

Edge Colouring
A k – edge colouring of a Graph G is an assignment of k colours to the edges of G such that no two adjacent edge receive the same colour.
F : E(G) → (1, 2...........k)
: f(x) = f(y) Whenever x and y are adjacent in G.
There is a close relationship between edge – Colourings and 4.C.C.
No edge of the same colour are incident with the same vertex. In other words, the set of edge of the same colour is independent.



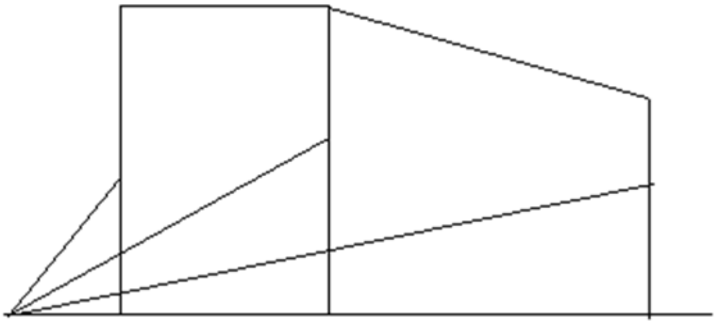
Bipartite Garphs in Terms of Colouring Graph
Theorem:
A Simple graph is bipartite if and only if it is possible to assign one of two different colours to each vertex of the graph so that no two adjacent vertices are assigned the same colour.





Claim : 3 – CNF – SAT ≤ 3 – COLOUR

Proof
Given 3 – SAT instance with n variables Xi and k clauses C_i
For each clause, and "gadget" of 6 new nodes and 13 new edges.



The Five Colour theorem
Statement:
Every planar graph is 5 vertex Colourable.

Proof
Proof by Contraduction
Suppose that the theorem is false. Then there exist a Six Critial Plane graph is simple. We know that,
If G is a simple planar graph then S ≤ 5. We know by Corollary that S ≤ 5.
On the otherhand by a theorem,
"If G is k – Critical them S ≥ k – 1
S = 5

Let v be a vertex of degree 5 in G.
Let (V_1, V_2..........V_5) be a planar 5 – vertex colouring of G – v.
Such a colouring exist, because G is 6 – Critical. Since,
G itself is not 5 – vertex Colourable must be adjacent to a vertex of each of 5– colours
We can arrange the neighbours of v in the clockwise order about v which are V_1, V_2..........V_5 Where V_i E V_i for 1 ≤ i ≤ 5. Denote by G_{ij} the Subgraph.

Now,
V_i and V_j must belong to the same component of G_{ij} for otherwise, 1 consider the component of G_{ij} that contains Vi by interchanging the colours i & j. In this Component. We obtains a new proper 5 – vertex colouring of G – v in which only four colours are assigned to the neighbour of v. We have already shown that this situation can't arise.
V_i and V_j must belong to the same component of G_{ij}.

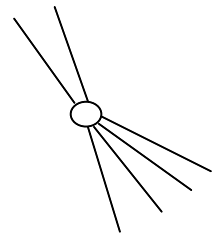
Let Pij be a (V_i, V_j) path in Gij and Let C denote the cycle V $e_{13}V_3$V.
Since, C seperators V_2 and V_4.
[V_2E int c and V_4 E ext c] it follows from Jordan curve them that the path (V_2, V_4) must meet C in some point, because G is a plane graph. This point must be a vertex. But this is impossible.

Since, the vertices of P_{24} have colour 2 and 4 Where as no vertex as c as either of these colours.

STATE AND PROVE VIZING'S THEOREM:

Statement:

If G is Simple then either X' = (or) X' = +Δ Δ



Proof:

Let G be a simple graph

X' ≥ Δ

We have to prove that

X' ≤ +1 Δ

Suppose that

X' > Δ+ 1

Let C = E_1, E_2..........E +1 be a optimal Δ edge colouring Δf G.

Let u be a vertex -) : C(u) < d (u)

If a colour i_0 & i_1 -) :i0 is not represented at u and i_1 is represented atleast twice at u.

Let uv_1 have colour i_1 as shown in figure since $d(V_1)$ < +1. Δ

Some colour i_2 is not represented at v_1. Now i_2 must be represented at u.

Otherwise by recolouring uv_2 by i_2, We would obtain and improvement on c.

Thus some edge uv_2 has colour i_2 again $d(V_2)$ < +1. Δ

Some colour i_3 is not represented at v_2 of i_3 must be represented at u.

Otherwise by recolouring uv_2 by i_2, We would obtain and improvement on c.

Thus some edge uv_2 has colour i_2 again c (V_2) < +1. Δ

Some colour i_3 is not represented at V_2 of i_3 must be represented at u.

Otherwise by recolouring uv_1, by i_2 and uv_2 by i_3 we would obtain at important on c.

Thus some edge uv_3 has colour i_3 containing is this way we have a sequence V_1, V_2......... of vertices and i_1, i_2......... of colour -):

i) Uv_j has colour ij

ii) ij + 1 is not represented at V_g but d(u) is finite. If a smallest integer l.

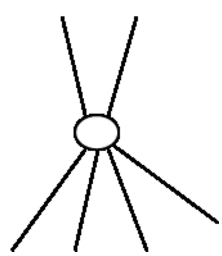
-) : for i ≤ j ≤ l – l

iii) il + 1 = i_k

We now recolour G has follows for i ≤ j ≤ k – l recolour uvj with colour ij + 1

Given a new (+1) edge Δolouring

C_1 = (E_1', E_2' ....E'_{+1}) as Δshow in figure.



Clearly $C'(V) \geq C(V)$ V V (- V —

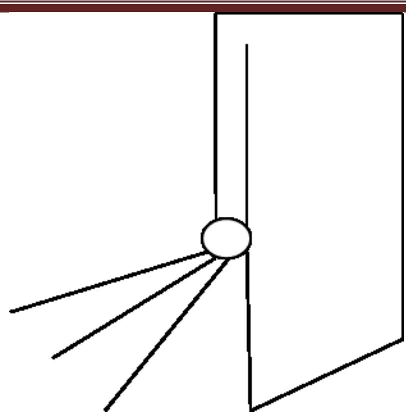
C1 is also an optimal (+1) edge cΔlouring.

The component u' of G [E_iO U E_ik'] contain u is an odd cycle.

Now in addition recolour uv_j with colour ij + 1

For and uvi by colour ik to obtain a (+1)Δdge colouring.

C'' = [E''1, E''2,.......... E''_{+1}] as show in fiΔure.



Now, C″(V) > (C) (V) V V (- V —
The component u'' of G [E″ iO U Eik″]
That contains u is an odd cycles
But d(Vk) = 2 in H'
d(Vk) = 1 in H''

This is Controdiction
Hence X' ≤ △+1
△≤ X' ≤ △+1
X' < △(or) X' = △1

Hence the vizing's theorem.
Some existing results for vector colouring:
If the graph G has n – vertices then X(G) ≤ n.
X(H) ≤ X(G), ifH is a Subgraph of graph G.
X(k_n) = n, for all n ≥ 1.

If the graph G has G_1, G_2........G_n as its connected components then X(G) = max X(G_i). 1≤ i ≤ n
For any non-empty graph G, X(G)=2 if and only if G is bipartite.
X(G) ≥ n when G contain Kn as a Subgraph

For any non-empty graph G, X(G) ≥ 3 if and only if G has an odd cycle.
Let G be a connected graph with (G) ≥ 3. If G△nt complete then X(G) ≤ (G). △
For any graph G, X(G) ≤ (G) +1. △
Some existing results about edge colouring:
Let G be a non-empty bipartite graph then X'(G) = (G). △
Let G=K_n, the complete graph on n vertices, n ≥ 2. Then
X'(G) = △ (G), if n is even,
△(G) +1, if n is odd

Let G be non-trivial graph. Then
△(G) ≤ X' (G) ≤ △(G) +1.

Conclusion

The Colouring graph have attracted many researchers to work on it. We have provided basis informations, termgology and existing results on the concepts. And Graph colouring enjoys many practical applications as well as theoretical challenges and that colouring graphs contain graphs are key objects studied in discrete mathematics. And it has even reached popularity with the general public in the form of the popular number puzzle Sudoku Graph colouring is still a very active field of research. This chapter was intended to report some existing result.

Acknowledgement
It is my privilege to express my earnest gratitude to those who guided any support me during my work pertaining the dissertation on thd subject of "A STUDY OF GRAPH COLOURING" for the awarded of degree of master of philosophy in the subject of mathemathics. And I am highly indedled to all those who have helped me to fullfill my academic desire.

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Graph Theory And Its Applications

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ABSTRACT: *The field of mathematics plays vital role in various fields. One of the vital areas in mathematics is graph theory that is employed in structural models. Graph theory was started with Euler idea. Graph theory is used for finding communities in network. We discuss about the title of graphs, Special graphs Bipartite graphs and Isomorphic graphs. The main idea of this paper is quite simple to know about graph theory.*

1. Introduction

Graph theory started with Euler who was ask to find unique path across the seven Konigsberg bridges. The (Eulerian) path should cross over every of the seven bridges exactly once Another early bird was Sir William Rowan Hamilton (1805-1865).ln 1859 he urban a modern based on finding a path visiting of a graph exactly once and sell it to a modern creator in Dublin. It never was a big success. But now graph theory is used for finding communition in networks.

Content

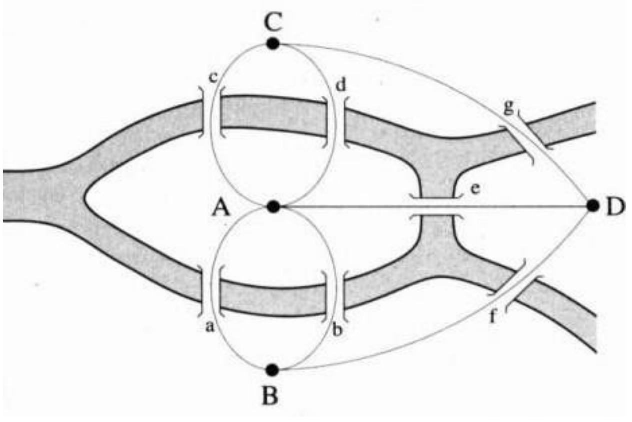
- Definition of graphs
- Complete graph
- Isomorphic
- Walks Trail and Paths
- Trees

Definition

A graph G consists of pair [V(G),X(G)] where V(G) is a non-empty finite set whose elements are called **points** or **vertices** and X(G) is a set of unordered pairs of distinct elements of V(G). The elements of X(G) are called **lines** or **edges** of the graph G. If x={u,v}■X(G), the line x is said to join u and v. We write x=uv and we say that the points u and v are **adjacent.** We also say that point u ns the line x are **incident** with other. If two distinct lines x and y are incident with common point then they are called **adjacent lines** . A graph with p points and lines is called (p,q) **graph.**
A **simple** graph has no self-loops or multiple edges.

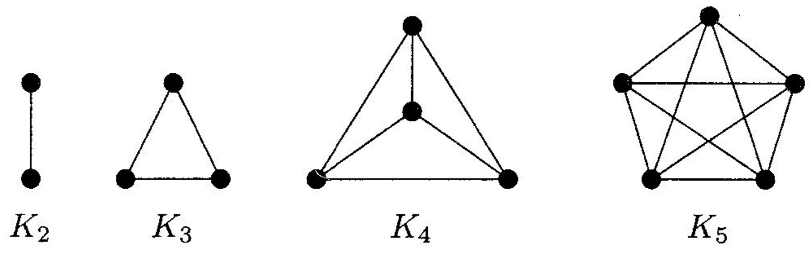
Proposition

The total of the degrees of a graph G = (V;E) equals 2|E| = 2m (trivial).



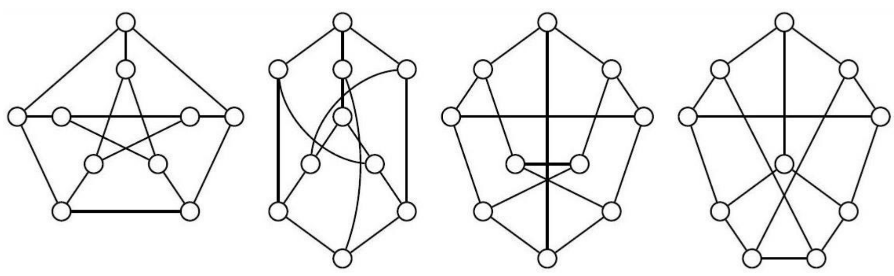
Complete Graph
A complete graph with in which any two distinct points are adjacent is called a complete graph. The whole graph with p points is denoted by K_p.

Example



A graph G is called a **bigraph** or **bipartite graph** if V can be partitioned into two disjoint subsets V_1 and V_2 such that every line of G joins a point of V_1 to a point V_2 .(V_1,V_2) is called a
bipartition of G. If further G contains every line joining the points of V_1 to the points V_2 then G is called **complete bigraph.**
If V_1 contains m points and V_2 contains n points then the complete bigraph G is denoted by $\mathbf{K_{m,n}}$.

Isomorphic
Two graphs G_1 and G_2 are **isomorphic** if there is a bijection between their respective nodes which make each edge of G_1correspond to exactly one edge of G_2, and vice versa.



Proposition 1
Let G be a k - regular bigraph with bipartition (V_1,V_2) and k>0. Prove that $|V_1|=|V_2|$.

Proof
Since every line of G has one end in V_1 and other end in V_2,it follows that
$\sum v\in V1\ (v) = \sum v\in V2(v)$=q.

Also d(v)=k for all $v \in V= V_1 \cup V_2$. Hence $\sum_{v\in V1} d(v)$=k$|V_1|$ and $\sum_{v\in V2} d(v)$=k$|V_2|$ so that k$|V_1|$ = k$|V_2|$.
Since k>0 We have $|V_1|=|V_2|$.

Walk , Trail and Path
A **walk** of a graph G is an alternating sequences of points and lines $v_0,x_1,v_1,x_2,v_2,\ldots,x_n,v_n$ beginning and ending with points such that each line xi is incident with v_{i-1} and v_1. The walks joins v_0 and v_n and it is called v_0-v_n walk. v_0 is called the **initial point** and v_n is called the **terminal point** of the walk.
A walk is called a **trail** if all its lines are distinct and is called a **path** if all its points are distinct.

Example
For the graph $v_1,v_2,v_3,v_4,v_2,v_1,v_2,v_5$ is a walk.
V_1,v_2,v_4,v_3,v_2,v_5 is a trail but no a path, v_1,v_2,v_4,v_5 is a path.

Note

1. Obviously every path is a trial and a trial need not be a path
2. The graph consisting of a path with n points is denoted by P_n.

Proposition 2

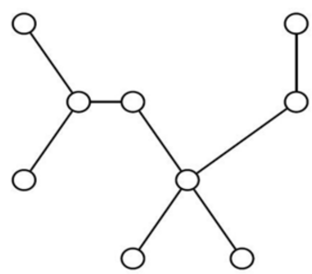
In a graph G, any u-v walk contains a u-v path.

Proof

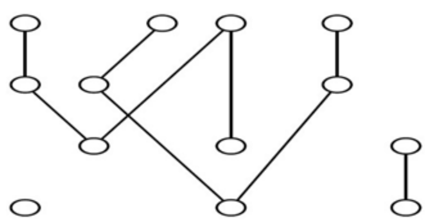
We prove the result y induction on the length of the walk.
Any walk of length 0 or 1 is obviously path.
Now, assume the result for all walks of length less than n.
Let $u=u_0, u_1 \ldots u_n=v$ be a u-v walk of length n.
If all the points of the walk are distinct is a already a path.
If not, there exists i and j such that 0■i■j■n and $u_i=u_j$.
Now, $u=u_0, u_1, \ldots u_i, u_{j+1}, \ldots u_n=v$ is a u-v walk of length less than n, which by induction hypothesis contains a u-v path.

Tree

A **tree** is an acyclic and connected graph.



A **forest** is an acyclic graph (and hence a union of trees)



Proposition 3

Every non-trivial tree G has atleast two vertices of degree 1.

Proof

Since G is non-trivial, d(v)≥ 1 for all points v.
Also $\sum (v)$ =2q=2(p-1)=2p-2.
Hence d(v)=1 for atleast two vertices.

Spanning Tree

A spanning tree sub graph of a graph that is tree is called a spanning tree.

Proposition 4

Every connected graph has a spanning tree.

Proof

Let G be a connected graph. Let T be a minimal connected spanning subgraph of G. Then for any line x of T,T-x is disconnected and hence x is a bridge of T.
Hence T is acyclic.
Further T is connected and hence is a tree.

Conclusion

The main aim of this paper is to present then importance of graph theory ideas in vital area of computer application for researches which can use graph theory concepts for the research an impression is accessible particularly to project, the ideal of graph theory

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Study on Trees and Spanning Tree

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ABSTRACT: *We deal with a particular type of connected graphs called Trees. These graphs are important for their application in different fields. Tree is the simplest graph which is convenient to study and to prove any result on Graph Theory.*

Keywords: *Tree , forest , eccentricity, radius, diameter.*

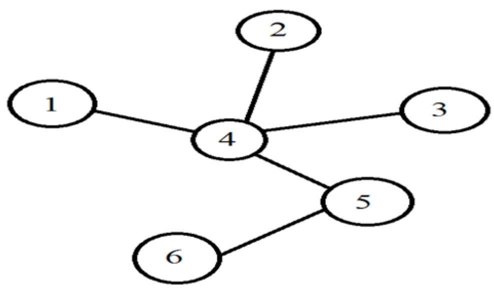
1. Introduction

Trees are very important for the sake of their application to many different fields. In this chapter we study a special class of graphs, known as Trees, which arose from the study of operator in differential calculus. Further a tree is the simplest non-trivial type of a graph and in typing to prove a general result or test a conjecture in Graph Theory, it is sometimes convenient to first study the situation for trees.

Tree

A connected that does not contain any cycle called an acyclic graph.
A connected acyclic graph is called a tree.

Example



Forest

Union of trees is called forest.

Theorem

Let G be a graph. The following statements are equivalent

1) G is a tree
2) Every two vertices of G are joined by a unique path.
3) G is connected and q = P-1
4) G is acyclic and q = P-1

Proof

(i)=>(ii)

If G is a tree, then by definition it is connected. Hence any two vertices are joined by atleast one path. If the vertices x and y are joined by two different paths, then a cycle is produced in G. but this a contradiction to the definition of G. hence there is a unique path between any vertices.

(ii) => (iii)

Suppose that g is a graph in which two vertices are joined by a unique path then the graph G must be connected. We shall show that q = P-1 using induction on P.

If $P = 1$, then $G = K$, and hence $q = 0 = P-1$. Hence the result is true when $P = 1$.
We now assume that the result is true for all trees of order $< P$ and let G be a tree of order $P \geq 2$.
Let $u = uv \in E(G)$. Since e is the path joining u and v there is no u-v path in G-e. So that G-e is disconnected and G-e has exactly 2 components say G_1 and G_2.

Both $G_1 = (P_1,Q_1)$ and $G_2 = (P_2,Q_2)$ are trees and each has order less than P.

Thus by induction $q_1 = P_1 - 1$ and $q_2 = P2 - 1$

Now
$q = q_1 + q_2 + 1$
$q = P_1 - 1 + P_2 - 1 + 1$
$q = P_1 + P_2 - 1$
$q = P - 1$

(iii) => (iv)
Suppose G is connected and $q = P - 1$. Then we have to show that G is acyclic.
Suppose G contains a cycle C and suppose e is an edge of C. Then G-e is connected and has order P and size P-2 **(** $q = P-1$ and further we have removed one edge so that number of edges in G-e is P-2 **)**. This is contradiction since $q = P - 1$.

(iv) => (v)
Suppose G is acyclic and $q = P-1$. We will show that G is a tree. Since G is acyclic, it is enough to show that g is connected.

Suppose G is disconnected, then there can be atleast two component for G. Since G is acyclic, the components c_1 and c_2 are also acyclic. Hence c_1 and c_2 are trees. If the order and size of c_1 and c_2 are P_1, q_1 and P_2, q_2 respectively.

Then $q_1 = P_1 - 1$ and $q_2 = P_2 - 1$.
$q = q_1 + q_2$
$q = P_1 - 1 + P_2 - 1$
$q = P_1 + P_2 - 2$
$q = P - 2$

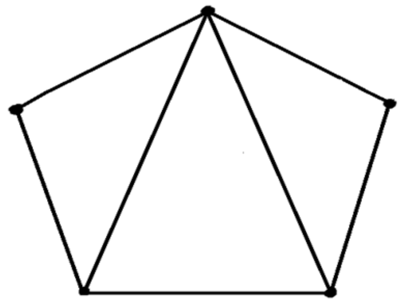
This is a contradiction to our hypothesis, hence the graph G must be connected and hence it is a tree.

Corollary
Every non – trivial tree G has atleast two vertices of degree 1.

Proof
Since G is non – trivial $d(\upsilon) \geq 1$ for all vertices.
Also $\Sigma\, d(\upsilon) = 2q = 2(P-1) = 2P - 2$
Hence $d(\upsilon) = 1$ for atleast two vertices.

Spanning
A spanning subgraph of a graph that is tree is called a spanning tree.

Example



Theorem
Every connected graph has a spanning Tree.

Proof
Let G be a connected graph.
Let H be a minimal connected spanning subgraph of G.
Then for any line x of H, H-x is disconnected and hence x is a bridge of H.
Hence H is acyclic.
So H is a connected acyclic spanning subgraph of G.
Hence G has a spanning tree H.

Definitions
Eccentricity
The Eccentricity $e(v)$ of a vertex v in a graph G is the distance from v to a vertex farthest from v.
$e(v)$ = { d(u,v) / u ϵ v (G) }

Radius
The radius rad G of a graph G is defined as the minimum eccentricity of vertices.
(i.e) rad G = min {e(v) / v ϵ V(G) }

Diameter
The diameter diam G of G is the maximum distance between two vertices of G. (i.e) diam G = max { e(v) / v ϵ V(G) }

Centre of a Tree
A Vertex is called a central point if e(v) = r(G) and the set of all central points is Called the centre of G.

Theorem
Every tree has a centre consisting of either paint or two adjacent points.

Proof
The result is obvious for the trees k_1 and k_2 . Now, let T be any tree with P ≥ 2 points. T has atleast two end points and maximum distance from a given point u to any other point v occurs only when v is an end point.
Now delete all the end points from T. The resulting graph T' is also a tree and the eccentricity of each point in T' is exactly one less than the eccentricity of the same point in T.

Hence T and T' have the same centre.
If the process of removing end points is repeated, we obtain successive trees having the same centre as T and we eventually obtain a tree which is either K_1 or K_2.

Hence the centre of T consists of either one point or two adjacent point.

Conclusion

The main aim of this paper is to present the importance of graph therotical ideas in various area of computer aplication for researches that they can use graph theroritical concepts for the research an overview is presented especially to project he idea of graph theory.

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An Study on Linear Transformation and Kernal of The Linear Transformation in Algebra

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ABSTRACT: *In this paper we introduced the concept of linear transformation on algebra and prove that the set of all linear transformation and kernal of linear transformation and also linear transformation matrix. Let us define the linear transformation of mapping T from v to w is called linear transformation. And also given some examples and we will proved some theorems. we shall study properties of linear transformation*

Keywords: *Linear transformation, kernel of linear transformation, linearity, zero map, linear transformation and matrix.*

1. Introduction

The significance of vector spaces arises from the fact that we can pass from one vector space to another by means of function that possess a certain property called is linearity. These function is known as linear transformation.

Definitions

Linear transformation:

Let S and S' be two vector spaces over the same field F. Then a map
T: S —>S' is said to be a linear transformation [or vector space homomorphism or linear map] if

1. $T(s_1+s_2)=T(s_1)+T(s_2)$, $s_1\ s_2 \in S$
2. $T(as)=a.T(s)$, $a\in F$, $s\in S$
3. The condition **1** and **2** can be combined into a single condition namely $T(as_1+bs_2)=a\,T(s_1)+b\,T(s_2)$, $a, b\in F$, $s_1\ s_2 \in S$

Example 1 (Zero Map)

1. Let s and s' be vector spaces over F. Define T: s—> s' by T(s)=0 for all $v\in V$. Then T is a linear transformation.

Sol:
Let $s_1, s_2 \in S, \alpha\in F$
$T(\alpha s_1)=0=\alpha 0$
$=\alpha T(s_1)$
T is a linear transformation.
This linear transformation is known as trivial linear transformation or zero map.

Example:-2

2. Define $T:R^2 \to R^2$ by $T(x,y)=(x+y,x)$; then T is linear transformation.

Sol:-
Let $X=(x_1,x_2),(y_1,y_2)$
Be any two element of R^2. Then
$X+Y=(x_1,x_2)+(y_1,y_2)=(x_1+y_1,x_2+y_2)$.
$T(X+Y)= T(x_1+y_1,x_2+y_2)$
$=(x_1+y_1+x_2+y_2,x_1+y_1)$
$=(x_1+x_2,x_1)+(y_1+y_2,y_1)$
$=T(x_1,x_2)+T(y_1,y_2)$ and
$T(\alpha X)= T(\alpha x_1,\alpha x_2)$
$=(\alpha x_1+\alpha x_2,\alpha x_1)$

$=\alpha(x_1+x_2,x_1)=\alpha T(x_1,x_2)$
$=\alpha\, T(X)$
T is a linear transformation.

Example:-3

3. Define $T:R^3 \rightarrow R^3$ by $T(x,y,z)=(x,y,0)$ (T is called the projection mapping of R^3 into xy-plane).
Then T is a linear transformation.

Sol:-
For let $s_1=(x,y,z)$, $s_2=(x_2,y_2,z_2) \in R^3$ and α is a scalar, then
$S_1+S_2=(x_1+x_2,y_1+y_2,z_1+z_2)$
$\alpha x_1=(\alpha x_1,\alpha y_1,\alpha z_1)$
$T(s_1+s_2)=T(x_1+x_2,y_1+y_2,z_1+z_2)$
$=(x_1+x_2,y_1+y_2,0)$
$=(x_1,y_1,0)+(x_2,y_2,0)$
$=T(s_1)+T(s_2)$ and
$T(\alpha s_1)=T(\alpha x_1,\alpha y_1,\alpha z_1)$
$=(\alpha x_1,\alpha y_1,0)=\alpha(x_1,y_1,0)$
$=\alpha\, T(s_1)$
T is a linear transformation.

Kernel of a linear transformation:-

Let $T:V \rightarrow V'$be a linear transformation. Then the subset H of V defined by $H=\{ v \in V/ T(v)=0\}$ is called the kernal of T and is denoted by ker T.

Example

Let $T:V \rightarrow V'$ be a liner transformation
Then (i) Ker T is a subspace of V

Proof:-
Sine $T(0)=0, 0 \in$ ker T.
Ker $T \neq \emptyset$
Let $\alpha,\beta, \in F, v_1,v_2 \in$ ker T.
Then $T(\alpha v_1+\beta v_2)=\alpha\, T(v_1)+\beta\, T(v_2)$
$=\alpha.0+\beta.0$
$=0$
$\alpha\, v_1+\beta\, v_2 \in$ ker T
Ker T is a subspace of V.

Linear transformation and matrix:-

Given the following system of linear equations,
$a_{11}x_1+a_{12}x_2+\ldots\ldots.+a_{1n}x_n=b_1$
$a_{12}x_1+a_{22}x_2+\ldots\ldots..+a_{2n}x_n=b_2$
……..
……..
$a_{m1}x_1+a_{m2}x_2+\ldots\ldots\ldots+a_{mn}x_n=b_m$

where $a_{ij} \in k$ a field, we may consider it as a linear transformation from $K^n=\{(x_1,x_2,\ldots,x_n) : x_i \in K.\}$ to $K^m=\{(b_1,b_2,\ldots,b_m) : b_i \in K$.
$AX=B$

The system of linear equations can be solved if and only if the vector B is in (A) Note that if the inverse A^{-1} of the matrix A exists, then the system of equation can be solved easily by
$X=A^{-1}B$

Properties of Linear Transformation
Let $T:S\longrightarrow S'$ be a linear transformation from vector space S to S'.
Then (1) $T(0)=0$
(2) $T(-s)=-T(s)$
(3) $T(s_1-s_2)= T(s_1)-(s_2)$

Proof
1.T is a linear transformation and $0,s\in S$
$T(0+s)= T(0)+T(s)$
i.e., $0+T(s)=T(0)+T(s)$
$0=T(0)$ by right cancellation law as S is a group with respect to addition.

2.By (1), $0=T(0)$
$=T(s+(-s))=T(s)+T(-s)$
$T(-s)$ is the additive inverse of $T(s)$
$T(-s)= -T(s)$

3.$T(s_1-s_2)=T(s_1+(-s_2))=T(s_1)+T(-s_2)$ (by(2))

THEOREM
Let S and S' be finite dimensional vector space over a field F. Let $(s_1,\ldots\ldots,s_n)$ be a basis of S and let $(s_1',\ldots\ldots,s_n')$ be any set of vectors in s' (not necessarily distinct) Then there exists a unique linear transformation $T:S\longrightarrow S'$ such that $T(s_i)=s_i'$ for $i=1,2,3,\ldots..,n$

Proof
Let $s\in S$, since $(s_1,\ldots..,s_n)$ is a basis of S, s has a unique representation of the form $s=\alpha_1s_1+\ldots..+\alpha_ns_n$ where $\alpha_1\ldots..\alpha_n \in F$.
Define $T:S\longrightarrow S'$ by
$T(\alpha_1s_1+\ldots..+\alpha_ns_n)=\alpha_1s_1'+\ldots..+\alpha_ns_n'$.
Since $\alpha_1\ldots..\alpha_n$ are unique T is well defined.

Now,
$S_i=0s_1+\ldots.+0s_{i-1}+1s_i+0s_{i+1}+\ldots.+0s_n$
$T(s_i) =0s'_i+\ldots.+0s'_{i-1}+1s'_i+0s'_{i+1}+\ldots..+0s'_n$
$=s'_i$ for all i.

Let $\alpha,\beta \in F$ and $u,u'\in S$
Let $u=\alpha_1s_1+\ldots\ldots.+\alpha_ns_n$
$u'=\beta_1s_1+\ldots..+\beta_ns_n$

Then
$T(\alpha u+\beta u')= T[(\alpha(\alpha_1s_1+\ldots.+\alpha_ns_n)+\beta(\beta_1s_1+\ldots..+\beta_ns_n)]$
$= (\alpha\alpha_1+\beta\beta_1)s_1'+\ldots\ldots+ (\alpha\alpha_n+\beta\beta_n)s'_n$
$= \alpha\ (\alpha_1s_{1'}+\ldots..+\alpha_ns_{n'}) + \beta(\beta_1s_{1'}+\ldots..+\beta_ns_{n'})$
$= \alpha\ T(\alpha_1s_1+\ldots\ldots+\alpha_ns_n) + \beta T(\beta_1s_1+\ldots..+\beta_ns_n)$
$= \alpha\ T(u)+\beta\ T(u')$

T is a linear transformation.
To prove T is unique.
Let $P:S\longrightarrow S'$ be a linear transformation such that $P(s_i)= s_i'$ for all i.
Let $u=\alpha_1s_1+\ldots..+\alpha_ns_n\in S$
Then $P(u)=P(\alpha_1s_1+\ldots..+\alpha_ns_n)$
$=\alpha_1P(s_1)+\ldots..+\alpha_nP(s_n)$

$=\alpha_1 s_1' + \ldots + \alpha_n s_n'$
$=T(\alpha_1 s_1 + \ldots + \alpha_n s_n)$
$=T(u)$

$P(u)=T(u) \forall u \in U$
$P=T$.
T is unique.

Conclusion

Here by I have explained the definition of linear transformation and kernel of linear transformation with suitable examples and zero map method. In addition to that I have also explained the linear transformation matrix with appropriate examples. Properties of linear transformation were also proved. More over there theorem were also explained thus I have brought the conclusion for the above mentioned topics.

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Solution of Fuzzy Assignment Problems Using Triangular Fuzzy Number Method

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ABSTRACT: *In this chapter, we discussed the fuzzy job and workers by Hungarian assignment problems. In this problem C_{ij} denotes the cost for assigning the n jobs to the n workers. The cost is usually deterministic in nature* $\widetilde{C_{ij}}$ *has been considered to be trapezoidal and triangular numbers denoted by* $\widetilde{C_{ij}}$ *which are more realistic and general in nature.*

Keywords: *Ranking techniques, fuzzy assignment, crisp assignment, k cut of the fuzzy number*

1. Introduction

For finding the best possible assignment, we must optimize total cost this problem assignment. The assignment problem is formulated to the crisp assignment problem in the linear programming problem form and solved by using Hungarian method and Robust's ranking method for the fuzzy numbers with suitable problems.

1.2 Robust's Ranking Techniques - Algorithms

The Assignment difficulty can be affirmed in the form of n x n cost matrix [C_{ij}] of real numbers as given in the following

	Job 1	Job 2	Job 3	. . . .	Job j	. . . . . .	Job N
Worker 1	C_{11}	C_{12}	C_{13}	. . . .	C_{1j}	. . . .	C_{1n}
Worker 2	C_{21}	C_{22}	C_{23}	. . . .	C_{2j}	. . . .	C_{2n}
. . . **Worker i** . .	. . . C_{i1} . . .	. . . C_{i2} . . .	. . . C_{i3} . . .	. . . .	. . . C_{ij} . . .	. . . .	. . . C_{ij} . . .
Worker N	C_{n1}	C_{n2}	C_{n3}	. . . .	C_{nj}	. . . . . .	C_{nn}

Mathematically assignment problem can be stated as minimize

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{i=1}^{n} X_{ij} = 1$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad x_{ij} \in [0,1] \qquad (1)$$

Where $x_{ij} = \begin{cases} 1 ; & \text{if the } i^{th} \text{ worker is assigned the } j^{th} \text{ job} \\ 0 ; & \text{otherwise} \end{cases}$

is the decision variable denote the assignment of the worker i to job j. $\widetilde{c_{ij}}$ is the cost of assigning the j^th job to the i^th worker. The purpose is to minimize the total cost of assigning all the jobs to the available persons (one job to one worker).

When the costs or time $\widetilde{c_{ij}}$ are fuzzy numbers, then the total costs becomes a fuzzy number

$$\breve{z}^* = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Hence it cannot be minimize directly for solving the problem. We defuzzify the fuzzy price coefficients into hard ones by a fuzzy number ranking method. Robust's ranking technique which satisfies compensation, linearity, and additivity properties and provides results which are consistent with human intuition. Give a convex fuzzy number $\breve{C}$, the Robust's Ranking index is defined by

$$\mathrm{R}(\breve{C}) = \int_0^1 0.5(C_k^L, C_k^U)dk$$

Where (C_k^L, C_k^U) is the k – level cut of the fuzzy number of $(\breve{C})$.

we use this method for ranking the objective values. The Robust's ranking index $R(\breve{C})$ gives the representative value of the fuzzy number $\breve{C}$, it satisfies the linearity and additive property:

If $\hat{P} = l\breve{E} + m\breve{Y}$ and $\hat{Q} = s\breve{C} - t\breve{N}$

where *l, m, s, t* are stable then we have

$\mathrm{R}(\hat{p}) = l\mathrm{R}(\breve{E}) + \mathrm{n}\,\mathrm{R}(\breve{Y})$ and $\mathrm{R}(\hat{Q}) = s\,\mathrm{R}(\breve{C}) - \mathrm{t}\,\mathrm{R}(\breve{N})$

on the source of this belongings the fuzzy assignment problem can be transformed in to a crisp assignment problem linear programming problem from. The ranking technique of the Robust's is

If $\mathrm{R}(\hat{G}) \le \mathrm{R}(\breve{H})$ then $\hat{G} \le \breve{H}$ i,e min $\{\hat{G}, \breve{H}\} = \hat{G}$ from the assignment problem(1), with fuzzy objective function

$$\min\ \breve{z}^* = \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{c}_{ij}x_{ij})$$

We apply Robust's ranking way (using the linearity and assistive property) to get the least objective value $\breve{z}^*$ from the formulation

$$R(\breve{z}^*) = \min \mathrm{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{c}_{ij}x_{ij})$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{n} x_{ij} = 1\,, \qquad x_{ij} \in [0,1] \qquad\qquad (2)$$

Where $x_{ij} = \begin{cases} 1\,; & \text{if the } i^{th} \text{ worker is assigned the } j^{th} \text{ job} \\ 0\,; & \text{otherwise} \end{cases}$

is the decision variable denoting the assignment of the worker i[th] to j[th] job. $\tilde{c}_{ij}$ is the cost of designing the j[th] job to the i[th] worker. The objective is to diminish the total cost of assigning all the jobs to the available workers.

Since R($\tilde{c}_{ij}$) are crisp values, this problem (2) is obviously the crisp assignment problem of the form (1) which can be solved by the conventional methods, namely the simplex method to solve the linear programming problem form of the problem. Once the optimal solution x^* of model (2) is found, the optimal fuzzy objective value $\breve{z}^*$ of the original problem can be calculated as

$$\breve{z}^* = \sum_{i=1}^{n}\sum_{j=1}^{n} R(\tilde{c}_{ij})\, x_{ij}^*$$

1.3.Numerical Examples

Example 1.3.1.

Let us consider a fuzzy assignment problem with rows representing 3 workers A,B,C and Columns representing the 3 jobs, Job1, Job2, and Job3 the cost matrix is given whose elements are Triangular Fuzzy numbers. The difficulty is to find the best possible assignment so that the total cost of job assignment becomes minimum.

$$[\tilde{C}_{ij}] = \begin{pmatrix} (1,5,9) & (8,9,10) & (2,3,4) \\ (7,8,9) & (6,7,8) & (6,8,10) \\ (5,6,7) & (6,10,14) & (10,12,14) \end{pmatrix}$$

Solution:

The fuzzy assignment difficulty can be formulated in the following mathematical programming from.

Min{$\Re(1,5,9)x_{11}+\Re(8,9,10)x_{12}+\Re(2,3,4)x_{13}$
$+\Re(7,8,9)x_{21}+\Re(6,7,8)x_{22}+\Re(6,8,10)x_{23}$
$+\Re(5,6,7)x_{31}+\Re(6,10,14)x_{32}+\Re(10,12,14)x_{33}$}

Subject to

$$x_{11}+x_{12}+x_{13}=1 \quad x_{11}+x_{21}+x_{31}=1$$
$$x_{21}+x_{22}+x_{23}=1 \quad x_{12}+x_{22}+x_{32}=1$$
$$x_{31}+x_{32}+x_{33}=1 \quad x_{13}+x_{23}+x_{33}=1 \quad x_{ij}\epsilon[0,1]$$

Now we calculate $\Re$ (1, 5, 9) by applying Robust's ranking method. The association function of the triangular fuzzy number (1, 5, 9) is

$$\mu(x) = \begin{cases} \frac{x-1}{5-1} & , 1 \le x \le 5 \\ 1 & , x = 5 \\ \frac{9-x}{9-5} & , 5 \le x \le 9 \\ 0 & , \text{otherwise} \end{cases}$$

$$\mu(x) = \begin{cases} \frac{x-1}{4} & , 1 \le x \le 5 \\ 1 & , x = 5 \\ \frac{9-x}{4} & , 5 \le x \le 9 \\ 0 & , \text{otherwise} \end{cases}$$

The k-Cut of the fuzzy number (1, 5, 9) is $(c_k^L, c_k^U) = (4k$ +1, 9-4k) for which Proceeding similarly, the Robust's ranking indices for the fuzzy costs $\tilde{c}_{ij}$ are calculated as:

$\Re(\tilde{c}_{11}) = \Re(1,5,9) = \int_0^1 0.5(c_k^L, c_k^U)\, dk = \int_0^1 0.5(10)\, dk = 5$

$\Re(c_{12}) = 9,\ \Re(c_{13}) = 3,\ \ \Re(c_{21}) = 8,$

$\Re(c_{22}) = 7,\ \Re(c_{23}) = 8,\ \Re(c_{31}) = 6,$
$\Re(c_{32}) = 10,\ \Re(c_{33}) = 12$

We replace these values for their corresponding $\tilde{c}_{ij}$ in the convenient assignment problem in the linear programming problem. We solve it by Simplex method to get the following best possible solution is

$(\tilde{c}_{ij}) = \begin{pmatrix} 5 & 9 & 3 \\ 8 & 7 & 8 \\ 6 & 10 & 12 \end{pmatrix}$

$= \begin{pmatrix} 2 & 6 & (0) \\ 1 & (0) & 1 \\ (0) & 4 & 6 \end{pmatrix}$

$x_{13}^* = x_{22}^* = x_{31}^* = 1$ and $x_{11}^* = x_{12}^* = x_{21}^* = x_{23}^* = x_{32}^* = x_{33}^* = 0$

With the optimal objective value $\Re(\breve{z}^*)$= 16 which represents a optimal total cost. In other words the optimal assignment is

A→3, B→2, C→1 $\Re(\breve{z}^*)$= 3+7+6= 16

The fuzzy optimal total cost is = $\tilde{c}_{13} + \tilde{c}_{22} + \tilde{c}_{31}$
= $\Re(2,3,4) + \Re(6,7,8) + \Re(5,6,7)$
= $\Re(13,16,19)$

Also
We find that $\Re(\breve{z}^*)$ = 16.

Example 1.3.2.
Let us consider a fuzzy assignment problem with rows representing 4 workers A, B, C, D and Columns representing the jobs, Job1, Job2, Job3 and Job4. The cost matrix $\tilde{c}_{ij}$ is given whose elements are Triangular Fuzzy numbers. The difficulty is to find the best possible assignment so that the total cost of job assignment becomes minimum.

$\begin{pmatrix} (3,5,6,7) & (5,8,11,12) & (9,10,11,15) & (5,8,10,11) \\ (7,8,10,11) & (3,5,6,7) & (6,8,10,12) & (5,8,9,10) \\ (2,4,5,6) & (5,7,10,11) & (8,11,13,15) & (4,6,7,10) \\ (6,8,10,12) & (2,5,6,7) & (5,7,10,11) & (2,4,5,7) \end{pmatrix}$

Solution:
The fuzzy assignment difficulty can be formulate in the following mathematical programming form.
Min{$\Re(3,5,6,7)x_{11}$+$\Re(5,8,11,12)x_{12}$+$\Re(9,10,11,15)x_{13}$+$\Re(5,8,10,11)x_{14}$+$\Re(7,8,10,11)x_{21}$+$\Re(3,5,6,7)x_{22}$+$\Re(6,8,10,12)x_{23}$+$\Re(5,8,9,10)x_{24}$
+$\Re(2,4,5,6)x_{31}$ +$\Re(5,7,10,11)x_{32}$ +$\Re(8,11,13,15)x_{33}$+$\Re(4,6,7,10)x_{34}$
+$\Re(6,8,10,12)x_{41}$ +$\Re(2,5,6,7)x_{42}$ +$\Re(5,7,10,11)x_{43}$+$\Re(2,4,5,7)x_{44}$

Subject to

$x_{11}+x_{12}+x_{13}+x_{14}=1$ $\quad$ $x_{11}+x_{21}+x_{31}+x_{41}=1$

$x_{21}+x_{22}+x_{23}+x_{13}=1$ $\quad$ $x_{12}+x_{22}+x_{32}+x_{42}=1$

$x_{31}+x_{32}+x_{33}+x_{34}=1$ $\quad$ $x_{13}+x_{23}+x_{33}+x_{43}=1$

$x_{41}+x_{42}+x_{43}+x_{44}=1$ $\quad$ $x_{14}+x_{24}+x_{34}+x_{44}=1$

$x_{ij}\epsilon[0,1]$

Now we calculate $\mathfrak{R}$ $(3,5,6,7)$ by applying Robust's ranking method. The membership function of the triangular fuzzy number $(3,5,6,7)$ is

$$\mu(x)=\begin{cases}\dfrac{x-3}{5-3}, & 3\le x\le 5\\ 1, & 5\le x\le 6\\ \dfrac{7-x}{7-6}, & 6\le x\le 7\\ 0\ , & \text{otherwise}\end{cases}$$

$$\mu(x)=\begin{cases}\dfrac{x-3}{2}, & 3\le x\le 5\\ 1\ , & 5\le x\le 6\\ \dfrac{7-x}{1}, & 6\le x\le 7\\ 0\ , & \text{otherwise}\end{cases}$$

The k - *cut* of the fuzzy number $(3,5,6,7)$ is

$(C_k^L,C_k^U)=$ (2k+3, 7-k) for which

$$\mathfrak{R}(\tilde{c}_{11})=\mathfrak{R}(3,5,6,7)=\int_0^1 0.5(c_k^L,c_k^U)\,dk=\int_0^1 0.5(10+k)\,dk=5.25$$

Proceeding similarly, the Robust's ranking indices for the fuzzy costs $\tilde{c}_{ij}$ arecalculated as:

$\mathfrak{R}(c_{12})=9,\ \ \mathfrak{R}(c_{13})=11.25,\ \ \mathfrak{R}(c_{14})=8.5, \mathfrak{R}(c_{21})=9,$

$\mathfrak{R}(c_{22})=5.25,\ \ \mathfrak{R}(c_{23})=9,\ \ \mathfrak{R}(c_{24})=8, \mathfrak{R}(c_{31})=4.25,$

$\mathfrak{R}(c_{32})=8.25,\ \ \mathfrak{R}(c_{33})=11.75,\ \ \mathfrak{R}(c_{34})=\ 6, \mathfrak{R}(c_{41})=9,$

$\mathfrak{R}(c_{42})=5,\ \ \mathfrak{R}(c_{43})=8.25,\ \ \mathfrak{R}(c_{44})=4.5$

We replace these values for their corresponding $\tilde{c}_{ij}$ in the convenient assignment problem in the linear programming problem. We solve it by Hungarian method to get the following optimal solution is

$$(\tilde{c}_{ij})=\begin{pmatrix}5.25 & 9 & 11.25 & 8.5\\ 9 & 5.25 & 9 & 8\\ 4.25 & 8.25 & 11.75 & 6.75\\ 9 & 5 & 8.25 & 4.5\end{pmatrix}$$

$$=\begin{pmatrix}0 & 1.5 & (0) & 1\\ 6 & (0) & 0 & 2.75\\ (0) & 1.75 & 1.5 & 0.25\\ 4.5 & 0.5 & 0 & (0)\end{pmatrix}$$

$x_{13}^*=x_{22}^*=x_{31}^*=x_{44}^*=1$ and $x_{11}^*=x_{12}^*=x_{14}^*=x_{21}^*=x_{23}^*=x_{24}^*=x_{32}^*=x_{33}^*=x_{34}^*=x_{41}^*=x_{42}^*=x_{43}^*=0$

With the optimal objective value $\mathfrak{R}(\check{z}^*)$= 25.25 which represents a optimal total cost. In other words the optimal assignment is

A→3, B→2, C→1, D→4 $\Re(\check{z}^*)$ = 11.25+5.25+4.25+4.5 = 25.25

The fuzzy best possible total cost is
= $\tilde{c}_{13} + \tilde{c}_{22} + \tilde{c}_{31} + \tilde{c}_{44}$
= $\Re(9,10,11,15) + \Re(3,5,6,7) + \Re(2,4,5,6) + \Re(2,4,5,7)$
= $\Re(16, 23, 27, 35)$

Also
We find that $\Re(\check{z}^*)$ = $\Re(16, 23, 27, 35)$
= 25.25

Conclusion

In this paper, the assignment expenses are measured as vague numbers described by fuzzy numbers which are more realistic and general in nature. furthermore, the fuzzy assignment problem has been transformed into crisp assignment problem using Robust's ranking indices. Mathematical examples show that by this method we can have the most favorable assignment as well as the hard and fuzzy optimal total cost. By using Robust's ranking method we have shown that the total cost obtained is best possible.

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An Analysis of Trace and Transpose

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ABSTRACT: *In this paper I have derived some properties of trace and transpose and few examples were solved.*

Keywords: *Trace, transpose, matrix, invertible, symmetric, skew symmetric*

1. Introduction

Concepts of trace and transpose have a wide application and I am very much interested in this concept. So I decided to solve a paper. In this paper it consists of some of the properties of trace and transpose. In addition to these I have solved few examples to it.

Trace

A Trace of "A" is the sum of the elements on the main diagonal of A. we shall write the Trace A as, tr(A) if $A=\alpha_{ij}$ Then,
$Tr(A) = \sum_{i=1}^{K} \alpha_{ij}$

Lemma 1.1 :
If $A,B \in Gn$. $\lambda \in F$.
$tr(\lambda A) = \lambda \, tr\, A$
$tr\,(A+B) = trA+trB$
$tr\,(AB) = tr(BA)$

Proof

Let $A = M_{ij}$, $B = N_{ij}$ Then, $AB = R_{ij.}$
Where, $R_{ij} = \sum_{k=1}^{n} m_{ik} N_{jk}$ and $BA = P_{ij}$
Where, $P_{ij} = \sum \lambda M_{ij}$
$= \lambda \sum M_{ij}$

$Tr(\lambda A)= \lambda\, tr(A)$
$tr(A+B) = \sum (M_{ii} + N_{ii}$
$= \sum M_{ii} + N_{ii}$

$tr(A+B) = tr\,(A) + tr(B)$
$tr\,(AB) = \sum N_{ij}$
$tr(AB) = \sum (\sum M_{ik} N_{ik}) \ldots\ldots\ldots\ldots\ldots(a)$

$tr(AB) = \sum P_{ij}$

$tr\,(BA) = \sum (\sum N_{ik} M_{ik})$

$tr\,(BA) = \sum\sum M_{ik} N_{kj} \ldots\ldots\ldots\ldots\ldots(b)$

From (a) and (b) we get,
$tr\,(AB) = tr\,(BA)$

Corollary

If k is invertible then $tr\,(KAK^{-1}) = tr(A)$

Proof

Let $C= AK^{-1}$; Then $tr(KAK^{-1}) = tr(KC)$
$= tr(CK)$
$= tr(AK^{-1}k)$
$= tr(A)$.

Theorem

For Every $K \in M$, $m \geq 2, (R_{i1,\ldots\ldots in})^{n}_{ij=1} \subset J$ and $x^{1}_{i1}\ldots\ldots x^{m}_{in} \in K^{m}_{l2}$
We have, $\sum R_{i1\ldots\ldots in} \sum x^{1}_{i1}(k) \mid \leq J_{G}^{m-1}$ Sup $|\sum (a_{i1\ldots\ldots in}).(t_{i1\ldots\ldots in})| \; |t_{ij}| \leq 1$

Proof

Let us consider,
$Z_{im} = \sum^{k}_{i1\ldots\ldots in-1 = 1} a_{i1\ldots\ldots in} \quad e_{i1\ldots\ldots\ldots} e_{in-1} \in l^{m}_{1\ldots\ldots\ldots\ldots} \in$
$||(Z_{im})^{m}_{in=1}||\omega = \text{Sup}\{ |\sum (a_{i1\ldots in})(t_{i1\ldots\ldots in})| : |t_{ij}| \leq 1 \}$.

For Every $j \in (1,2,3,\ldots\ldots\ldots n-1)$ we definen then operater $,U_j : l^{m}_{2} \rightarrow l^{m}_{2,}$
By, $U_j(e_{ij}) = x^{j}_{ij} \quad (1 \leq ij \leq m)$.

We know that,
$\sum ||v_{n-1}(Z_{in})|| \leq K^{n-1}_{G} \; ||(Z_{in})^{m}_{in=1}||\omega$

Finally,
$\sum ||v_{n-1}(z_{in})|| \geq ||\sum (v_{n-1}(z_{in}), x^{n}_{in})||$
$= |\sum a_{i1\ldots in} \sum x^{1}_{i1}(k),\ldots\ldots\ldots x^{n}_{in}(k)|$

If X_j is an Ⱡ$_{\infty, \lambda j}$- Space $(1 \leq j \leq n)$, Then every multi linear form,
$T : X_1 * \ldots\ldots\ldots * X_n \rightarrow K$. is $(1:2\ldots\ldots2)$- summing and $||T||\;(1;2\ldots..2) \leq K^{n-1}_{G} \; ||T||$holds.

Proof

We know that,
The local behaviour of the (1;2……2) summing Multi linear operators, it is enough
To prove that $T: l^{m}_{\infty} * \ldots\ldots l^{m}_{\infty} \rightarrow K$ is Multi linear form,
Such that, $||(x^{j}_{r})^{m}_{r=1}||w \leq 1$.
$\sum |T(x^{1}_{r\ldots\ldots\ldots} x^{n}_{r})| \leq K^{n-1}_{G} ||T||$

Let us define, $T_{i1\ldots\ldots\ldots in} = T(e_{i1\ldots\ldots\ldots} e_{in})$ and,
$H_r = \sum T_{i1\ldots\ldots\ldots in} \; x^{1}_{r}(i_1)\ldots\ldots\ldots\ldots x^{n}_{r}(i_n)$,
$\theta_r = \{ \frac{|hr|}{hr}, \quad h_r \neq 0$
$0, \quad h_r=0$

If $y^{1}_{i1}(r) = \theta_r \; x^{1}_{r}(i_1)$ and $y^{j}_{ij}(r) = x^{j}_{r}(i_j)$ for $j \geq 2$,

There fore, $y^{j}_{ij} \in B^{m}_{l2}$ and so,
By we know that,
$|\sum T_{i1\ldots\ldots\ldots in} \sum y^{1}_{i1}(r)\ldots\ldots\ldots y^{n}_{in}(r)| \; \sum K_{E}^{m-1} \; ||T||$

But,
$|\sum T_{i1\ldots\ldots in} \sum y^{1}_{i1}(r)\ldots\ldots\ldots y^{n}_{in}(r)| = |\sum \theta_r \sum T_{i1\ldots\ldots in} x^{1}_{r}(i_1)\ldots\ldots x^{n}_{r}(i_n)|$
$= \sum |T(x^{1}_{r}\ldots\ldots x^{n}_{r})|$

Transpose

The new matrix obtained by interchanging the rows and columns of the original matrix is called as the transpose of the matrix. If $L = [m_{ij}]$ be an p*q matrix, then the
It is denoted by L' or (L^T). In other words,
If $L=[m_{ij}]_{p*q}$, Then, $L^T=[m_{ji}]_{q*p}$.

Example

$$L = \begin{bmatrix} 2 & 7 \\ 5 & 3 \\ 1 & 8 \end{bmatrix}$$

$$L^T = \begin{bmatrix} 2 & 5 & 1 \\ 7 & 3 & 8 \end{bmatrix}$$

Properties of the Transpose
$(AB)^T = B^T A^T$

If A is invertible, so is A^T, and $(A^T)^{-1} = (A^{-1})^T$
For any matrix A, rank (A) = rank (A^T).

Theorem
If A is skew symmetric, then A is a square matrix and $a_{ij} = 0$, i= 1,2,….n.

For any matrix $A \in M_n(F)$
$A - A^T$ skew- symmetrc while $A + A^T$ IS symmetric.

Every matrix $A \in M_n(F)$ can be uniquely written as the sum of a skew-symmetric and symmetric matrix

Proof

1) If $A \in M_{m,n} (F)$, then $A^T \in M_{n,m}(F)$. so if $A^T = -A$

We,must have m=n. Also,
$a_{ii} = -a_{ii}$
for i= 1,2,……n. so, $a_{ii} = 0$ for all i.

2) Since, $(A-A^T) = A^T - A = -(A-A^T)$, follows that,
$A-A^T$ is skew-symmetric,

Let A= B+C be a second such decomposition. Subtraction gives
$\frac{1}{2} (A+A^T) - B = C \frac{1}{2}(A-A^T)$

The left matrix is symmetric while the right matrix is matrix is skew symmetric, Hence, both are the zero matrix.

$A = \frac{1}{2} (A+A^T) + \frac{1}{2} (A+A^T)$.

Conclusion

In this paper I have derived some properties of trace and transpose and few examples were solved. In this paper I have solved.
The transpose of a matrix is the sum of its diagonal entires. Also, Note that, the diagonal entires of the transposed matrix are the same as the original matrix.

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A Study on Calculus of Integration By Parts

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ABSTRACT: *In this concept of integration by parts is an solving and find the product of two functions. Bernouillis formula is a n^{th} term of integration by parts. In this formula I want to apply the new technique in a footnote. Use a tabular method to perform integration by parts is very easily.*

Keywords: Integration by parts, bernaouillis formula, applications, tabular integration.

1. Introduction

In this section you will study an integration technique is called integration by parts. This technique can be applied to a wide variety of function and is particularly useful for integrands involving products of algebraic and transcendental function.

Integration By Parts

Let v and w be differentiable function . then $\int v\,dw = v\,w - \int w\,dv$

We know that,
d/dx (v w) = v d/dx + w d/dx

integrating, we get
$\int v\,dw = v\,w - \int w\,dv$

The method of evaluating a given integral by using the above theorem is called integration by parts. In applying this method, we must choose u and v carefully so that the resulting integral is simpler than the given integral.

Bernouillis Formula

Let v and w be differentiable function. suppose that there exist a positive integer n such that $v^{(n)}=0$ then
Where $w_1 = \int w\,dx$, $w_2= \int w_1\,dx$

Proof

Using integration by parts formula,
$= v\,w- \int v'\,d(w_1)$
$= v\,w - v'\,w_1 + \int w_{1'}\,dv'$
$= v\,w - v'\,w_1 + \int v''\,d(w_2)$
$= v\,w - v'\,w_1 + v''\,w_2 - \int w_2\,dv''$

Proceeding like this we get, the required formula.

Integration By Parts

Integration by Parts is a method of integration that is useful when two functions are multiplied together, but is also helpful in other ways.
$\int u\,v\,dx = u \int v\,dx - \int u' (\int v\,dx)\,dx$

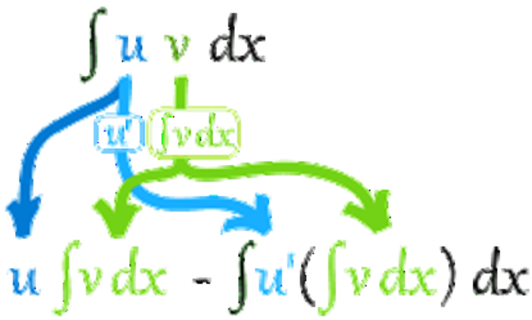
- **u** is the function u(x)
- **v** is the function v(x)

Footnote: Where Did "Integration by Parts" Come From?

It is based on the product of the derivatives,
$(uv)' = uv' + u'v$
Integrate both sides and rearrange:

∫(uv)' dx = ∫uv' dx + ∫u'v dx
u v = ∫uv' dx + ∫u'v dx
∫uv' dx = uv − ∫u'v dx
∫uv dx = u ∫v dx − ∫u'(∫v dx) dx

As a diagram



or more compactly

∫ u dv = u v - ∫ v du

Applications
Polynomials and trigonometric functions
In order to calculate, I = ∫ y cos y dy
Let, y = u, du = dy
dv= cos y dy, v = sin y

then
∫ y cos y dy = y sin y - ∫ sin y dy
= y sin y + cos y +c

where *C* is a constant of integration.

Functions multiplied by unity
The examples are when integration by parts is applied to a function expressed as a product of 1 and itself. if the derivative of the function is known, and the integral of this derivative times *y* is also known.
The first example is ∫ ln(*y*) d*y*. We write this as:

I = ∫ ln (y) .1 dy

Tabular integration by parts
The process of the above formula can be summarized in a table; the resulting method is called "tabular integration" and was featured in the film Stand and Deliver.
For example, consider the integral
1) ∫ x^3 cos x dx,

$u^{(0)} = x^3$. $v^{(n)} = \cos x$

Using repeated integration by parts,
Column A the function $u^{(0)} = x^3$ and its derivatives $u^{(i)}$
And column B the function $v^{(n)} = \cos x$ and its subsequent integral $V^{(n-i)}$ until the size of column B is the same as that of column A.

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# i	Sign	A: derivative u(i)	B: Integral V(n-i)
0	+	X^3	Cos x
1	−	$3x^2$	Sin x
2	+	6x	−Cos x
3	−	6	−Sin x
4	+	0	Cos x

$= (+1)\ (x^3)\ (\sin x) + (-1)\ (3x^2)\ (-\cos x) + (+1)\ (6x)\ (-\sin x) + (-1)\ (6)\ (\cos x) +$
$\int (+1)\ (0) \cos x\, dx$

This yields,
$\int x^3 \cos x\, dx = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + c$

Conclusion
The main aim of this paper is to present the importance of integral calculus. This is very useful to find the product of two functions and more than functions. This paper will be useful in mathematical Derivation.

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A study on Euler Graph and its applications

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ABSTRACT: *The main purpose of this paper is to study Euler graph and its various aspects in our real world. Now the generation Euler graph got height of achievement in many situations that occur in computer science, physical science, communication science, economics and many other areas can be analysed by using techniques mathematics. To realize the purpose first we need to study the basic concepts of graph theory, after that we summarize the methods that are adopted to find Euler path and Euler cycle.*

Keywords: *Graph theory, Konigsberg bridge problem, Eulerian circuit*

1. Introduction

A graph G consists of a set V called the set of points (nodes, vertices) of the graph and a set of edges such that each edge e Є E is associated with ordered or unordered pair of elements of V, i.e., there is a mapping from set of edges E to set of ordered or unordered pairs of elements of V .The set V(G) is called the vertex set of G and E(G) is the edge set. The graph G with vertices V and edges E is written as G = (V, E) or G (V, E). Types of graphs are simple graph, multi graph, pseudo graph. Then

$$\sum_{v\in V} deg\,(V) = 2e$$

i.e., the sum of degrees of the vertices in an undirected graph is twice the number of edges **(Handshaking theorem).**

If G = (V, E) be a directed graph with e edges,

Then

$$\sum deg^{+}(v) = \sum deg^{-}(v) = e$$

i.e., the sum of the out degrees of the vertices of a digraph G equals the sum of in degrees of the vertices which equals the number of edges in G.

1) Origin and Importance of Eulerian graph (Konigsberg problem)

The river called Pregel flows through the city Konigsberg (located in Russia) dividing the city into four land regions, two are river banks and two are islands or delta formation. There was an entertaining or interesting exercise for the citizens of Konigsberg. Start from any land regions and come back to the starting point after crossing each of the seven bridges exactly once without repeating same path.

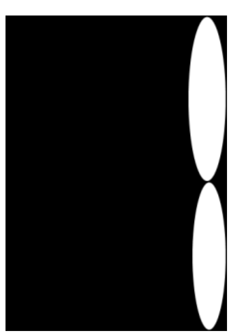
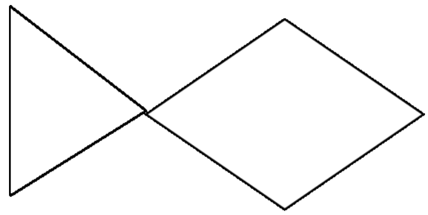


Fig 1: The bridges of Konigsberg problem

This simplifies the problem to great extent. Then Euler proposed that any given graph can be traverse with each edge traversed exactly once if and only if it had, zero or exactly two nodes with odd degrees. The graph is called **Eulerian circuit** or **path**

Using Euler's theorem we need to introduce a path to make the degree of two nodes even. And other two nodes can be of odd degree out of which one has to be starting and other at another the end point.



Fig(2)

Trail that visits every edge of the graph once and only once is called **Eulerian trail.** Starting and ending vertices are different from the one on which it began. A graph of this kind is said to be crossable

An Eulerian circuit is an Eulerian trail that is a circuit i.e., it begins and ends on the same vertex. A graph is called Eulerian when it contains an **Eulerian circuit.**

2) Existence of an Euler path

There are several ways to find the existence of Euler path. Considering the existence of an Euler path in a graph is directly related to the degree of vertices in a graph. Euler formulated the theorems for which we have the sufficient and necessary condition for the existence of an Euler circuit or path in a graph respectively.

Theorem

An undirected graph has at least one Euler path if and only if it is connected and has two or zero vertices of odd degree.

Theorem

An undirected graph has an Euler circuit if and only if it is connected and has zero vertices of odd degree.

Let us take a graph having no odd vertices, the path can begin at any vertex and will end there vice versa in the case of two odd vertices. Any finite connected graph with two odd vertices is negotiable. A negotiable trail may begin at either odd vertex and will end at the other odd vertex.

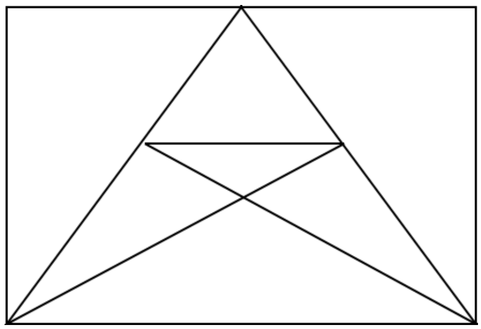
Finding an Euler path is a relatively simple problem it can be solve by keeping few guidelines in our mind:

- Always leave one edge available to get back to the starting vertex (for circuits) or to the other odd vertex (for paths) as the last step.
- Don't use an edge to go to a vertex unless there is another edge available to leave that vertex (except for the last step).

3) Constructive Algorithm

Constructive algorithm used to the prove Euler's theorem and to find an Euler cycle or path in an Eulerian graph. A graph with two vertices of odd degree. The graph with its edges labelled according to their order of appearance in the path found. Steps that kept in mind while traversing Euler graph are first to choose any vertex u of G. Start traversing through edges that not visited yet until a cycle is formed. If there is an unvisited edge's then start the first step until whole graph is traversed.

Example
Illustrations of Constructive algorithm to find Euler cycle, consider the graph



Fig(3)

Check to start that the graph is connected and all vertices are of even degree, find cycle of length 6 such as ((a ,a), (a ,b), (b ,c), (c ,a), (d ,f), (f ,a))

Theorem
A connected graph has an Euler trail, but not an Euler cycle, if and only if it has exactly two edges of odd degree.

Examples
An Euler trail exists for the graph in fig. 4

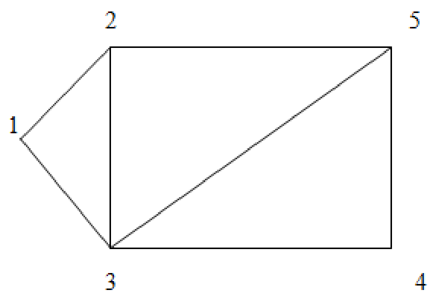


Fig (4)

Euler trail
((1,5)=(5,4),(4,3),(3,1),(2,1),(4,1),(1,2))

Eulerization and Semi- Eulerization
In case Eulerian circuit or path does not exist for the graph. There is a simple process of Eulerization which provides a solution for this type of problem. Eulerization nothing but it is the process of adding phantom (or duplicate) edges to the graph so that the resulting graph has not any vertex of odd degree (and thus contains an Euler).
A comparable problem rises for obtaining a graph that has an Euler path. This is a called Semi-Eulerization and ends with the creation of a graph that has exactly two vertices of odd degree.
Eulerization can be achieved in a different set of edges to duplicate.

4) Application
Line drawings
A graph has a unicursal tracing if it can be traced retracing any line perceptibly, a closed unicursal tracing of a line drawing is equivalent to an Euler circuit in the corresponding graph correspondingly, an open unicursal tracing is equal to an Euler path.

- A line drawing has a closed unicursal tracing iff it has no points if intersection of odd degree.

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➢ A line drawing has an open unicursal tracing iff it has exactly two points of intersection of odd degree.

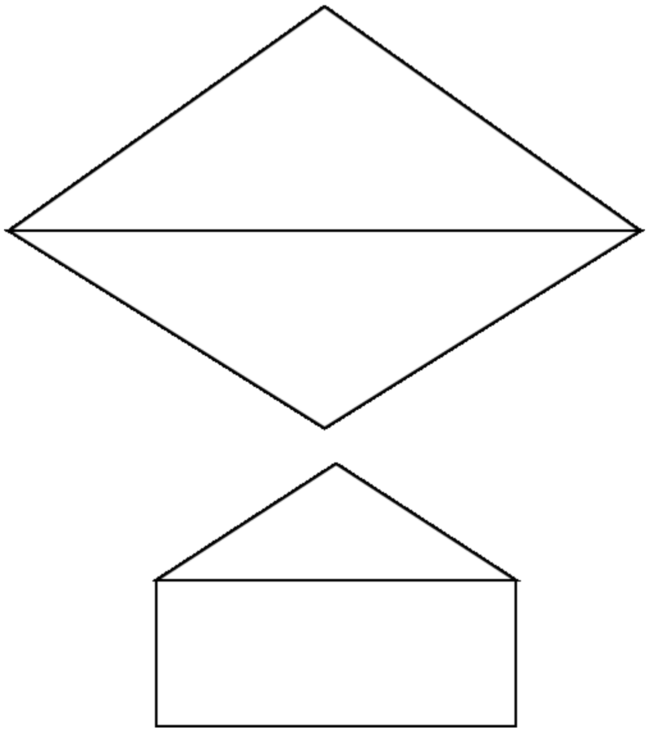

Fig (5)

Conclusion

Eulerian graph is applicable in many real situations. Applications of Eulerian circuits abound. For example, Eulerian circuits are obviously desirable in the deployment of street sweepers, snow plow, buses and mail carriers. Finding an Euler path is a relatively simple problem it can be solve by keeping few guidelines in our mind.

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A Study on Span of Vector Space

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ABSTRACT: *In this paper we will give various definition of span, linear combination and span set of vectors and we will give the deep explanation about vector span and some examples of span and theorem based on span and vector and matrix and problem based on spanning vector.*

Keywords: *Span, vector, matrix, linear span, spanning vector.*

1. Introduction

In this paper we introduce another algebraic system known as vector spaces. The idea of a vector arises in the study of various physical applications. Many physical entities like mass, temperature etc., are characterised in terms of a real number and are scalars. Such entities are called scalars.

Linear Combination

Let V be a vector space over a field F. Let $v_{1,}v_2\ldots\ldots v_n \in V$. Then an element of the form $\alpha_1 v_{1+}\alpha_2 v_{2}+\ldots..+\alpha_n v_n$ where $\alpha_i \in$ F is called a **linear combination** of the vectors $v_{1,}$ $v_{2,}\ldots v_n$.

Linear Span

Let S be a non-empty subset of a vector space V. Then the set of all linear combinations of finite sets of elements of S is called the **linear span** of S and is denoted by **L(S).**

Note

Any element of L(S) is of the form $\alpha_1 v_{1+}\alpha_2 v_{2}+\ldots..+\alpha_n v_n$ where $\alpha_{1,}\alpha_2\ldots.\alpha_n \in$ F.

If A Vector Is In the Span

How to know if a vector is in the vector is in the span$\{v_1 ,v_2,\ldots v_n\}$.Let
A= $[v_{1,}v_2\ldots.v_n]$ using the linear combination.

We can written AX.
This testing if b is in span $\{v_{1,\ldots\ldots\ldots}v_{n\}}$ is equivalent to testing if the matrix equation AX=b as a solution.

Example

- Span{}

The span {} have only one vector the zero vector.

- Span {[1,1][1,0]} over R

The span [ ] over R of the following linear combination.
1[1,1]+1[1,0]=[1,1]
1[1,1]+0[1,0]=[1,0]

- Span {3,2} over R
- Span of two vectors

The span of two vector is a plane contain the origin.

Problem

Show that if A is an nxn so that Dy =A is consistent for nx1 matrix A, then column vectors of D span R^m.

Sol:

Let V=$\{v_1 \ldots..v_m\}$ be the set of column vector D and A be any nx1 vector in R^m.

We note that Dx=A.So there exist a vector y= $\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix}$ such that

$y_1 v_1+y_2 v_2+\ldots+y_m v_m = A$.

So A is the linear combination of the column vector of D .So vector span of R^m.

Theorem
Let $w_1,w_2, \ldots w_n$ be vector in R^m. Then { $w_1,w_2, \ldots w_n$ }span R^m if only if for the matrix A={ $w_1,w_2, \ldots w_n$ } the linear system AX=w is consistent for every $w \in R^m$.

Sol:
Let the vector $w_1=(1,-1,4)$;$w_2=(-2,1,3)$ span R^3.
To prove:-
Solve the linear system AX=w

Where A= $\begin{bmatrix} 1 & -2 & 4 \\ -1 & 1 & -3 \\ 4 & 3 & 5 \end{bmatrix}$ X $\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$ for an arbitrary $w \in R^3$ if w=(x,y,z).

Reduce the argument matrix to $\begin{bmatrix} 1 & -2 & 4 & x \\ 0 & 1 & -1 & -x-y \\ 0 & 0 & 0 & 7x+11y+z \end{bmatrix}$

This has a solution when, 7x+11y+z=0.
Thus the span of these three vector is a plane, they did not span R^3.

EXAMPLE:
1) Vector $e_1=(0,1,0)$; $e_2=(1,0,0)$ from spanning set of R^2 as $(x,y)=xe_1+xe_2$.

2) Matrices $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ form a sapnning set

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} +b \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} +c \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + d \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Spanning Vector:

Let $v_1=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $v_2=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ then $\{v_1, v_2\}$ span R^2.

Sol:

Let $\begin{bmatrix} a \\ b \end{bmatrix} \in R^2$.

We can write

$X_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} +X_2\begin{bmatrix} 1 \\ -2 \end{bmatrix}=\begin{bmatrix} a \\ b \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \quad \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}=\begin{bmatrix} a \\ b \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & a \\ 1 & -2 & b \end{bmatrix}$

$E_2=E_2=E_1$

$\sim \begin{bmatrix} 1 & 1 & a \\ 1 & -3 & b-a \end{bmatrix}$

$E_2 = \frac{-1}{3} E_2$

$\sim \begin{bmatrix} 0 & 1 & a \\ 0 & 1 & \frac{a-b}{3} \end{bmatrix}$

$E_1 = E_1 - E_2$

$\sim \begin{bmatrix} 0 & 1 & a - \frac{a-b}{3} \\ 0 & 1 & \frac{a-b}{3} \end{bmatrix}$

$X_1 = a - \frac{a-b}{3}$

$X_2 = \frac{a-b}{3}$

Let $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$X_1 = a - \frac{a-b}{3} \quad = 4 - \frac{[4-(-2)]}{3} \quad = 2$

$X_2 = \frac{a-b}{3} \quad = \frac{[4-(-2)]}{3} = 2$

$2\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

Hence the proof.

Conclusion

From this we conclude that if $w_1, w_2, \ldots w_n$ be vector in R^m.
Then$\{w_1, w_2. \ldots w_n\}$ spans R^m.

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A Study on Application of Laplace Transform

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ABSTRACT: *The Laplace transform is powerful method for solving differential equations, complex differential equations are solved using laplace transform. A partial differential equation system which has two real dependent and two real independent variable an application of laplace transform. In this paper latest we obtain real and imaginary parts of solution using inverse laplace transform*

Keywords: *Laplace transform, properties of Laplace transform derivative of Laplace transform*

1. Introduction

The general solution of some equation especially type of elliptic are not found independent variables are even can be transform to a complex differential equation. for example,

$$U_{XX}+U_{YY}=0 \quad (1)$$

Laplace equation has not general solution but it can be witten

$u_{z\bar{z}}=0$

with the relation

$D = \frac{\partial}{\partial z \partial \bar{z}}$

and the solution of the equation is given as

$u = f(z)+g(\bar{z})$

where f is analytic and g is anti analytic function. Partial differential equation system has one and two real independent variable ,for examples

$u_x - v_{y=0}$

$U_Y + V_{X=0}$

Cauchy Riemann system transforms to complex equation

$\mathbf{W}_{\bar{Z}} = 0$

where w=u+iv ,z=x+iy solution of this complex equation aare analytic function, any one complex differential equation can be transform to partial differential equation two independent variables are separated the real and imaginary parts.

Application of the Laplace Transform For Solving Differential Equation

The laplace transform allow to transform of ordinary differential equation into algebraic equation .for example a continuous function f(t) derivatives f'(t) and f''(t)

$Af''(t) + bf'(t) + cf(t) = 0$

If we apply the Laplace Transform:

$L[af''(t) + bf'(t) + cf(t)] = 0$

Some properties of the Laplace Transform

1. L {0} = 0
2. L {f (t) g(t)} = L {f (t)} L {g(t)}
3. L {c f (t)} = c L {f (t)}, for any stable c.

4. The derivative of Laplace transforms
L {(−tf (t)} = F ′(s) or, equally L {t f (t)}=−F′

Laplace transform of elementary functtions :-

1. Laplace transforms of 1

L{f(t)}=$\int_0^\infty e^{-st}\ f(t)\,dt$
F(t)=1
L(1)=$\int_0^\infty e^{-st}\ 1\,dt$
= $\int_0^\infty e^{-st}\ \ dt$
=[$e^{-s}/-s$]
=$e^{-\infty}/s+e^0/s$

L[1]=$\frac{1}{s}$

Basic definitions and theorems
Let f(t) be a function of t>0.laplace transform of f(t) is defined as
L(F(t))=f(s)=$\int_0^\infty e^{-st}$f(t)dt
Then we can write $L(F(t)) = f(s)$.

Theorem 1. *If* $F(n)\ (t)$ *is partial continuous, then*

$L(F(n)(t)) = s^n f(s)-s^{n-1}F(0)-s^{n-2}F'(0)-sn-2F''(0)-\ldots-F(n-1)(0)$

where $L(F(t)) = f(s)$.

Complex derivatives
Let w=w(z,$\bar{z}$) be a complex function. Here z=x+iy, w(z, $\bar{z}$)=u(x,y)+iv(x,y).then the first order derivatives according to z and $\bar{z}$ on defined as

$$\frac{\partial w}{\partial z} = \frac{1}{2}\left(\frac{\partial w}{\partial x} - i\frac{\partial w}{\partial y}\right)$$

$$\frac{\partial w}{\partial z} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y}\right)$$

Example : Co efficient of equation are $A = 2, B = -1, C = 0$ and $F(z, z) = 4z+1$. therefore

$F*1\ (x, s) = L[F1(x,y)] = (4x+1)/s$
$F*2\ (x, s) = L[F2(x,y)] = 4/s2$

$u(x,y) = L-1[\frac{\frac{\partial}{\partial x}\left(\frac{8x+2}{s}\right)-3s\left(\frac{8}{s^2}-3(x^2+5x)\right)}{D^2+S^2}$
$=L^{-1}[\frac{[\frac{8}{s}-\frac{24}{s}+9s(x^2+5x)}{D^2+9S^2}$
$=L^{-1}[\frac{1}{9S(1+\frac{d}{9S})}(\frac{-16}{S}+9sx^2+45sx)]$
$=L^{-1}[\frac{1}{9s^2}(\frac{-16}{s}+9sx^2+45sx-\frac{18}{9s})]$
$=L^{-1}[-\frac{2}{s3}+\frac{x^2+5x}{s}]=x^2+5x-y^2$

Similarly,

$V(x,y)=L^{-1}[\frac{1}{9s2(1+\frac{d2}{9s2}}(-6x-15+24x+6)]$

$=L^{-1}[\frac{1}{9s}(18x-9)$

$= 2xy-y$.

Consequently

$w = u+iv$

$= x2 +5x-y2+i\,(2xy-y)$

$= x2 +2ixy-y2+3(x-iy)+2(x+iy)$

$= z2 +3z+2z$.

Conclusions

In this communication we successfully applied laplace transform of differential equation .it gives a simple and a powerful mathematical tool the results reveal that the method is very effective and simple.

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The Basic Concepts of Eigen Value and Eigen Vector

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ABSTRACT: *We review here the basics of computing eigen values and eigen vectors. E Eigen values and eigenvectors play a prominent role in the study of ordinary differential equations and in many applications in the physical sciences. Except to see them come up in a variety of contexts.*

Keywords: *Eigen values, matrix, polynomial, root, multiple.*

1. Introduction

Eigen values have their greatest importance in dianomic problems. The solution of dv/dt =Av is changing with time growing or decaying or oscillating. The eigen value problem is a problem of considerable theoretical interest and wide ranging application. The eigen vectors are a lineal combination of atomic movements which indicates global movements of the proteins (the essential deformation on models).

Definition

Let A be an n×n matrix . the number Computing eigen value and eigen vector
We can rewrite the condition Av=λv as
(A-λI)v=0

where I is the n×n identity matrix. Now in order for a non-zero vector v to satisfy this equation, A-λI must not be invertible.
That is, the determinant of A-λI must equal 0.We call p(λ)=det(A-λI) the characteristic polynomial of A. The eigen values of A are simply the roots of the characteristic polynomial of A.

Computing Eigen values and Eigenvectors

We can rewrite the condition Av= λv as
(A- λI)=0
where I is the n×n identity matrix. Now, in order for a non-zero vector v to satisfy this equation, A- λI must not be invertible.
That is, the determinant of A-λI must equal0. We call p(λ)=det(A-λI) the characteristic polynomial of A. The eigen values of A are simply the roots of the characteristic polynomial of A.

Example

Let A=[2 -1 -4 -1].Then p(λ)=det[2-λ-1-4-1-λ](2-λ)(-1-λ)-(-4)(-1)λ_2-λ-6(λ-3)(λ+2)
Thus, λ_1=3 and λ_2=-2 are the eigenvectors of A.
To find eigenvectors v=[$_{v1\ v2\ \ vn}$]corresponding to an eigen value λ, we simply solve the system of linear equations given by
(A-λI)v=0

Definition

Any non-zero vector uϵv such that Au=λv is called an eigenvector of A corresponding to eigen value λ.

Example

The matrix A=[2 -1 -4 -1]of the previous example has eigen values λ_1=3 and λ_2=-2.Let's find the eigenvectors corresponding to λ_1=3.Let v=[v2v1].Then(A-λI)v gives us
[2 -3 -1 -4 -1 -3][v1v2]=[0 0]

From which we obtain the duplicate equations
-v1-4v2-v1-4v_2=0 0

If we let v_2=t, then v_1=-4t.All eigenvectors corresponding to λ_1=3 are multiples of [1 -4] and thus the eigen space corresponding to λ_1=3 is given by the span of [1 -4].That is,{[1 -4]} is a basis of the eigen space corresponding to λ_1=3

Repeating this process with λ_2= -2,we find that
$4v_1-4v_2-v_1+v_2=0$

If we let v_2=t then v_1=t as well. Thus, an eigenvector corresponding to λ_2=-2 is[11] and the eigen space corresponding to λ_2=-2 is given by the span of [11].{[11]} is a basis for the eigen space corresponding to λ_2=-2.
In the following example, we see a two dimensional eigen space.

Definition
Let A = $[a_{ij}]$ be an n x m matrix. The determinant
$f(\lambda) = \det(\lambda I_n - A)$
is called the characteristic polynomial of A. The equation

$f(\lambda) = \det(\lambda I_n - A) = 0$ is called the equation of A.

Theorem
The eigen value of A are the real roots of the characteristic polynomial of A.

Proof
Let λ be an eigen value of A with associated eigen vector X. Then Ax = λx , which can be rewritten as
$Ax = (\lambda I_n)X$

A homogeneous system of n equations in n unknowns. The system has a non-trivial solution if and only if the determinant of its coefficient matrix vanishes.
Conversely, if λ is a real root of the characteristic polynomial of A, then $\det(\lambda I_n - A) = 0$, so the homogeneous system (5) has a nontrivial solution X. Hence λ is an eigen value of A. Two results that are sometimes useful in this connection are as follows: The product of all the roots of the polynomial
$f(\lambda)= \lambda^n+a_1 \lambda^{n-1}+...+a_{n-1} \lambda+a_n$

The value of λ in Equation and solving the resulting homogeneous system

Conclusion
From this paper we conclude that we can find the eigen vectors corresponding to an eigen value λ and we can simply solve the system of linear equation given by simplest way as (A- λI)V =0 here A is the matrix, I is the Identity matrix.

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Basic Concept of Ring Homomorphism

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ABSTRACT: *The algebraic system is serving as the building blocks for the structure comparing the subject known as modern algebra. Abstract concept of a group has its origins in the set of mappings are permutations, of a set onto itself. In contrast, rings steam from another and more familiar source. In this operations are usually called addition and multiplication. We shall require the appropriate analogs of homomorphism, normal subgroups, factors groups etc. The ideas and result in this section are closely interwoven with those of the preceding one. If there is one center idea which is common to all aspects of modern algebra it is the notation of homomorphism.*

1. Introduction

Modern algebra is a study of sets with operators defined on them. It begins with the observation that certain familiar rules hold for different operations on different sets. Let us consider the set of natural numbers

$(a + b)+c = a + (b + c)$ ------------------------→ (1)
$(a\, b)\, c = a\, (b\, c)$ --------------------------→ (2)
$(a + b) = (b + a)$ -------------------------→ (3)
$(a.b) = (b.a)$ --------------------------→ (4)
$a(b + c) = a\, b + a\, c$ -------------------------→ (5)
$1.\, a = a$ --------------------------→ (6)
$0+a = a$ --------------------------→ (7)
$0a = 0$ --------------------------→ (8)
$a + (-a) = 0$ --------------------------→ (9)
$a(1/a) = 1$ ---------------------------→ (10)

At this stage you should note that the following pairs of rules are very similar same: (1), (2) and (3) , (4) and (6), (7), (9) and (10) only difference is that they reference operators. In algebra one studies an abstract set with one or more operations defined on it. These operations are assumed to satisfy some basic properties and the aim is to study to consequences.

Definition

- Let R=(R, +R, .R) and (S, +S, .S) be ring. A set map Ψ: R→S is a ring homomorphism if

1) $\Psi\, (a + R\, b)= \Psi(a)+S\, \Psi(b)$ for all a , b ∈ R
2) $\Psi\, (a.\, R\, b) = \Psi\, (a).S\, \Psi(b)$ for all a , b ∈ R
3) $\Psi\, (1_R)=1s$

- A map f: R→S between rings is called a ring homomorphism.

For simplicity we will often conditions (1) and (2) as $\Psi\, (a + b) = \Psi\, (a) + \Psi\, (b)$ and
$\Psi\, (a.\, b) = \Psi\, (a).\, \Psi\, (b)$ with the particular addition and multiplication implicit.
$f\, (x\, y) = f(x)\, f(y)$ ∀ **x, y** ∈ R if $f(x + y) = f(x) + f(y)$

- A ring homomorphism which is bisection is called the ring homomorphism. If f : R→S is such an isomorphism, we call the ring . R and S isomorphic and write R=S.

Remark 1

If Ψ(R, +,.)→(S, +,.) Is a ring homomorphism then Ψ (R.+) →(S,+) is a group homomorphism.

Lemma

Let Ψ: R→S be a ring homomorphism.
Then i) $\Psi\, (0_R) = 0_{S.}$
ii) $\Psi\, (-x) = -\, \Psi(x)$ for all r ∈ R.

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iii) If $x \in R^*$ then $\Psi(x) \in S^*$ and $\Psi(r^{-1}) = \Psi(r)^{-1}$ and
iv) If $R' \subset R$ is a sub ring then $\Psi(R')$ is a sub ring of S.

Proof

Statement (i) and (ii) hold by of Remark1. Again we say the proof here for the completeness of the theorem.
Since $0_R + 0_R = 0_R$
$\Psi(0_R) + \Psi(0_R) = \Psi(0_R)$
Since S is a ring,
$\Psi(0_R)$ has an additive inverse which we may add to both sides.
Thus we obtain $\Psi(0_R) = \Psi(0_R) + \Psi(0_R) + (-\Psi(0_R))$
$= \Psi(0_R) + (-\Psi(0_R))$
$= 0_S$

.Let $x \in R$ since $x + (-x) = -x + x$
$= 0_R$

We have $\Psi(x) + \Psi(-x) = \Psi(-x) + \Psi(x)$
$= \Psi(0_R)$
$= 0_S$

We have this equality comes from (i).
i.e) $\Psi(-x) = -\Psi(x)$ as additive inverse are unique.
Now let $x \in R^*$ then there exist $x^{-1} \in R$ such that $xx^{-1} = x^{-1}x = 1_R$.

Then since Ψ is a ring homomorphism we have
$\Psi(x)\Psi(x^{-1}) = \Psi(x^{-1})\Psi(x)$
$= \Psi(1_R)$
$=1_S$.

Thus $\Psi(x)$ has a multiplicative inverse and it is $\Psi(x^{-1})$.
Finally let $R' \subset R$ be a sub ring. To show that $\Psi(R')$ is a sub ring. We must such that $1_S \in \Psi(R')$ and for all S_1, $S_2 \in \Psi(R')$, S_1-S_2 and S_1S_2 are also in $\Psi(R')$.
Since S_1, $S_2 \in \Psi(R')$ there exist $r_1r_2 \in R'$ such that $\Psi(r_1) = S_1$ and $\Psi(r_2) = S_2$.

Thus $S_1 - S_2 = \Psi(r_1) - \Psi(r_2)$
$= \Psi(r_1) + \Psi(r_2)$
$= \Psi(r_1 - r_2)$ and
$S_1S_2 = \Psi(r_1)\Psi(r_2)$
$= \Psi(r_1r_2)$

Since R^1 is a sub ring $r_1 - r_2$ and r_1r_2 and contained in R'.
Hence S_1-S_2 and S_1S_2 are in $\Psi(1_R) \in \Psi(R')$.
Furthermore $1_R \in R'$ so1s $= \Psi(1_R) \in \Psi(R')$.
Hence $\Psi(R')$ is a sub ring of S.

Conclusion

In this course we have learnt that modern algebra is a study of sets with operations defined on them. As the main example we have discussed about the ring homomorphism. Ring homomorphism is one of the most important areas of contemporary mathematics. Further study of rings can be undertaken in the appropriate honour modules.

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Concept of Vector Space and Co-Ordinates

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ABSTRACT: *A coordinate system is a system that user one or more number, or coordinates to uniquely determine the position of the points or other geometric elements on a manifold such as Euclidean space [1] [2] the order of the coordinates is significant and they are sometimes identified by their position in an ordered tipple and sometimes by a letter to be real numbers in elementary mathematics, binary be complex numbers or elements of a more abstract system such as a commutative ring.*

1. Introduction

One of the useful features of a basis B in an n-dimensional space V is that it essentially enables one to introduce co-ordinates in V analogous to the natural co-ordinates x; of a vector $\alpha = (x_1, - - x_n)$ in the space F^n. In this scheme, The co-ordinates of a vector α in V relative to the basis B will be the scalars which serve to express α as a linear combination of the vectors in the basis.

Co-Ordinates

Definition

If V is an n-dimensional vector space, we know that V has a basis S with N vectors in it; so for we have not paid much attention to the order of the vectors in S. However in the discussion of this section we speak of an ordered basis $S = \{V_1, V_2, \text{----}\ V_n\}$ for V; thus $S_1=\{V_2, V_1, \text{----}\ V_n\}$ is a different ordered basis for V.

If $S =\{V_1, V_2, \text{----}\ V_n\}$ is an ordered basis for the n-dimensional vector space V, the every vector V in V can be uniquely expressed in the form

$V=C_1 V_1 + C_2 V_2+ \text{--------}+ C_n V_n$

Where $C_1, C_2 \text{-----}\ C_n$ are real numbers we shall refer to

$$[V]_S=\begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}$$

As the co-ordinate vector of V with respect to the ordered basis S. the entries of $[V]_s$ are called the co-ordinates of V with respect to S.

Co-ordinate systems

Theorem 6.1

Let $S = \{ V_1, V_2, \text{----------}\ V_n\}$ be a basis of a vector space V. Then for each vector v in V. There exists a unique set of scalars $C_1, C_2 \text{------}\ C_n$ such that

$V= C_1 V_1+ C_2 V_2+ \text{------}+ C_n V_n$

Proof

Let $B= \{ V_1, V_2, \text{----------}\ V_n\}$ be a basis of a vector space V. Then every v of v has a unique expression

$V= C_1 V_1+ C_2 V_2+ \text{------}+ C_n V_n$

The scalars $V= C_1,C_2,\ldots\ldots C_n$ are called the co-ordinates of v relative to the basis B (or B co-ordinates of V) and the vector.

$$[V]_B=\begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}$$

Is called the co-ordinate vector of v relative to B (or the B – co-ordinate vector of v) we may write

$V= C_1 V_1+ C_2 V_2+ \text{------}+ C_n V_n$

$= [V_1,V_2 \text{------} V_n] \begin{bmatrix} C_1 \\ \vdots \\ C_n \end{bmatrix}$

=B [V]B

Example

Any two linearly independent vectors of R^2 from a basis of R^2 for instance the set

$B= \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

Is basis of R^2 the vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ has the

B co-ordinate vector $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ however the co-ordinate vector of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is just itself under the standard basis.

$E=\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

Conclusion

It's a statement, decision reached by the researcher based on findings in the research. On basic difference an abstract is always at the beginning of a academic paper. A conclusion is always at the end.

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An Application of the Selected Graph Theory Domination Concepts to Transportation Networks Modelling

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ABSTRACT: *One of the possibilities when modelling a transport network is to use a graph with vertices and edges. They represent the nodes and arcs of such a network respectively. The main aim of this paper has been to show the possibilities of the application of the selected domination-oriented concepts to modelling and improving the transportation and logistics networks. The edge-subdivision and bondage number notations and their implementations to the transportation network description and modelling were then proposed.*

Keywords: *Cubic graph, graph coloring, transport network, vertex set, edge set.*

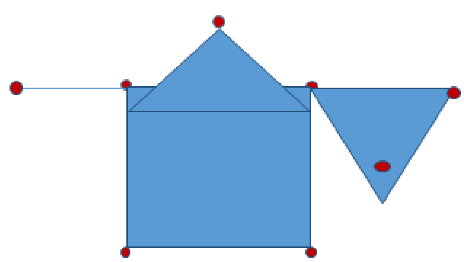
1. Introduction

Transportation systems are the basis of today's national and world economies. It is a major component of each country's Gross Domestic Product. Therefore, it is recognized in many countries as one of the critical infrastructures. a transportation system can be defined as the combination of elements and their interactions, which produce the demand for travel within a given area, and the supply of transportation services to satisfy this demand. These items are means of transport, infrastructure, and people.
By Euler's formula,

"If the Sum of the degree of vertices is a graph in G is twice the number of edges in G.
Then G $\Sigma d(v)=2E$"

Theorem: Every cubic graph k is regular of degree p-1
Proof: Let G be a cubic graph with p points.Then $\Sigma \deg v = 3p$ which is even. Hence p is even.
Remark, .A theorem of Erd˝os and T. S´os describing the odd case was a starting point of the theory of Ramsey Tur'an problems.

Example



2. Graph Theory Topics Review

This section contains the basic notations of graph theory, some definitions and parameters of domination and edge-subdivision terms based on results.

Basic notations

In the whole article, we have considered the connected, simple, undirected graph where V is the set of vertices and E is the set of edges. In other words, for a graph G, V(G) and E(G) respectively denote its vertex-set and the edge- set. These assumptions are very important, because the connectivity of transport or logistics networks is fundamental to the functioning of these networks.

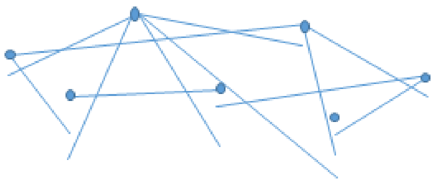
3. Arrangeability And Graph Coloring

The admissibility of relative to L is then defined as the maximum size of a subset S that is admissible for x, and the admissibility of L is the maximum value of the admissibility of the nodes relative to L.

Domination and bondage numbers in graphs

The definitions of two dominating sets and domination numbers have been introduced in this subsection. Generally, a set D ⊆ V(G) is a dominating set of graph G, if for any v ∈ V either v ∈ D or NG(v) ∩ D = ∅. While the minimum cardinality of a dominating set of graph G is called the domination number of G and denoted as γ (G).

Example



Domination and edge-subdivision in graphs

Another, interesting concept in the theory of domination is edge-subdivision. It was introduced by approach is based on the operation of subdividing graph G, which was defined in the following way.
Let G be a graph and uv be an edge of G. By subdividing the edge uv we mean forming a graph H from G by adding a new vertex w and replacing the edge uv by uw and wv.
Moreover, this concept is used under the assumption that the domination number of graph S(G), obtained from G by subdividing every edge exactly once is more than that of G,is called the subdivision number of G and is denoted by ξ(G).

Implementations of domination-related concepts in transportation networks

The results presented in Section Introduction can be applied to the analysis and modelling of the transportation networks. This section presents the most efficient way to do this. Moreover, the connected bondage number and the bondage-connected number are defined as the author's new concepts in domination-related problems.Thus, the author has proposed two new bondage numbers, i.e. a connected bondage number for the connected dominating set, and a bondage connected number for the dominating set.

Applications

We have taken into account two cases:
Case 1: The transportation network has fixed main nodes, which correspond to the dominating set in the represented graph.
Case 2: The transportation network can be designed by finding the main nodes, i.e. the minimal dominating set.

Conclusions

The paper has presented the possibilities for modelling a transport network with the graph theory approach using the domination parameters and the edge-subdivision concept. First, a review of the literature on the methods of modelling and the optimization of technical and transport systems, with particular emphasis on maritime transport, was conducted.
Next, the basics of domination in graph theory were introduced. The domination number, bondage number, and the author's new concepts of the connected bondage number and the bondage-connected number have been proposed. The edge-subdivision methods for vertex-domination in graphs have been described and implemented for the transportation and logistics networks.
Finally, the application of the previously mentioned and defined methods has been presented as used on the exemplary transportation network in two cases.

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A Study on Plane and Planar Graphs

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ABSTRACT: *In this paper we consider the embedding (drawing without crossings) of graphs on surfaces, especially the plane. We study the properties of planar graphs. The graphs that can be drawn in the plane so that none of their edges intersects other than the vertices. We introduce concepts such as thickness, crossing number and outer planarity of graphs.*

Keywords: *Plane, planar graph, dual plane graph.*

1. Introduction

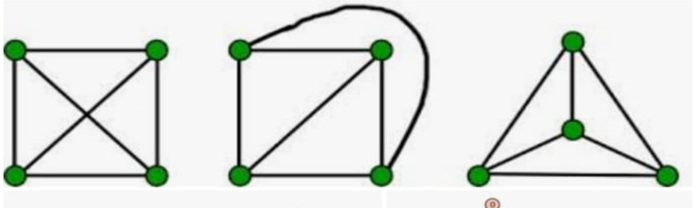
A graph is said to be embeddable in the planar if it can be drawn in the plane so that it edges intersects only at there end vertices such a drawing of a planar graph G is called planar embedding of G. A graph may be planar even if it is drawn with crossings, because it may be possible to draw it in a different way without crossing.

Plane Graph

A graph G (V, E) is called plane if,
V is a set of points in the plane
E is a set of curves in plane

Planar Graph

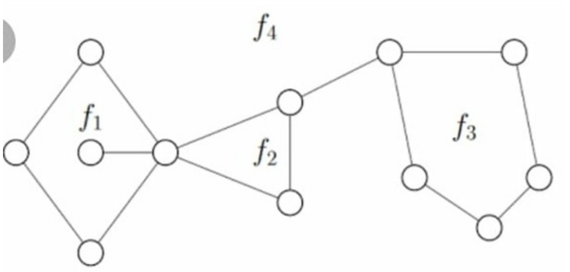
A planar graph is a graph, if it is isomorphic to a plane graph. The plane graph which is isomorphic to a given planar graph G is said to be embedded in the plane.



Plane graph (G) Planar graph

Faces of A Graph

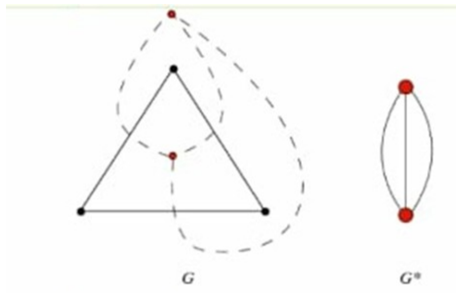
A plane graph G (V, E), a face of G is a maximal region of the plane if the vertices and the edges of G are removed.
An unbounded faces of G is called exterior or outer face. The vertices and the edges of G that are incident with a face F from the boundary of F.



Dual Plane Graph

Let G be a plane graph. The dual graph G* of G is a new plane graph having a vertex for each face in G and the edges that correspond to the edges in the following manner.

If e an edges of G which separates two faces X and Y, then the corresponding dual edge $e^* \epsilon$ E (G*) is an edge joining the vertices x and y that correspond to X and Y respectively.



Theorem: 1

Euler's formula:

If G is a connected plane graph then $\gamma - \epsilon + \phi = 2$

Proof:

Let us prove that the formula by applying induction on ϕ the number of faces.
If $\phi = 1$ then such a graph cannot have a cycle. In this case G is a cyclic graph.
Also, each edge of G is a cut edge of G, as G is connected it must be a tree.
In this case, by known then

$\epsilon = \gamma - 1$
$\therefore\ \gamma - \epsilon + \phi = \gamma - (\gamma - 1) + 1$
$= \gamma - \gamma + 1 + 1$
$\gamma - \epsilon + \phi = 2$

Thus the result is true in this case suppose that it is true for all connected plane graph with fewer than n – face.

Let G be a connected plane graph with n $\geq$2 faces choose an edge e of G. That is not result for a cut edge. Then G-e is a connected plane graph and has (n-1) faces.
$\therefore$ The two faces of G generated by e contained to form one face of G – e.

By induction hypothesis, we must have,
V (G-e) - ϵ (G-e) =2

But V (G-e) = V (G)
ϵ (G-e) = ϵ (G) – 1
ϕ (G–e) = ϕ (G) – 1
V (G) - ϵ (G) +1+ ϕ (G)-1=2
V (G)-ϵ (G) + ϕ (G) = 2
The result follows by principle of induction

Theorem: 2

If G is a planar graph (no parallel edges) with vertical t edges ($t \geq 2$) then $t \leq 3s - 6$. In addition G is bipartite there $t \leq 2s-4$.

Proof:

Let u be the number of faces of G and Let mi be the number of edges in the boundary of the i^{+n} face (i=1,. . . . . . n)

Since every face contains at least three edges,
$3r \leq \Sigma^{n}_{i=1}$ mi

Thus $3u \leq 2t$ and by Euler's theorem, $s-t + 2t/3 \geq 2$, implying $t \leq 3s-6$.
If G is bipartite, the shortest cycle of length at least 4. Thus
$4u \leq \Sigma^{n}_{i=1}$ mi
Together with $\Sigma^{n}_{i=1}$ mi $\leq 2t$ and $s-t + u=2$, we get the second of the theorem.

Corollary: 1
If G is a plane graph with $\delta(G) \geq 3$ then there is a face in G of degree ≤ 5.

Proof:
Let G be a plane graph with $\delta(G) \geq 3$ we have to show that there is a face in G of degree $\leq$ 6
(ie) $d(f) = 6$
$\Sigma_{f\epsilon F}\, d(f) \geq \epsilon 6$
$\epsilon 6 = 6 + 6 + \ldots\ldots. + 6$ (ϕ terms)
$2\epsilon \geq 6\,\phi$
$3\,\phi \leq \epsilon$
$\phi \leq \epsilon/3$
$\delta\,(G) \geq 3$
$d\,(v) \geq \delta(G) \geq 3\ \ \forall\, v \in V$
$\Sigma_{v\,\epsilon V}\, d\,(v) \geq 3\,\epsilon$
$3\,\epsilon = 3 + \ldots. + 3$ (γ terms)
$2\,\epsilon \geq 3\,\gamma$
$\gamma \leq 2\,\epsilon/3$
By Euler's formula for planar graphs,

We have,
$2 \leq \gamma + \phi - \epsilon$
$2 \leq 2\,\epsilon/3 - \epsilon + \epsilon/3$
$2 \leq {}^{2\varepsilon\, -\, 3\varepsilon\, +\, \varepsilon}/_{3}$
$2 \leq 0$

which is absurd
This is contradiction
G has a face of degree ≤ 5

Corrollary: 2
k_5 is non-planar.

Proof:
We have $\gamma\,(K_5) = 5$, $\epsilon(K_5) = 10$.
We have to prove that k_5 is non-planar.
On the contrary assume
k_5 is planar.
We know that,
If G is simple planar graph with $\gamma \geq 3$ then $\epsilon \leq 3\gamma-6$

From this
$\epsilon \leq 3\gamma-6$
$10 \leq 3(5) - 6$
$10 \leq 9$
which is a contradiction
k_5 is non-planar.

Corrolary: 3
Prove that $k_{3,3}$ is non polar.

Proof:
We have γ ($k_{3,3}$) =6
ϵ ($k_{3,3}$) =9

Suppose that $k_{3,3}$ is planar and Let G be a planar embedding of $k_{3,3}$.
We know that,
A graph G is bipartite if it has no odd cycles.
$k_{3,3}$ is bipartite it has no odd cycle.
Hence it has no cycles of length less than four.
Every face of G must degree at least four then,
$2\epsilon = \Sigma_{f\epsilon F}$ d (f)$\geq$4+4+4………+4(ϕ terms)
$2\epsilon \geq 4\ \phi$
$18 \geq 4\ \phi$
$\phi = 4$
Euler formula on planar graph

We have,
$2 \leq \gamma - \epsilon + \phi \leq 6 - 9 + 4 \leq 1$
$2 \leq 1$

which is absurd
This contradiction proves that $k_{3,3}$ is non – polar.

Conclusion
In this paper we are willing to summarize types of planar graph and also types of graph along with persons related to respective topic. We hope this paper will help researchers to look into the field of graph at glance.

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Vector Space - Subspace, Linear Span and Basis & Dimension

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ABSTRACT: *In this paper, vector space owe their importance to the fact that so many models arising in the solution of specific problems turn out to be vector space. Among these fundamentals notations or those of linear combinations, linear dependence, basis and dimensions which will be developed. In this section, we shall define the mathematical object which experience has shown to be the most useful abstractions of this types of algebraic system.*

Keywords: *Introduction, Vector space, Subspace, Linear span, Linearly Independent or Linearly dependent, Basis and Dimensions, Finite dimension Vector space, Inner product space.*

1. Introduction

The idea of vector space arises in the study of various physical applications. The concept of direction is given vector and whose length represents the magnitude of vector. This introduced a new concept of multiplication of a vector by a scalar, the resulting product being a vector.

Definition: Vector Space

A vector space is a not empty set V of objects, called vector space.
They are the following operation are called addition and multiplication.

- ✓ $x + y$ is in V
- ✓ $x + y = y + x$
- ✓ $x + (-x) = 0$
- ✓ $\alpha (x + y) = \alpha x + \alpha y$
- ✓ $(\alpha + \beta) x = \alpha x + \beta x$

Theorem – 1

Suppose V is a vector space, $x, y \in V$ and β is a non-zero scalar from A. If $\beta x = \beta y$ then $x = y$.

Proof:

$x = 1x$
$= (1/ \beta * \beta) x$
$=1/ \beta (\beta x)$
$=1/ \beta (\beta y)$
$= (1/ \beta * \beta) y$
$x = y$
Hence proved.

Theorem – 2:

Suppose V is a vector space, $v \neq 0$ is a vector in V and $\beta, \gamma \in C$. If $\beta v = \gamma v$ then $\beta = \gamma$.

Proof

$0 = \beta v + (- \beta v)$
$= \gamma v + (- \beta v)$
$= \gamma v + (-1) (\beta v)$
$= \gamma v + ((-1) \beta) v$
$= \gamma v + (-\beta) v$
$= (\gamma - \beta) v$
By hypothesis, $v \neq 0$

$0 = \gamma - \beta$

$\beta = \gamma$
Hence proved.

Definition: Subspace
The subspace S of a vector space V is a subset W of V. It has three properties:
- The zero vector of V is in W.
- For each x and y are in W, x + y is in W.
- For each x in W and each scalar α, αx is in W.

If the subset W satisfies these three properties, then W itself is a vector space.

Theorem:
If $u_1, u_2 ... u_p$ are any vectors in R^n, then space $\{u_1, u_2 ... u_p\}$ is a subspace of R^n.

Proof:
We have to verify that three properties,
- The zero-vector

$0 = 0u_1 + 0u_2 + ... + 0u_p$ is in the span.
- If $v = a_1u_1 + a_2u_2 + ... + a_pu_p$

$w = b_1u_1 + b_2u_2 + ... + b_pu_p$ are in span $\{u_1, u_2 ... up\}$ then
$v + w = (a_1 + b_1) u_1 + ... + (a_p + b_p) u_p$ is also in span $\{v_1, v_2 ... v_p\}$
- If $u = a_1u_1 + a_2u_2 + ... + a_pu_p$ is in span $\{u_1, u_2, ... u_p\}$ and α is a scalar , then

$\alpha u = \alpha a_1u_1 + \alpha a_2u_2 + ... + \alpha a_pu_p$ is also in span $\{u_1, u_2 ... u_p\}$
Since, span $\{u_1, u_2 ... u_p\}$ satisfies the three properties of a subspace.
Hence, it is a subspace.

Definition: Linear Span
The span of a non-empty subset is S of a vector space V. Then the set of all linear combinations of finite series set of S is called Linear span (or) Span of a set. It is denoted by L(S).
Then L(S) is of the form,
$\beta_1v_1 + \beta_2v_2 + ... + \beta_nv_n$ where $\beta_1, \beta_2 ... \beta_n \in F$
Another form of Linear span (or) span of a set,
$L(S) = \{\sum_{j=1}^{n} \beta j vj : vj \in S \text{ and } \beta j \in F, 1 \leq j \leq n\}$

Theorem
Let V be a vector space over a field F and S be a non-empty subset of V. Then L(S) is a subspace of V.

Proof:
Let u, s $\in$ L(S) and $\alpha, \beta \in F$.
Then $u = \alpha_1u_1 + \alpha_2u_2 + ... + \alpha_nu_n$ where $u_i \in S$ and $\alpha_i \in F$.
Also, $S = \beta_1s_1 + \beta_2s_2 + ... + \beta_ms_m$ where $s_j \in S$ and $\beta_j \in F$.
Now, $\alpha u + \beta s = \alpha (\alpha_1u_1 + \alpha_2u_2 + ... + \alpha_nu_n) + \beta (\beta_1s_1 + \beta_2s_2 + ... + \beta_ms_m)$
$= (\alpha\alpha_1) u_1 + (\alpha\alpha_2) u_2 + ... + (\alpha\alpha n) u_n + (\beta\beta 1) s_1 + (\beta\beta_2) s_2 + ... + (\beta\beta_m) s_m$
$\therefore$ $\alpha u + \beta s$ is also a linear combination of a finite number of elements of s.
Hence $\alpha u + \beta s \in L(S)$.
$\therefore$ L(S) is a subspace of V.

Definition: Linearly Independent or Linearly Dependent
In a vector space, a set of vectors is linearly independent if none of its elements in a linear combination of the others from the set. Otherwise the set is linearly dependent.

Example
In V (A) = $\{e_1, e_2 ... en\}$ is a linearly independent set of vectors for,
$\beta_1e_1 + \beta_2e_2 + ... + \beta_ne_n = 0$
$\beta_1(1,0,...,0) + \beta_2(0,1,...,0) + ... + \beta_n(0,...,1) = (0,0,...,0)$
$\rightarrow (\alpha_1, \alpha_2 ... \alpha_n) = (0, 0 ... 0)$

$\rightarrow \alpha_1 = \alpha_2 = \ldots = \alpha_n = 0$

Definition: Basis and Dimension

Suppose $S = \{u_1, u_2 \ldots u_n\}$ is a set of vector from the vector space V. Then S is called a basis for V if both of the following conditions old.

1. Span (S) = V, that is, S spans the vector space V.
2. S is a linearly independent set of vectors.

Definition: Finite Dimension Vector Space

A vector space V (A) is said to be finite dimensional vector space if there exist a finite subset of V that spans it.

A vector space which is not finite dimensional may be called an infinite dimensional vector space.

Theorem

Every finite dimensional vector space has a basis.

Definition: Inner Product Space

A Vector space V along with an inner product defined on it is called an inner product space. It associates each pair of vectors a, b with a scalar $\langle a, b\rangle$, and which satisfies:

- $\langle a, b\rangle = \langle b, a\rangle$
- For any $a \in V$, $\langle a, a\rangle \geq 0$ and with equality if and only if a = 0.

Conclusion

- This topic representations allow us to model rotational and translational symmetries.
- This introduced a new concept of multiplication of a vector by a scalar, the resulting product being a vector.

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A Study on Matrix Multiplication

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ABSTRACT: *In this paper we represent some matrix with rows and columns and give deep explanation about the product of matrices and theorems and some examples based on matrix and matrix multiplication.*

Keywords: Matrix multiplication, row, column, field, real, real numbers and complex numbers.

1. Introduction

In this chapter we have 2x2, 3x3 and mxn matrices. We have represented linear transformation by these matrices. Also we have developed the general theory of matrices. Throughout this chapter we dealt with matrices whose entries are from the field F of real or complex numbers.

Definition

Let A be an m x n matrix over the field F and let B be an n x p matrix over F. The product AB is the m x p matrix C whose i,j entry is

$C_{ij}=\sum^{n}_{r=1} A_{ir} B_{rj}$

Theorem

If A, B, C are matrices over the field F such that the products BC and A(BC) are defined, then so are the product AB, (AB)C and A(BC)=(AB)C

Proof

Suppose B is an nxp matrix since BC is defined, c is a matrix with p rows, and BC has n rows. Because A(BC) is defined we may assume A is an mxn matrix . thus the product AB exists and is an mxp matix, from which if follows that the product (AB)C
exists. To show that A(BC)=(AB)C means to show that$[A(BC)]_{ij}=[(AB)C]_{ij}$

For each i,j by definition $[A(BC)]_{ij}=\sum_r A_{ir}(BC)_{rj}$

$=\sum_r A_{ir} \sum_s B_{rs} C_{sj}$

$=\sum_r\sum_s A_{ir}B_{rs}C_{sj}$

$=\sum_s\sum_r A_{ir}B_{rs}C_{sj}$

$=\sum_s(\sum_r A_{ir} B_{rs})C_{sj}$

$=\sum_s(AB)_{is} C_{sj}$

$[A(BC)]_{ij} =[(AB)C]_{ij}$

Example: 1

The matrix A, B are

$A = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{bmatrix}$

Calculate AB & BA

Solution

Using the rules of matrix multiplication

$AB = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{bmatrix}\begin{bmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

BA = $\begin{bmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{bmatrix} \begin{bmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

The Matrix B is the inverse of the matrix A, and this is usually written as A^-1
Equally, the matrix A is the inverse of the matrix B. The equation AA^-1 =A^-1A =I is always true.

Example: 2
Let A and B be the 2x2 matrices

A = $\begin{bmatrix} -2 & 4 \\ 5 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 4 \\ 5 & 3 \end{bmatrix}$

The Matrix AB is,

AB = $\begin{bmatrix} -2 & 4 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4+20 & -8+12 \\ -10+15 & 20+9 \end{bmatrix}$

AB = $\begin{bmatrix} 24 & 4 \\ 5 & 29 \end{bmatrix}$

Conclusion
From this paper we conclude that If A,B,C are matrices then so are the products AB,(AB)C and A(BC)=(AB)C.

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A Study on Concept of Matrix in A Linear Transformation

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ABSTRACT: *We have been discussing matrix of linear transformation for sometime, it has always been in a detached and impersonal way. We have really never come face to face with specify matrix of linear transformations. At the same time it is clear that if one were to pursue the subject further there would often arise the need of making a through and detailed study of a given matrix of linear transformation.*

Keywords: *Basis, vector spaces, linear transformation, array, matrix.*

1. Introduction

In a matrix of linear transformations play an important role in many area of mathematics, as well as in number our applied problems in the physical sciences the social sciences and economics and we will give various definition and some example of matrix of linear transformation.

In this paper we shall give a brief introduction form a geometric point of view to certain functions mapping A onto B, is called a matrix of linear transformation.

Definition

Let A and B be finite dimensional vector spaces over a field F. let dimA=m and dimB=n. Fix an ordered basis $\{a_1, a_2, \ldots, a_m\}$ for A and an ordered basis $\{b_1, b_2, \ldots, b_n\}$ for B.

Let $T:A \rightarrow B$ be a linear transformation we have seen that T is completely specified by the elements, $T(a_1)$, $T(a_2)$, …, $T(a_m)$. Now let,

$T(a_1) = p_{11}b_1+p_{12}b_2+\ldots+p_{1n}b_n$
$T(a_2) = p_{21}b_1+p_{22}b_2+\ldots+p_{2n}b_n$
$T(a_m) =p_{m1}b_1+p_{m2}b_2+\ldots+p_{mn}b_n$

Hence $T(a_1)$, $T(a_2)$, … , $T(a_m)$ are completely specified by the mn elements p_{ij} of the field F. These a_{ij} can be conveniently arranged in the form of m rows and n columns as follows.

P_{11} p_{12} … p_{1n}
P_{21} p_{22} … p_{2n}
P_{m1} p_{m2} … p_{mn}

Such an array of mn elements of F arranged in m rows and n columns is known as mxn matrix over the field F and is denoted by p_{ij} . Thus to every linear transformation T there is associated with it an mxn matrix over F. Conversely any mxn matrix over F defines a linear transformation $T:A \rightarrow B$ given by the formula (1).

Theorem

Let A and B be two finite dimensional vector spaces over a field F. Let dimA =B and dimB =A. Then L(A,B) is a vector space of dimension mn over F.

Proof

By the known results L(A,B) is a vector space over F. Now, we shall prove that the vector space L(A,B) is isomorphic to the vector space $M_{mxn}(F)$ is of dimension mn, it follows that L(A,B) is also of dimension mn. Fix a basis $\{a_1, a_2, \ldots, a_m\}$ for A and a basis $\{b_1, b_2, \ldots, b_n\}$ for B. We know that any linear transformation, T €(A,B) can be represented by an mxn matrix over F.

Let T be represented by M(T). This function $M : L(A,B) \rightarrow M_{m\,x\,n}(F)$ is clearly 1-1 and onto.
Let T_1, T_2 € L(A,B) and $M(T_1) =(p_{ij})$ and $M(T_2) = (q_{ij})$.

$M(T_1) = (p_{ij}) \Rightarrow T_1(a_i) = \sum_{j=1}$
$M(T_2) = (q_{ij}) \Rightarrow T_2(a_i) = \sum q_{ij}w_j$

Therefore, $(T_1+T_2) = \sum_{j=1} (p_{ij}+q_{ij})w_j$

$M(T_1+T_2) = (p_{ij}+q_{ij})$
$= p_{ij}+q_{ij}$
$= M(T_1)+M(T_2)$

Similarly,
$M(kT_1) = k\, M(T_1)$.

Hence, M is the required isomorphism from L(A,B) to $M_{m \times n}(F)$.

Example: 1
Obtain the matrix representing the linear transformation T: $A_3(R) \rightarrow A_3(R)$ given by T(p, q, r) = (3p, p-q, 2p+q+r) with respect to the standard basis $\{e_1, e_2, e_3\}$.

Proof
$T(e_1) = T(1,0,0)$
$= (3, 1, 2)$
$= 3e_1 + e_2 + 2e_3$

$T(e_2) = T(0,-1,1)$
$= (0, -1, 1)$
$= -e_2+e_3$

$T(e_3) = T(0,0,1)$
$= (0, 0, 1)$
$= e_3$

Thus the matrix representing T is

Example: 2
Find the linear transformation T: $A_3(R) \rightarrow A_3(R)$ determined by the matrix

Proof
$T(e_1) = e_1+3e_2 + e_3$
$= (1, 3, 1)$

$T(e_2) = 0e_1+2e_2 + e_3$
$= (0, 2, 1)$

$T(e_3) = -e_1+4e_2+3e_3$
$= (-1, 4, 3)$

Now,
$(p, q, r) = p(1, 0, 0)+q(0, 1, 0)+r(0, 0, 1)$
$=pe_1+qe_2+re_3$

Therefore, we have
$T(p, q, r) = T(pe_1+qe_2+re_3)$
$= pT(e_1)+qT(e_2)+rT(e_3)$
$= p(1, 3, 1)+q(0, 2, 1)+r(-1, 4, 3)$

T(p, q, r) =(p-q, 3p+2q+4r, p+q+3r)

This is the required matrix of linear transformation.

Conclusion

From this paper we conclude if A and B be two finite dimensional vector space. Then, L(A,B) is a vector space of dimension mn over F.

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The Maximal Regular Ideal of Some Commutative Rings

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ABSTRACT: *In **ring theory**, a branch of abstract algebra, a commutative ring is a ring in which the multiplication operation is **commutative**. where multiplication is not required to be commutative and an Ideal is a special subset of a ring. **Ideals** generalize certain subsets of the integers. An ideal can be used to construct a quotient ring in a **group theory**, a normal subgroup can be used to construct a quotient group.*

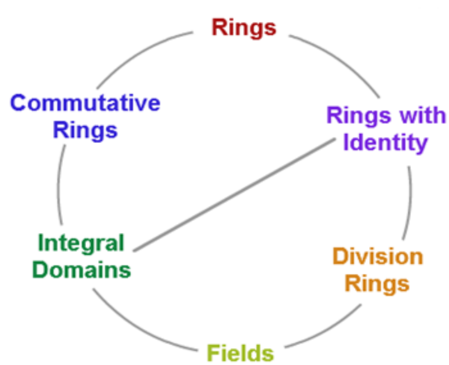
Keywords: *Rings, normal group, ideal, group theory , commutative ring*

1. Introduction

A ring addition is commutative, ring multiplication is not required to be commutative: ab need not necessarily equal b a . Rings satisfy the commutativity for multiplication are called commutative rings. On commutative algebra often adopt the convention that **"ring"** means **"commutative ring"**, to simplify terminology.

A nonempty subset I of R is then said to be a **left ideal** in R if, for any z, y in I and r in R, z+ y and r z are in I. similarly, I is said to be **right ideal** if IR in I. A subset I is said to be a **two-sided ideal** if it is both a left ideal and right ideal.

Commutative Ring



Definition

A ring is a set R with two binary operations addition (denoted +) and multiplication (denoted .). These operations satisfy the following axioms:

1. Addition is associative: If a, b, c in R , then

a + (b + c) = (a + b) + c

2. There is an identity for addition, denoted 0. It satisfies.

0+ a = a and a + 0 = a for all a in R.

3. Addition is commutative: If a, b in R , then

a + b = b + a.

4. Multiplication is associative: If a, b, c in R , then

a. (b . c) = (a . b) .c.

5. Multiplication distributes over addition: If a, b, c in R, then

a .(b + c) = a. b + a. c.

6. Every elements of R has an additive inverse. That is, if a in R, there is an element a in R which satisfies

a + (-a) = 0 and (-a) + a = 0.

Remarks
a. The additive identity is called the **zero of the ring** and is denoted by **'0'**.
b. The operation . is not necessarily commutative. If it is, we call R is a **commutative ring**.

Example
1. The integers Z with the usual addition and multiplication is a commutative ring with identity. The only elements with (multiplicative) inverses are **±1**.
2. The sets Q, R, C are all commutative rings with identity under the appropriate addition and multiplication. In these every non-zero element has an **inverse**.

Ideal
Definition
A non-empty subset I of R is said to be a **Ideal** of R.
✓ If A is a subgroup of R under addition(+).
✓ For every a in I and r in R both (a r) and (r a) are in I.

Types of Ideal
Maximal Ideal
A **proper ideal** I is called a maximal ideal if there exists no other proper ideal J with I a proper subset of J. The factor ring of a maximal ideal is a field for commutative rings.

Minimal Ideal
A **non-zero ideal** is called minimal if it contains no other non- zero ideal.

Prime Ideal
A proper ideal I is called a prime ideal if for any a and c in R, if ac is in I, then at least one of a and b is in I. The factor ring of a prime ideal is an integral domain for commutative rings.

Theorem 1
If R is a commutative ring with one prove that every maximal ideal in R is a prime ideal.

Proof
Let m be a maximal ideal in R
⟹ R/m is a field
⟹ R/m is an integral domain
⟹ m is a prime ideal.

Theorem 2
If R is finite commutative ring then prove that every prime ideal is maximal.

Proof
Let p be a prime ideal of R
∴ R/p is an integral domain

But R is finite
R/p is finite integral domain
R/p is field
∴ P is maximal

Problem
1. Prove that intersection of two ideals is an ideal.

Solution
Let I.J be two ideals of R
⟹ I and J are addition sub-group of R

$\Rightarrow I \cap J$ is addition sub- group of R

Let $r \in R$ and $a \in I \cap J$
$\Rightarrow r \in R$ $a \in I$ and $r \in R$ $a \in J$
$\Rightarrow ra, ar \in I$ and $ra, ar \in J$
$\Rightarrow ra, ar \in I \cap J$
$\therefore I \cap J$ is an ideal of R
$\therefore$ Hence it is proved.

Conclusion

In this article we discussed **ring theory**, a commutative ring Ideal . An ideal can be used to construct a quotient ring in a **group theory**, a normal subgroup can be used to construct a quotient group.

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Basic Concept of Vector Space and Linear Transformation

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ABSTRACT: *In this paper we will give defn about vector space and linear transformation and problems based on linear transformation and introduction about some special subspaces and give deep explanation about independent sets and basis and the theorem based on linear dependent and linear combination.*

Keywords: *Vector space, Scalar,Euclidean space, Subspace,Linear Transformation.*

1. Introduction

In this paper we introduce linear transformation system known as vector space & we will give definition some example of vector space & briefly explain about linear transformation. We introduce some special subspaces B,and definition and theorem based on I.

Vector Space

A vector space is a nonempty set V, whose objects are called vectors, equipped with two operations, called addition and scalar multiplication. For any two vector u,v in V and a scalar c, there are unique vector u+v and cu in V such that the following properties are satisfied.

1) u+v = v+u
2) (u+v)+w = u+(v+w)
3) u+ 0 = u

Ex:

The Euclidean space R^m is a vector space under the ordinary addition and Scalar multiplication.

Introduction Linear Transformation

Functions Occur in almost every application of mathematics. In this section we shall give a brief introduction from a geometric point of view to certain functions mapping R^n into R^m . Since we wish this special class of function called Linear transformation.

Defn for Linear Trnsformation

Let V and W be vector spaces over the field F. A linear transformation from
V into W such that T(cα+β)=c(Tα)+Tβ for all α and β in V and all scalars c in F.

Ex:

$L:R^3 \to R^2$ be a linear transformation for which we know that L(1,0,0)=(3,-2),L(0,1,0)=(2,2),L(0,0,1)=(-1,3) find L(8,6,3)

Soln:

Given L(1,0,0)=(3,-2),L(0,1,0)=(2,2),L(0,0,1)=(-1,3)
$(8,6,3)=8\bar{i}+6\bar{j}+3\bar{k}$
L(8,6,3)=8(3,-2)+6(2,2)+3(-1,3)
= (24,-16)+(12,12)+(-3,9)
L(8,6,3)= (33,5)

Some Special Subspace:

Let B an m×n matrix .The null space of B,denoted by Nul B , is the space of solutions of the linear system Bx = 0.that is
Nul B=$\{x \in R^n; Bx=0\}$

Let $T:R^n \rightarrow R^m$, $T(x)=Bx$, be a linear transformation. Then Nul B is the set of inverse image of 0 under T and col B is the image of T, that is ,
Nul B $=T^{-1}(0)$ and col $B=T(R^n)$

Independent Set and Bases
Vectors $u_1,u_2,\ldots\ldots u_n$ of a vector space V are called linearly independent if, whenever there are constants $c_1,c_2,\ldots.c_n$ such that
$c_1u_1+c_2u_2+\ldots\ldots+c_nu_n=0$

We have,
$C1=C2=\ldots\ldots=Cn=0$
The vectors $u_1,u_2,\ldots u_n$ are called linearly independent if there exist constant $c_1,c_2,\ldots,c_n$ not all zero such that $c_1u_1+c_2u_2+\ldots..+c_nu_n=0$

Any family of vectors that contains the zero vector 0 is linearly dependent .A single vector u is linearly independent iff u not equal to zero.

Theorem:
Vectors $V_1,V_2,\ldots.,V_r$ $(r>=2)$are linearly dependent iff one of the vectors is a linear combination of the others that is there is one i such that
$V_i=a_1v_1+\ldots..+a_{i-1}v_{i-1}+a_{i+1}v_{i+1}+\ldots+a_rv_r.$

Proof:
Since the vectors $V_1,V_2,\ldots..,V_r$ are linearly dependent there are constants $c_1,c_2,\ldots.,c_r$ not all zero such that
$C_1V_1+C_2V_2+\ldots.+C_rV_r=0$
Let C_i not equal to zero .then,
$V_i= (-c_1/c_i)v_1+\ldots\ldots\ldots+(c_r/c_i)v_r$

Dimensions of Vector Space
A vevtor space V is said to be finite dimensional if it can be spanned by a set of finite number of vectors. The dimension of V denoted by dim V ,is the number of vectors of a basis of V . The dimension of the zero vector space {0}is zero.

Rank
The rank of a rectangular matrix B is the number pivot positions of B ,that is the dimension of the row space and the column space of B.For a linear transformation $R : V \rightarrow W$,the rank of R is the dimension of the subspace R(V).

Theorem
Let Y be an m×m invertible matrix .Then
dim row Y = dim col Y= rank Y =m dim nul Y=0

Proof:
The invertibility of Y implies that the number of pivot positions of Y is m,so rank Y=n and dim nul Y =0.

Matrices of Linear Operator
Let U be an n –dimensional vector space with a basis $B = \{u_1,u_2,\ldots\ldots,u_n\}$.
A linear transformation $T:U \rightarrow U$ is called a linear operator on U.The matrix
$A = \{ [T (u_1)]B,[T(u_2)]B,\ldots\ldots.,[T(u_n)]B \}$ is called the matrix of T relative to the basis B.

Conclusion
From this paper we conclude that $T:V \rightarrow W$ be a linear transformation from vector space V with basis $B=\{v_1,\ldots\ldots\ldots.,v_n\}$ .If Y be an m×m is invertible matrix then
dim nul Y=0.

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Observer Design For (Max,+) Linear Systems

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ABSTRACT: *This paper deals with the state estimation for max-plus linear systems. This estimation is carried out following the ideas of the observer method for classical linear systems. The system matrices are assumed to be known, and the observation of the input and of the output is used to compute the estimated state. The observer design is based on the Residuation theory which is suitable to deal mapping inversion in idempotent Semiring.*

Index Terms: *Discrete Event Dynamics Systems, Idempotent Semirings, Max-Plus Algebra, Residuation Theory,Timed Event Graphs, Dioid, Observer, State Estimation.*

1. Introduction

Many discrete event dynamic systems, such as transportation networks [21], [12], communication networks, manufacturing assembly lines [3], are subject to synchronization phenomena.

1.Timed event graphs (TEGs) are a subclass of timed Petri nets and are suitable tools to model these systems. A timed event graph is a timed Petri net of which all places have exactly one

Up stream transition and one downstream transition. Its description can be transformed into a (max, +) or a (min, +) linear model and vice versa [5], [1]. This property has advantaged the emergence of a specific control theory for these systems, and several control strategies have been proposed, e.g., optimal open loop control [4], [20], [16], [19], and optimal feed back control.

2 .The (max, +) algebra is a particular idempotent semiring, therefore section II reviews some

algebraic tools concerning these algebraic structures. Some results about the residuation theory and its applications over semiring are also given. Section III recalls the description of timed event graphs in a semiring of formal series. Section IV presents and develops the proposed observer. It is designed by analogy with the classical Luenberger [17] observer for linear systems. It is done under the assumption that the system behavior is (max, +)-linear. This assumption means the model represents the fastest system behavior, in other words it implies that the system is unable to be accelerated, and consequently the disturbances can only reduce the system performances i.e., they can only delay the events occurrence. They can be seen as machine breakdown in a manufacturing system, or delay due to an unexpected crowd of people in a transport network.

In the opposite, the disturbances which increase system performances, i.e., which anticipate

the events occurrence, could give an upper estimation of the state, in this sense the results obtained are not equivalent to the observer for the classical linear systems. Consequently, it is assumed that the model and the initial state correspond to the fastest behavior (e.g. ideal behavior of the manufacturing system without extra delays or ideal behavior of the transport network without traffic holdup and with the maximal speed) and that disturbances only delay the occurrence of events. Under these assumptions a sufficient condition allowing to ensure equality between the state and the estimated state is given in proposition 4 in spite of possible disturbances, and proposition 3 yields some weaker sufficient conditions allowing to ensure equality between the asymptotic slopes of the state and the one of the estimated state, that means the error between both is always bounded.

II. Algebraic Setting

An idempotent semiring Sis an algebraic structure with two internal operations denoted by $\oplus$ and $\otimes$. The operation $\oplus$is associative, commutative and idempotent, that is, $a\oplus a=a$. The operation $\otimes$is associative (but not necessarily commutative) and distributive on the left and on the right with respect to $\oplus$. The neutral elements of $\oplus$and $\otimes$are represented by ε and e respectively, and ε is an absorbing element for the law $\otimes$($\forall a\in S$, $\varepsilon \otimes a=a\otimes\varepsilon=\varepsilon$).As in classical algebra, the operator $\otimes$will be often omitted in the equations, moreover, $a^i=a\otimes a^{i-1}$and $a^0=e$. In this algebraic structure, a partial order relation is defined by $a\preceq b\Leftrightarrow a=a\oplus b\Leftrightarrow b=a\wedge b$ (where the greatest lower bound of b), therefore an idempotent semiring S is a

partially ordered set (see [1], [12] for an exhaustive introduction). An idempotent semiring S is said to be complete if it is closed for infinite ⊕-sums and if ⊗distributes over infinite ⊕-sums.
In particular >= Lx nu the greatest element of S(>is called the top element of S).

Example 1:
(Z max): Set Z max =Z∪ {−∞,+∞} endowed with the max operator as sum and the classical sum +as product is a complete idempotent semiring, usually denoted Zmax, of which ε=−∞ and e= 0.

Theorem 1:
(see [1], th. 4.75): The implicit inequality x≥ax ⊕b as well as the equation x=a*x ⊕b defined over S, admit x=a∗b as the least solution, where a∗=L i∈N ai(Kleene star`o) i∈N.

Theorem 2:
(see [2],[1]): Let f: (D,≤)→(C,≤)be an order preserving mapping. The following statements are equivalent (i) f is residuated.
(ii) there exists an unique order preserving mapping f]:C → D such that f◦ f] f]and f≥IdD.

Definition 2
(Restricted mapping);
Let f:D → C be a mapping and B ⊆ D. We will denote by f|B :B → C the mapping defined by f|B =f◦ Id|B where Id|B :B → D, x 7→ xis the canonical injection. Identically, let E ⊆ C be a set such that Imf⊆ E. Mapping E| f:D → E is defined by f=Id|E ◦ E| f, where Id|E :E → C, x 7→ x.

Proposition 1 (see [6]): Let f:D → D be a closure mapping. Then, Imf|fis a residuated mapping whose residual is the canonical injection Id|Imf.

Remark 1: According to equation (4), (a∗)+=a∗, therefore ImK⊂ImP.

Definition 4 (Reducible and irreducible matrices): Let Abe a n×n matrix with entries in a semiring S. Matrix A is said reducible, if and only if for some permutation matrix P, the matrix PTAP is block upper triangular. If matrix A is not reducible, it is said to be irreducible.
III. TEG DESCRIPTION IN IDEMPOTENT SEMIRING

Timed event graphs constitute a subclass of timed Petri nets i.e. those whose places have one and only one upstream and downstream transition. A timed event graph (TEG) description can be transformed into a (max, +) or a (min, +) linear model and vice versa. To obtain an algebraic
model in Z max, a "dater" function is associated to each transition. For transition labelled xi,xi(k) represents the date of the kth firing (see [1],[12]). A trajectory of a TEG transition is then a firing date sequence of this transition. This collection of dates can be represented by a formal series x(γ) = Lk∈Zxi(k) γkwhere xi(k)∈Zmax and γis a backward shift operator1in the event domain (formally γx(k) = x(k−1)). x1,x2and x3are related as follows over Zmax:x1(k)=4⊗x1(k−1) ⊕1⊗x2(k)⊕6⊗x3(k).

Their respective γ-transforms, expressed over Zmax[[γ]], are then related as:
x1(γ) = 4γx1(γ)⊕1x2(γ)⊕6x3(γ).

In this paper TEGs are modelled in this setting, by the following model :
x=Ax ⊕Bu ⊕Rw y=Cx, (13) where u∈(Zmax[[γ]])p,y∈(Zmax [[γ]])m and x∈(Zmax[[γ]])n are respectively the controllable in-put, output and state vector, i.e., each of their entries is a trajectory which represents the collection of firing dates of the corresponding transition. Matrices A∈(Zmax[[γ]])n×n,B∈(Zmax [[γ]])n×p, C∈(Zmax[[γ m×n represent the links lbetween each transition, and then describe the structure of the graph. Vector w∈(Zmax [[γ]]) represents uncontrollable inputs (i.e. disturbances2). Each entry of w corresponds to a transition which disables the firing of internal transition of the graph, and then decreases the performance of the system. This vector is bound to the graph through matrix R∈(Zmax[[γ]])n×l.
Afterwards, each input transition ui(respectively wi) is assumed to be connected to one and only one internal transition xj, this means that each column of matrix B(resp. R) has one entry

equal to e and the others equal to ε and at most one entry equal to eon each row. Furthermore, each output transition yi is assumed to be linked to one and only one internal transition xj ,i.e each row of matrix Chas one entry equal to e and the others equal to ε and at most one entry equal to eon each column. These requirements are satisfied without loss of generality, since it is sufficient to add extra input and output transition. Note that if R is equal to the identity matrix, w can represent initial state of the system x(0) by considering w=x(0)γ0⊕... (see [1], p. 245, for a discussion about compatible initial conditions). By considering theorem 1, this system can be rewritten as :

$x=A*Bu \oplus A*Rw$ (14)

$y=CA*Bu \oplus CA*Rw$, (15)

Lemma 1: The greatest matrix Lsuch that $(A \oplus LC)*B=A*B$is given by:

L1= (A∗B)◦

/(CA∗B).(27)

Proof: First let us note that $L=\varepsilon \in Z^{n\times m}_{max}$ is a solution, indeed $(A \oplus \varepsilon C)*B=A*B$.

V. Conclusion

- This paper5has proposed a methodology to design an observer for (max, +) linear systems.
- The observer matrix is obtained thanks to the residuation theory and is optimal in the sense that it is the greatest which achieves the objective. It allows to compute a state estimation lower than or equal to the real state and ensures that the estimated output is equal to the system output. As a perspective, this state estimation may be used in state feedback control strategies as proposed in [6], [19], and an application to fault detection for manufacturing systems may be envisaged. Furthermore, in order to deal with uncertain systems an extension can be envisaged by considering interval analysis as it is done in [15],[11] and more recently in [8]. The authors are grateful to V.

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Matching in Bipartite Graphs

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ABSTRACT: *We consider the concepts of independent set of points, independent set of lines, independence number α and independence number α′. The set of matchings of matching in G. Which graphs are matching graphs of some graph is not known in general. We determine several forbidden induced sub graphs of matching graphs and add even cycles to the list of known matchings.*

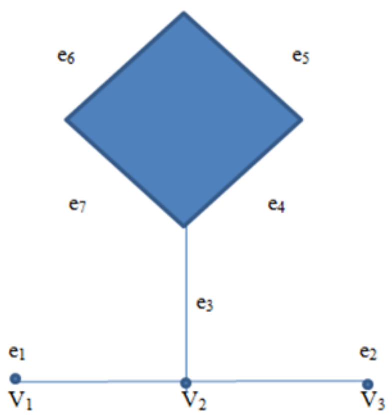
Keywords: *Vertex, edge, bipertite*

1. Introduction

This paper assumes basic knowledge of definitions and concepts as they pertain to graph theory. With that in mind, let's begin with the main topic of these notes matching. Later we will look at matching in bipartite graphs then Hall's Marriage Theorem.

Definition:

A matching of graph G is a sub graph of G such that every edge shares no vertex with any other edge. That is, each vertex in matching M has degree one.



Bipartite graph in matching

A Bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets and such that every edge connects a vertex in-to-one-in

Hall's Theorem

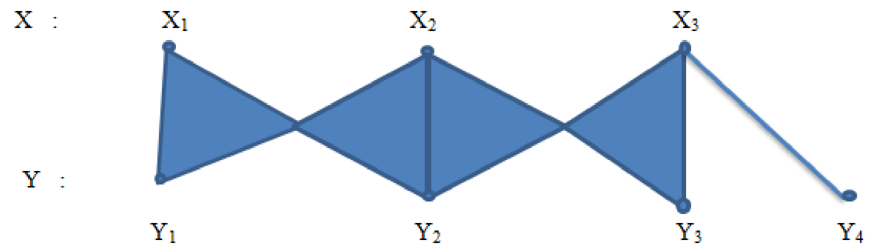
Statement:

Let G be a bipartite graph with bipartition (x, y) then prove that G contains a matching that saturated every vertex in x if |N(S)| ≥ |S| for every S contained or equal to x.

Proof

Suppose that G has matching M which saturated every vertex in X.

Let S be a subset of x. Now, the vertices in x are matched under M with distinct vertices in N(S). X



To Prove:
❖|N(S)| ≥ |S| for every S contained or equal to X

Conversely,
Suppose that G is a bipartite such that |N(S)| ≥ |S|
for every S contained or equal to X (1)
We have to prove that for every a matching which saturated every vertex in x. Let M* be a maximum matching in G.
By our choice M* does not saturated all vertices in X. Let u be an M*-unsaturated vertex in X.
Let Z be denote the set of all vertices connected to u by M* alternative paths.
❖ $Z = \{ u_1,v_2,u_2,v_1,u_1,u_4,v_4,v_3,u_3 \}$

But M* is a maximum matching,
❖ u is the only M* unsaturated vertex in Z.

Put S = Z∩X and T= Z∩Y
$S = \{u, u_1, u_2, u_3, u_4\}$
$T = \{v_1, v_2, v_3, v_4\}$

Clearly, the vertices in S-{u} are matched under M with the vertices in T.
❖ |T| = |S| - 1 (2)
But we have N(S) = T (3)
|N(S)| = |S| - 1
|N(S)|<|S| is a contradiction.
Hence G contains a matching saturating all vertex in X.

Perfect Matching
A perfect matching is a matching in which each node has exactly one edge incident on it. One possible way of finding out if a given bipartite graph has a perfect matching is to use the above algorithm to find the maximum matching an checking if the size of the matching equals the number of nodes in each partition.

If G is a K- regular bipartite graph n with K>0 then, Prove that G has a perfect matching.
Proof
Let G be a K—regular bipartite with bipartition (X,Y)
❖K|X|=|E|=K|Y|
K|X|=K|Y|
|x|=|Y|
Now, let s be a subset of X denoted by E_1 and E_2.The sets of edges incident with the vertices in S and n(s).
By definition of N(S), we have
E_1cE_2 and
$K|N(S)|=|E_2|\geq|E_1|=K|S|$
|N(S)|≥|S|

By Halls Theorem,
G has a matching saturating all vertex in X.

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|X|=|Y|
G has a perfect matching.
Hence proved.

M-alternating Path
Let M be a matching in G. An M-alternating path in G,whose edges are alternating in M.

For what values of n does the complete graph k_n have a perfect matching.
Solution
Clearly any graph with p odd has no perfect matching. Also the complete graph k_n has a perfect matching if n is even.

For example.
If $V(k_n)$ =1,2,……n. Then 12,34,….(n-1)n is a perfect matching of k_n.
Thus, k_n has a perfect matching if and only if n is even.

Show that a tree as atmost one perfect matching.
Proof
Suppose that a tree T has tree distinct perfect matching M_1 and $M_{2.}$
∴T has perfect matching |V| |is even say 2n.
∴Each vertex of H is saturated by an edge of M_1 and M_2.
But these 2 edges are different and each vertex of H of degree 2 only.
∴Components of H are cycles, a contradiction in a tree.
∴A tree has atmost one perfect matching.
Hence proved.

Conclusion
In this paper we conclude that the types of matching and also along with persons related to respective topic. We hope this paper will help of researches to look the field of graphs.

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Equivalent Solutions to Some Integral Calculus Problems

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ABSTRACT: *This paper approach to determine the equivalent solution to some Integral Calculus problems.The students are grouped and are subjected to the solve same Integral calculus problem.They came up with various solution to the given same Integral calculus problem.Each of the solutions obtained in each group was evaluated on specified integral to determain the numeric value in order to draw an equivalent solution. The Matlab software was used to determine if the solutions from each group are equivalent or differs from each other.*

Keywords: *Equivalent solution, integral calculus, numerical value, integration by parts, substitution.*

1. Introduction

Mathematics application is usefull in every fact of life. In day to day use of Mathematics in science and technology is complex in every area of science, technology and business enterprise is incomplete without mathematics. The significance of Mathematics has made it indispensable in both Ordinary level and Advanced level of studies.

There are numerous branch of Mathematics like Calculus is one of the main sub-division. Calculus as a branch of Mathematics has very wide areas of applications in Physics, Sciences,astronomy, ,Engineering and so on.The major advance in Integration come in the 17^{th} century with discovery of fundamental theorem of calculus by Newons and Leibniz. The Fundamental Theorem of Calculus connects two branches of calculus: differential calculus and integral calculus. In early it was known as infinitesimal calculus. Integral calculus depends on two fundamentally important concepts, that of a continuous function and that of the derivative of a continuous function .Today's valuable tool in mainstream economics is integral calculus.

This study is restricted to integral calculus focusing on determaining the equivalent set of solutions to a given integrable function over the specified intervals. We apply Matlab to enhance the teaching and learning process of the integration after the group presentation of the solutions by the students on integration problems. Several Mathematical software like Mathematica, Matlab, GeoGebra, Maple, etc that enhances the teaching and learning process inside and outside the classroom are available.

The focus of this paper is to present simplyfied approach to determine the equivalent solution set or difference in the integral of a given function. The two ways presented to determine equivalent solutions are:

1) Numerical evaluation of the integral solutions over the specified interval of integration after the application of various techniques and methods of integration.

2) Graphical representation of solution set .All the graphical representation of the equivalent solutions set presented here were plotted using matlab .

2) Techniques of Integration of Functions:

Many researchers have contributed to the learning of methods of integration and applications in real life situations. Some of the methods of integration are substitution method which concentrated on change of variables, integration by parts method of integrating product of functions, resolving into Power series approach, resolving into partial fraction method, numerical approximations of integrals such as: Simpsons rule, Vegas method, Trapezoidal rule, Riemann sum, Cuhre method, Suave method, Divonne method etc.

For the better understanding of various integration techniques of the function's.

The below two conditions are necessary condition ,Integral calculus depends on two fundamental important concepts

1. Continuous function.
2. Derivative of a continuous function.

∴ If a function is continuous on (a , b) then it is integrable on interval (a , b).

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For example : A function h= $|g(t)dt|$ is integrable for all values of t and not differential at t = 0.

3) Definition:

Indefinite integral

Indefinite integral of a function ƒ is differentiable function F whose derivative is equal to original function ƒ.In calculus an antidervative ,primitive function, primitive integral or indefinite integral this can de stated symbolically as F΄=ƒ.

Definite integral

Definite integral is defined informally as the signed area of the region in the xy-plane that is bounded by the graph of ƒ,the x-axis and the vertical lines x=a and x=b symbolically as

$F(x) = \int_a^b f(x)dx$

4) Theorems

Fundamental theorem of calculus [part -1]

If a function is continuous on the interval [a,b] then the function g defined by

$g(x) = \int_a^x f(t)\,dt \quad a \le x \le b$

is continuous on (a,b) and differentiable on (a,b) and g΄(x)=ƒ(x).

Proof

we know that $g'(x) = \lim_{h\to 0} \frac{g(x+h)+g(x)}{h}$

First we will focus on putting the quotient on the right hand side into a form for which we can calculate the limit. Using the definition of the function g(x), we get

$$\frac{g(x+h)-g(x)}{h} = \frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)\,dt}{h}$$

$$= \frac{1}{h}\int_x^{x+h} f(t)\,dt$$

If ƒ(x) > 0 the integral $\int_x^{x+h} f(t)\,dt$ is that area between the curve y = ƒ(t) and the t-axis, over the interval from t = x to t = x + h.

Since ƒ is continuous on the interval [x; x + h], we can use the Extreme Value Theorem to show that ƒ achieves a maximum, M and a minimum m, on that interval.That is, for all values of t in the interval [x,x + h], m ≤ ƒ(t) ≤M

and by the laws of definite integrals, we have

$$m(x + h - x) \le \int_x^{x+h} f(t)\,dt \le M(x + h - x)$$

Dividing across by ' h ' we get

$$m \le \frac{1}{h}\int_x^{x+h} f(t)\,dt \le M$$

The minimum and maximum are not necessarily at the endpoints of the interval they may be some where in the interior.

However the Extreme Value Theorem (which applies because the function is continuous) guarantees that there is a number c1 in the interval with ƒ(c1) = m ≤ƒ(t) for all t ∈ [x; x + h] and there is a number c2 ∈[x; x + h] for which ƒ(c2) = M ≥ ƒ(t) for all t ∈ [x; x + h]. So this gives us

$$f(c1) \le \frac{1}{h}\int_x^{x+h} f(t)\,dt \le f(c2)$$

where c1,c2 $\in$ [x , x + h]. Now taking limits we get

$\lim_{h\to 0} f(c1) \le \lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)dt \le \lim_{h\to 0} f(c2)$

As h$\to$ 0, c1 $\to$ x and c2 $\to$ x because the width of the interval is going to 0.
Because ƒ(t) is continuous
$\lim_{h\to 0} f(c1)$ ≤ ƒ(x) ≤ $\lim_{h\to 0} f(c2)$
And ƒ(x) $\le \lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)\,dt$ ≤ ƒ(x)
$\lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)\,dt$ = ƒ(x)

This proves that
g′(x) = $\lim_{h\to 0} \frac{g(x+h)+g(x)}{h}$
= $\lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)\,dt$

ǵ(x)= ƒ(x)
Hence the proof

Fundamental theorem of calculus [part -2]

If ƒ is continuous on [a,b] then $\int_a^b f(x)dx$ = F(b) – F(a) where F is any antiderivative of ƒ that is a function ƒ such that F́′ = ƒ.

Proof

Let g(x) = $\int_a^x f(t)dt$
we know that g(x) is an antiderivative of ƒ. Hence if F(x) is another antiderivative for ƒ, we have
F(x) = g(x) + C
for some constant C and a < x < b . Since F and g are continuous function , we take limits that
F(a) = g(a) + C and F(b) = g(b) + C.
Now g(a) = $\int_a^a f(t)\,dt$ and g(b) = $\int_a^b f(t)\,dt$

Therefore
F(b) - F(a) = (g(b) + C) - (g(a) + C)
= g(b) - g(a)

F(b) - F(a) = $\int_a^b f(t)\,dt$
This makes the calculation of integrals much easier for the function for which we can find an antiderivative.
Hence the proof

Theorem :1

Let ƒ(x) be a function bounded in interval [a,b] then a necessary and sufficient condition for the existence of the $\int_a^b f(x)$dx is that the set of discontinuous of ƒ(x) have measure Zero.

Theorem: 2

If ƒ is a continuous function on [a , b] then the function g defined by g(x) = $\int_b^x f(t)dt$,
a $\le x \le b$ is continuous on [a , b] and differentiable on [a , b] and g′(x) = ƒ(x) [or] $\frac{d}{dx}\int_b^x f(t)dt$ = ƒ(x).

Theorem: 3

Any two antiderivative F and G of ƒ(x) differ at most by a constant, then is
F(x) – G(x) = C.

Theorem: 4
Let f(x) be an integral function over an interval [a,b] , there exist at least solution set $\{y_1, y_2\}$ which are equal in numerical value evaluated on [a,b] but differ in constant of integration only.

5) Some properties of definite integral
Let y(x) and h(x) be orbitrary integrable function in an interval [a,b] then the following rules hold

(i) **Zero :** $\int_a^a y(x)dx = 0$

(ii) **Order of integration :** $\int_a^b y(x)dx = -\int_b^a y(x)dx$

(iii) Constant multiples : $\int_a^b \pm C\,[y(x)]dx = \pm C \int_a^b y(x)dx$ Where C is constant .

(iv) Sum and Difference: $\int_a^b [y(x) \pm h(x)]dx = \int_a^b y(x)dx \pm \int_a^b h(x)dx$

(v) Additivity : $\int_a^b y(x)dx + \int_a^c y(x)dx = \int_a^c y(x)dx$

(vi) Two parts:
- $\int_0^{2a} y(x)\,dx = \int_0^a y(x)dx + \int_0^a y(2a - x)dx$ if y(2a - x) = y(x)
- $\int_0^{2a} y(x)\,dx = 0$ if y(2a - x) = - y(x)

(vii) Two parts:
- $\int_0^{2a} y(x)\,dx = 2\int_0^a y(x)dx$ if y(- x) = y(x) (Even function)
- $\int_0^{2a} y(x)\,dx = 0$ if y(- x) = -y(x) (Odd function)

(viii) Maximum(M) and Minimum(m) inequality:
Consider an interval $a \le x \le b$, $m \le x \le M$ then

$M(b-a) \ge \int_a^b y(x)dx \ge m(b-a)$

(ix) Domination
Consider an interval $a \le x \le b$, $y(x) \le h(x)$ then

$\left|\int_a^b y(x)dx\right| \le \left|\int_a^b h(x)dx\right|$

(x) $\int_a^b y(x)dx = \int_a^b y(a+b-x)dx$

(xi) $\int_0^b y(x)dx = \int_0^a y(a - x)dx \int_a^b y(x)dx$

6) Some example of result
Example (1): Use any integration technique to determain the solution of $\int \sin x \cos x\, dx$?
Solution (1): Use integration by parts method
Let us use integration by parts method $\int udv$ **= uv -** $\int vdu \rightarrow (1)$
Let u = $\sin x$ then du = $\cos x dx$ and dv= $\cos x dx$ then v= $\sin x$
Substitute in equation (1) ,we have
$\int \sin x \cos x dx = \sin x \;.\sin x - \int \sin x \cos x\, dx$
$=(\sin x)^2 - \int \sin x \cos x\, dx$

This implies that $2\int \sin x \cos x\, dx = (\sin x)^2$
Hence $\int \sin x \cos x dx = \frac{1}{2}(\sin x)^2 + k$
Where k is the constant of integration .

Solution(2) : Use substitution method
Let u = $\cos x$. Then du = - $\sin x\, dx$
$dx = -\frac{du}{\sin x}$
Then $\int \sin x \cos x\, dx = \int \sin x\,.\,u.(-\frac{du}{\sin x})$
$= -\int udu$

$= -\frac{u^2}{2} + k$

$\int \sin x \cos x \, dx = -\frac{(\cos x)^2}{2} + k$

Solution(3) : Use substitution method
Use different choice of function
Let $u = \sin x$. Then $du = - \cos x \, dx$

$dx = -\frac{du}{\cos x}$

Then $\int \sin x \cos x \, dx = \int \cos x \, . \, u . (-\frac{du}{\cos x})$

$= - \int u du$

$= -\frac{u^2}{2} + k$

$\int \sin x \cos x \, dx = -\frac{(\sin x)^2}{2} + k$

Hence Solution(3) coincides with Solution(1)

Solution(4): Use Trigonometric identity
Sine double angle formula $\sin 2x = \sin(x + x) = 2 \sin x \cos x$

Then $\sin x \cos x = \frac{1}{2} \sin 2x$

Therefore $\int \sin x \cos x \, dx = \int \frac{1}{2} \sin 2x \, dx$

$= \frac{1}{2} \int \sin 2x \, dx$

$= \frac{1}{2} (-\frac{\cos 2x}{2}) + k$

$\int \sin x \cos x \, dx = -\frac{1}{4} \cos 2x + k.$

The solutions of $\int \sin x \cos x dx$ are :

$y_1 = \frac{1}{2} (\sin x)^2 + k$

$y_2 = -\frac{(\cos x)^2}{2} + k$

$y_3 = -\frac{1}{4} \cos 2x + k$

The following two steps were applied.

Step 1: We evaluated each solution set on the same interval of integration is we changed the problem to definite integral form and evaluate each of the solution to obtain numerical value.

Step 2: We graphed the solution set to the integration problem on the same interval to determine the behavior of the solution.

Assuming the solution exists on the interval $x \in \left[\frac{\pi}{4} , \frac{\pi}{2}\right]$

The solution(1) :

$\Rightarrow y_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x \cos x \, dx = \left[\frac{1}{2} (\sin x)^2\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \frac{1}{2} \left[(\sin \frac{\pi}{2})^2\right] - \left[(\sin \frac{\pi}{4})^2\right]$

$= \frac{1}{2} [(\sin 90)^2 - (\sin 45)^2]$

$= \frac{1}{2} [1 - \frac{1}{2}]$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x \cos x \, dx = \frac{1}{4}$

Similarly the solution(2) :

$y_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x \cos x \, dx$

$= \left[\frac{1}{2}(\cos x)^2\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \frac{1}{2} \left\{ \left[\left(\cos\frac{\pi}{2}\right)^2\right] - \left[\left(\cos\frac{\pi}{4}\right)^2\right] \right\}$

$= \frac{1}{2} \left[\frac{1}{2} \right]$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x \cos x \, dx = \frac{1}{4}$

Similarly the solution(3) :

$y_3 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x \cos x \, dx$

$= \left[-\frac{1}{4}(\cos 2x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= -\frac{1}{4}[\cos 180 - \cos 90]$

$= -\frac{1}{4}(-1-0)$

$= \frac{1}{4}$

The graphical representation of the solution on various intervals shows that the solution set { y_1, y_2, y_3 } differs in constant of integration but are equivalent to each other this conforms the fact that the integral of the function's is not unique.

[Using theorem :4]From the numerical result of the three solution set { y_1, y_2, y_3 } of the integral problem given as $\int \sin x \cos x \, dx$, $y_1 = y_2 = y_3 = \frac{1}{4}$ we see that without loss of generality showed clearly the existence of an equivalent solutions to an integral problem .This shows the evidence of parity in the solution set to the integral problem.

Example (2):

Use any integration technique of your choice to determain the solution of $\int -\frac{1}{2x}$ dx?

Solution(1):

$\int -\frac{1}{2x}$ dx = $-\frac{1}{2}\int \frac{1}{x}$dx

$= -\frac{1}{2}\log x$ + k

Where k is constant

Solution(2):

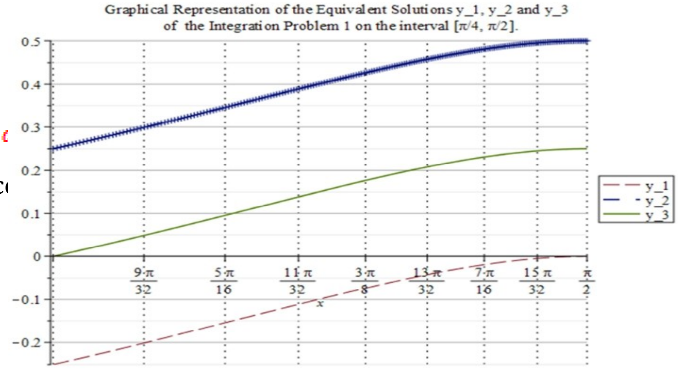
By following the integration rule $\int \frac{1}{ax+b}$

$\int -\frac{1}{2x}$ dx = $-\frac{1}{2}\log(-2x)$ +k [Where k is c

$=-\frac{1}{2}\log(-2) - \frac{1}{2}\log(x)$ +k

$= -\log\frac{1}{\sqrt{2}} - \frac{1}{2}\log(x)$ +k

$=-\frac{1}{2}\log(x)$ + c [where c= $-\log\frac{1}{\sqrt{2}}$ +k]



Like above example this solution (1) and (2) are differ in constant term only .Therefore the solution is equivalent solution.

Example (3): Use any integration technique of your choice to determain the solution of $\int \frac{e^x}{[e^x+e^{2x}]}$ dx?

Solution(1):Use substitution method

Let u=e^x and then du=$e^x\,dx$

$\int \frac{e^x dx}{[e^x+e^{2x}]} = \int \frac{du}{u+u^2}$

Use partial fraction method

$\frac{1}{u+u^2} = \frac{A}{u} + \frac{B}{1+u^2}$

By solving we get A=1 and B=-1

$\int \frac{du}{u+u^2} = \int \frac{du}{u} + \int \frac{-du}{1+u}$

$= \log u - \log(1+u) + k$

$= \log \frac{u}{(1+u)} + k$

$= \log \frac{e^x}{[1+e^x]} + k$

Solution(2): Multiply and divide by e^{2x}

$\frac{e^x}{[e^x+e^{2x}]} \times \frac{e^{2x}}{e^{2x}} = \frac{e^{-x}}{(1+e^{-x})}$

$\int \frac{e^x dx}{[e^x+e^{2x}]} = \int \frac{e^{-x}}{(1+e^{-x})} dx$

Use substitution method

u = $1+e^{-x}$ and then du= $-e^{-x}dx$

$\int \frac{-du}{u} = -\log u + k$

$= -\log(1+e^{-x}) + k$

The above two solutions are not same but they posses same value in each interval.

7) Conclution

In this paper, we evaluated the solution of definite integral of the given problem over a specific interval of integral problem solved in different method then we obtain different solution and same numeric value . And we have proved that existence of an equivalent solution to the continuous integral problem at some specific interval we obtain numeric value as solution .

Thus the integral calculus problem has equivalent solution which differ in contant of integration and the solutions are not unique.

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An Extension of Fermat's Last Theorem in Nine Dimensional Euclidean Space

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ABSTRACT: *The Fermat's last theorem states that there is no integer triple (a,b,c) such that $a^n + b^n = c^n$ for n>2 and our attempt is an extension of Fermat's last theorem in nine dimensional Euclidean space show that $a^n + b^n + c^n + d^n + e^n + f^n + h^n + h^n + i^n = j^n$ is true for n = 2.*

Keywords: *Integer nonuple, Integer decuple, Fermat's last theorem, unit sphere, Euclidean space.*

1. Introduction

Pierre- Fermat (1601-1665) wrote a comment by the side while reading Dedekind book of Pythagoras triple that there is no integer triple (x,y,z) for which $x^2 + y^2 = z^2$ for n>2. This is called Fermat's last theorem.
Now, in this paper an attempt is made to 9th dimension of Fermat's last theorem on integer triple into integer decuple.
There exists an natural number 'n' for which $a^n + b^n + c^n + d^n + e^n + f^n + h^n + h^n + i^n = j^n$
We are able to show that is true for n =2, where n is a natural number.

Euclidean Space

In geometry, Euclidean space encompasses the two dim Euclidean plane, the three dim space of Euclidean geometry and higher dimensional.

Preliminary Result

We present few results on integer Decuple

Result 1:
If (a, b, c, d, e, f, g, h, i, j) is an integer Decuple then, multiples of any integer n with this integer decuple is again an integer decuple. (na, nb, nc, nd, ne, nf, ng, nh, ni, nj)

Proof :
$(na)^2+(nb)^2+(nc)^2+(nd)^2+(ne)^2+(nf)^2+(ng)^2+(nh)^2+(ni)^2$
$= n^2 a^2+ n^2 b^2+ n^2 c^2+ n^2 d^2+ n^2 e^2+ n^2 f^2+ n^2 g^2+ n^2 h^2+ n^2 i^2$
$= n^2 (a^2+ b^2+ c^2+ d^2+ e^2+ f^2+ g^2+ h^2+ i^2)$
$= n^2 (j^2)$ ($\therefore$ using 5)
$(na)^2 + (nb)^2 + (nc)^2 + (nd)^2 + (ne)^2 + (nf)^2 + (ng)^2 + (nh)^2 + (ni)^2 = n^2(j^2)$

Result 2:
For any integer Duple (a, b, c, d, e, f, g, h, i, j) if a, b, c, d, e, f, g, h, i are even. Then j must be also even.

Proof
Let a= 2p, b = 2q, c = 2r, d= 2s, e = 2t, f = 2u, g = 2v, h = 2w, I = 2x.
$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2$
$= (2p)^2 + (2q)^2 + (2r)^2 + (2s)^2 + (2t)^2 + (2u)^2 + (2v)^2 + (2w)^2 + (2x)^2$
$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2$
$= 2(2p^2 + 2q^2 + 2r^2 + 2s^2 + 2t^2 + 2x^2 + 2v^2 + 2w^2 + 2x^2)$
Which is an even number.
Thus we get *j* must be even.
Hence the proof.

Result 3 :
For any Decuple (a, b, c, d, e, f, g, h, i, j). if a, b, c, d, e, f, g, h even and i is odd then j. must be odd.

Proof
Let a = 2p, b= 2q, c = 2r, d = 2s, e = 2t, f =2u, g = 2v, h = 2w and i = 2x+1.
$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2$
$= 4p^2 + 4q^2 + 4r^2 + 4s^2 + 4t^2\ 4u^2 + 4v^2 + 4w^2 + 4x^2 + 4x+1$
j^2 = an odd number.
Thus J is an odd.
Hence the proof.

Main Result
Integer Decuple and the nine dimensional Euclidean space.
Now we are going to relate
(a, b, c, d, e, f, g, h, i, j) to points on a sphere a rational solution is obtain for the equation.
$x^2+y^2+z^2+r^2+s^2+t^2+u^2+v^2+w^2 = 1$.

From which we obtain a general solution for integer
(a, b, c, d, e, f, g, h, i, j) for which
$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = j^2$
Consider *a, b, c, d, e, f, g, h, i, j* be integer, for which
$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = j^2 \rightarrow (2)$
Divide by j^2 on both sides in equation (2).

We get
$$\left(\frac{a}{j}\right)^2 + \left(\frac{b}{j}\right)^2 + \left(\frac{c}{j}\right)^2 + \left(\frac{d}{j}\right)^2 + \left(\frac{e}{j}\right)^2 + \left(\frac{f}{j}\right)^2 + \left(\frac{g}{j}\right)^2 + \left(\frac{h}{j}\right)^2 + \left(\frac{i}{j}\right)^2 = 1$$
So that the, $\left(\frac{a}{j}, \frac{b}{j}, \frac{c}{j}, \frac{d}{j}, \frac{e}{j}, \frac{f}{j}, \frac{g}{j}, \frac{h}{j}, \frac{i}{j}\right)$ is a solution of the equation (1)
It is well known that the equation.
$x^2 + y^2 + z^2 + r^2 + s^2 + t^2 + u^2 + v^2 + w^2 = 1$ is that of a sphere S of radius 1 centre at (0,0,0,0,0,0,0,0,0).

We shall we geometry of the sphere s to find all points on S where (x,y,z,r,s,t,u,v,w) co-ordinates are rational number.
Notice that it has Eighteen obvious points with co-ordinates,
(±1,0,0,0,0,0,0,0,0) , (0, ±1, 0,0,0,0,0,0,0), (0,0, ± 1,0,0,0,0,0,0)
(0,0,0, ±1,0,0,0,0,0), (0,0,0,0, ±1,0,0,0,0), (0,0,0,0,0, ±1,0,0,0)
(0,0, 0,0,0,0, ±1,0,0), (0,0,0,0,0,0,0, ±1,0), (0,0,0,0,0,0,0,0,±1)
Suppose we consider a vector b and the line L going through the point (-1, 0, 0, 0, 0, 0, 0, 0, 0) having b as its direction.
The line L is given by the vector equation.
L : r = - i + tb
Where, b= $b_1\bar{i} + b_2\bar{j} + b_3\vec{k} + b_4\bar{l} + b_5\vec{m} + b_6\vec{n} + b_7\vec{o} + b_8 p + b_9\vec{k}$
The Cartesian equation of L is given by,
$$\frac{x+1}{b_1} = \frac{y}{b_2} = \frac{z}{b_3} = \frac{r}{b_4} = \frac{s}{b_5} = \frac{t}{b_6} = \frac{u}{b_7} = \frac{v}{b_8} = \frac{w}{b_9}.$$

To find the intersection of S and L, we need to solve the equation.
$x^2+y^2+z^2+r^2+s^2+t^2+u^2+v^2+w^2 = 1$ and
$$\frac{x+1}{b_1} = \frac{y}{b_2} = \frac{z}{b_3} = \frac{r}{b_4} = \frac{s}{b_5} = \frac{t}{b_6} = \frac{u}{b_7} = \frac{v}{b_8} = \frac{w}{b_9} \rightarrow (3)$$

From Equation (3) comparing the terms and we get,

$$\left.\begin{array}{l} y = \frac{b_2(x+1)}{b_1}, z = \frac{b_3(x+1)}{b_1}, r = \frac{b_4(x+1)}{b_1}, s = \frac{b_5(x+1)}{b_1} \\ t = \frac{b_6(x+1)}{b_1}, u = \frac{b_7(x+1)}{b_1}, v = \frac{b_8(x+1)}{b_1}, w = \frac{b_9(x+1)}{b_1} \end{array}\right\} \rightarrow (4)$$

Equation (4) sub in 1 we get,

$$x^2 + \left(\frac{b_2(x+1)}{b_1}\right)^2 + \left(\frac{b_3(x+1)}{b_1}\right)^2 + \left(\frac{b_4(x+1)}{b_1}\right)^2 + \left(\frac{b_5(x+1)}{b_1}\right)^2$$
$$+ \left(\frac{b_6(x+1)}{b_1}\right)^2 + \left(\frac{b_7(x+1)}{b_1}\right)^2 + \left(\frac{b_8(x+1)}{b_1}\right)^2 + \left(\frac{b_9(x+1)}{b_1}\right)^2 = 1$$

$$\Rightarrow \left.\begin{array}{c} b_1^2x^2 + b_2^2(x+1)^2 + b_3^2(x+1)^2 + b_4^2(x+1)^2 + b_5^2(x+1)^2 + b_6^2(x+1)^2 \\ + b_7^2(x+1)^2 + b_8^2(x+1)^2 + b_9^2(x+1)^2 \end{array}\right\} = b_1^2$$

$$\left.\begin{array}{l} (b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)x^2 + \\ 2(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)x + \\ \quad b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2 \end{array}\right\} = b_1^2$$

$$(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)(x+1)\} = 2b_1^2$$

$$(x+1) = \frac{2b_1^2}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

$$x = \frac{b_1^2 - b_2^2 - b_3^2 - b_4^2 - b_5^2 - b_6^2 - b_7^2 - b_8^2 - b_9^2}{b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2}$$

Substituting this value of *x* in equation – 4 we get,

$$y = \frac{2b_1b_2}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

$$z = \frac{2b_1b_3}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

$$r = \frac{2b_1b_4}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

$$s = \frac{2b_1b_5}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

$$t = \frac{2b_1b_6}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

$$u = \frac{2b_1b_7}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

$$v = \frac{2b_1b_8}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

$$w = \frac{2b_1b_9}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}$$

Thus every point $(x, y, z, r, s, t, u, v, w)$ on the sphere $x^2 + y^2 + z^2 + r^2 + s^2 + t^2 + u^2 + w^2 = 1$ can be represented by,

$$\begin{pmatrix} \dfrac{b_1^2 - b_2^2 - b_3^2 - b_4^2 - b_5^2 - b_6^2 - b_7^2 - b_8^2 - b_9^2}{b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2}, \\ \dfrac{2b_1b_2}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}, \\ \dfrac{2b_1b_3}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}, \\ \dfrac{2b_1b_4}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}, \\ \dfrac{2b_1b_5}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}, \\ \dfrac{2b_1b_6}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}, \\ \dfrac{2b_1b_7}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}, \\ \dfrac{2b_1b_8}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)}, \\ \dfrac{2b_1b_9}{(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2 + b_8^2 + b_9^2)} \end{pmatrix}$$

This result gives way of obtaining $(a, b, c, d, e, f, g, h, i, j)$

Theorem

For any integer p, q, r, s, t, u, v, w, x with a = $p^2- q^2- r^2- s^2- t^2- u^2- v^2- w^2- x^2$, b =2pq, c=2pr, d=2ps, e = 2pt, f= 2pu, g = 2pv, h = 2pw, i= 2px then,

$j = p^2+ q^2+ r^2+ s^2+ t^2+ u^2+ v^2+ w^2+ x^2$ → 5

will satisfies the equation $a^2+ b^2+ c^2+ d^2+ e^2+ f^2+ g^2+ h^2+ i^2 = j^2$

Proof

Now,

$a^2+ b^2+ c^2+ d^2+ e^2+ f^2+ g^2+ h^2+ i^2$

$= (p^2 - q^2 - r^2 - s^2 - t^2 - u^2 - v^2 - w^2 - x^2)^2$

$+ (2pq)^2 + (2pr)^2 + (2ps)^2 + (2pt)^2 + (2pu)^2 + (2pv)^2 + (2pw)^2 + (2px)^2$

$= (p^2 - (q^2 + r^2 + s^2 + t^2 + u^2 + v^2 + w^2 + x^2))^2 + 4p^2q^2 + 4p^2r^2 + 4p^2s^2 + 4p^2t^2 +$
$4p^2u^2 + 4p^2v^2 + 4p^2w^2 + 4p^2x^2$

$= p^4 - 2(q^2 + r^2 + s^2 + t^2 + u^2 + v^2 + w^2 + x^2)p^2$

$+ (q^2 + r^2 + s^2 + t^2 + u^2 + v^2 + w^2 + x^2)^2 + 4p^2q^2 + 4p^2r^2 + 4p^2s^2 + 4p^2t^2 +$
$4p^2u^2 + 4p^2v^2 + 4p^2w^2 + 4p^2x^2$

$= p^4 + 2(q^2 + r^2 + s^2 + t^2 + u^2 + v^2 + w^2 + x^2)p^2$

$+ (q^2 + r^2 + s^2 + t^2 + u^2 + v^2 + w^2 + x^2)^2$

$= (p^2 + q^2 + r^2 + s^2 + t^2 + u^2 + v^2 + w^2 + x^2)^2$

$a^2+ b^2+ c^2+ d^2+ e^2+ f^2+ g^2+ h^2+ i^2 = j^2$ (∴ using 5)

$a^2+ b^2+ c^2+ d^2+ e^2+ f^2+ g^2+ h^2+ i^2 = j^2$

Hence the proof.

Conclusion

In this paper we extend the Fermet's last theorem and an attempt made to produce the result for $a^n + b^n+ c^n+ d^n+ e^n+ f^n+ h^n+ h^n+ i^n= j^n$ for n = 2, thus we call an extension of Fermat's last theorem in ninth dimensional Euclidean space.

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An Application on Laplace Transform and Its Properties

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ABSTRACT: *The aim of this paper is to discuss about laplace transform definitions based on examples, related properties and derivatives are explained below. They are explained in a easy manner. And also discuss about laplace based theorems.*

History of Laplace Transform

The laplace transform was first introduced by pierrelaplace in 1779 in his research on probability. G.Doetch develop the use of laplace transform to solve differential equation. His work in 1930s serve to justify the operational calculus earlier used by Oliver Heaviside. The importance of laplace transform lies in converting a differential equation into an algebraic equation. We first solve the simple algebraic equation and using the inverse Laplace transform technique the solution to the differential equation is easily obtained.

Introduction of Laplace

The laplace transform used to solve differential equation. The direct LAPLACE transform or the Laplace integral of a function f(t) (or) g(t). The f(t) (or) g(t) is called to be piecewise continuous on a finite interval. Then \`t\`s limit interval [0,∞).

Definition 1:

Let f(t) is a real valued function of the variable defined for t>0.
Then the laplace transform f(t) is denoted by L{f(t)} is defined by
$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$ where,

- L – Is Laplace transform
- F(t) – Is the real valued function,
- t – is the variable

L{f(t)} being clearly a function of \`S\`, can be written as $\bar{f}(s)$
Ie.,$L\{f(t)\} = \bar{f}(s)$

$$= \int_0^\infty e^{-st} f(t)\,dt$$

Note

- ✓ The "transform" $f(t) \to F(s)$ where t is integrated and s is variable.
- ✓ Conversely $F(s) \to f(t)$ where t is variable and s is integrated.

Formula Table

S. No	f (t)	$\bar{f}$(s) (or) L{f(t)}
1.	1	1/S , S>0
2.	e^{-at}	1/S+a , S>-a
3.	e^{at}	1/S-a , S>0
4.	T	1/s^2 , S>0
5.	t^n	n!/s^{n+1} , S>0
6.	$\sin bt$	b/s^2+b^2 , S>0
7.	$\cos at$	S/s^2+a^2 $, S > o$
8.	$\sinh bt$	b/s^2-b^2 , S> |a|

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9.	$\cosh at$	S/a^2-s^2
10.	$e^{at}\sin bt$	b/$(S-a)^2$+b^2
11.	$e^{bt}\cos at$	S-a/$(S-b)^2$+a^2

Example1:
prove that L[$\sin bt + e^{ct}] = \frac{b}{s^2+b^2} + \frac{1}{s+c}$

Proof:
Given that, LHS=L[$\sin bt + e^{ct}$]
We know that formula,
L{f(t)}=$\int_0^\infty f(t)\, e^{-st} dt$
LHS=L[$\sin bt] + L[e^{ct}]$
=$\int_0^\infty \sin bt e^{-st}\, dt + \int_0^\infty e^{-(s-c)t}\, dt$
=[$\frac{b}{s^2+b^2}(\sin bt + b\cos bt)]_0^\infty + [\frac{e^{-(s-c)t}}{-(s-c)}]_0^\infty$
= 0+0+$\frac{b}{s^2+b^2} + \frac{e^0}{s}$
=$\frac{b}{s^2+b^2} + \frac{1}{s}$
=RHS
LHS=RHS
Hence the Proof

EXAMPLE 1:
Find the Laplace transform of the following function
(I) t^3-4t+6 (ii)$\sinh 4t$

Proof:
(i) Given that, t^3-4t+6 → (1)
Taking laplace transform equation (1) we get
L{t^3-4t+6}=L[t^3]-L[4t]+L[6] → (2)
Using formula:
$L[t^n]$=n!/s^{n+1}

From equation (2)
=[3!/s^{3+1}]-4[1/s^2]+6[1/s]
=6/s^4-4/$s^2 + 6/s$
= $\frac{6+s^3-4s^2}{s^4}$

(ii) Given that, $\sinh 4t \rightarrow (1)$
Laplace transformeqn (1) we get,
$\text{Sinh } 4t = L\{\sinh 4t\} \rightarrow (2)$

Using formula;
$L[\sinh at] = a/s^2 - a^2$

From eqn (2)
$= \frac{4}{s^2-4^2}$
$= \frac{4}{s^2-16}$

Float Chart

$t^5 + \sin 2t - e^6$

↓ (Taking laplace trsnsform)

$L\{t^5 + \sin 2t - e^6\}$

↓ $(Seperate\ the\ equation)$

$L[t^5] + L[\sin 2t] - L[e^6]$
$\frac{5!}{s^6} + \frac{2}{s^2+4} - \frac{1}{s-6}$

↓ (Subtuting formula)

Theorem (First Shifting Rule):
Let g(t) be of exponential order and $-\infty < b < \infty$. Then
L{e^{bt} g(t)} = L(g(t))|w→(w−b) .

Proof
We Know that,
L{f(t)}=F(s)
L{f(t)}=$\int_0^\infty e^{-st} f(t)\, dt$
=F(s)

LHS=L{$e^{bt} g(t)$}
=$\int_0^n g(t) e^{bt} e^{-wt} dt$

We kown the exponential addition ,
$e^A e^B = e^{A+B}$

=$\int_0^n g(t)\, e^{-(w-b)t} dt$

Then $n \to \infty$ $we\ get,$
=$\int_0^\infty g(t)\, e^{-(w-b)t} dt$
=F(w-b)
Hence The Proof

Theorem (t-Derivative Rule):
If g(t) is continuous, lim t→∞ f(t)e^{-wt} = 0 for all large values of w and g'(t) is piecewise continuous, then L(g'(t)) exists for all large s and L(g'(t)) = wL(g(t)) – g(0).

Proof:
LHS=$L(g'(t))$
=$\int_a^b g'(t) e^{-wt} dt$
We Known the formula,
$\int u dv = uv - \int v du$

Let u=e^{-wt} dv=$g'(t)\, dt$
du=$e^{-wt}(-w)dt$v=g(t)

LHS=[g(t)$e^{-wt}]_a^b$-$\int_a^b g(t)(-w)e^{-wt}(t)dt$
=g(b)$e^{-wb} - g(a)e^{-wa} + w\int_a^c e^{-wt} g(t)\, dt$

Taking limit change to [a,b] to [0,n] on both side we get

$$\int_0^n g'(t)e^{-wt}dt = g(n)e^{-wn} - g(0)e^{-w0} + w\int_0^n g(t)e^{-wt}dt$$

Then n$\rightarrow \infty$on both side we get,

$$\int_0^\infty g'(t)e^{-wt}dt = g(\infty)e^{\infty} - g(0)e^{0} + w\int_0^\infty g(t)e^{-wt}dt$$

L{$g'(t)$}=wL{$g(t)$}$g(0)$

Hence the proof

Conclusion

In this paper, we discussed the laplace transform definition based examples and their properties in easy way. And also explained some theorems based on laplace. Properties of laplace transform are also explained in a easy manner. They are very easy to compute the problems.

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A Review on Binomial Distribution and It's Structures

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ABSTRACT: *The aim of thu paper is to construct the general binomial distribution and definition of binomial distribution and moment of generating function and the mean, variance and standard deviation and to study their theorem of Binomial distribution.*

Keywords: ** probability, distribution, parameter, success ,failure*

1. Introduction

In probability theory and statistics the Binomial distribution with parameter's ***n*** and ***p*** is the discrete probability distribution of the number of successes in a sequence of ***n*** independent experiments success with probability p and failure with probability q =1-p .A single success/ failure experiment is also called a **Bernoulli trail** or **Bernoulli experiment** on a sequence of outcomes is called a Bernoulli process for a single trial n=1,The Binomial distribution is 9 Bernoulli distribution .The Binomial distribution is the basis for the popular binomial test of statistical significance .
The Binomial distribution was discovered early in the **eighteenth century** by the swiss mathematician

Jakob Bernoulli
(1654 to1705).It was published posthumously **1713**.

1.1 Binomial Distribution:

The random variable x is called the binomial random variable with parameter **p** and **q.** If its probability density function is the form,
$f(x)= nc_x\, p^x (q)^{n-x,}$ for all x=0,1,2,……..n.

where 0<p<1 is the probability of success.
N = Total number of events
x=Total number of success full events
p =probability of success on a single
p(x) gives the probability in n binomial trail.
$nc_x= n!/x!(n-x)!$
1-p=q=probability of failure
nc_x is a combination.

2. Defnition:*Moment generating function:*

Let x be a discrete random variable with probability distribution $p(x=x_i)$, i=1,2……n. The function $M_x(t)$ called the moment generating function of x is defined by
$M_x\,(t)=\sum e^{txi}p(xi)$, if it exits,.

Example 2.1:
For a binomial description with parameters n=5 p=0.4 find the probabilities of gelling
1) atleast 3 success
2) exactly failures

Solution:
The Binomial distribution ,
$p=nc_x\, p^x q^{n-x}$
Given n=5,
p=0.4
q=1-p

=1-0.4
q=0.6

$p(x)=5c_x(0.4)^x(0.6)^{5-x}$

1) The probability of atleast 3 success
$p(x)=p(x=3)+p(x=4)+p(x=5)$
$=5c_3(0.4)^3(0.6)^{5-3}+5c_4(0.4)^4(0.6)^{5-4}+5c5(0.4)^5(0.6)^{5-5}$
$=10(0.064)(0.6)^2+5(0.0256)(0.6)+(1)(0.01024)$
=10(0.064)(0.36)+0.0768+0.01024
=0.2304+0.0768+0.01024
p(x)=0.3174

2) The probability for exactly 3 failure
= The probability of a exactly 2 success
p(x)=p(x=2)
$=5c_2(0.4)^2(0.6)^{5-3}$
$=10(0.16)(0.6)^3$
=10(0.16) (0.36)
=0.576
P(x)=0.576

Example 2.2:
The probability of winning a match for team A is 0.3.find the probability of winning 3 matches out of 5.

Solution:
Given n=5
The probability of winning p=0.3
Probability of losing q=1-p
=1-0.3
q=0.4
The binomial distribution,

$\mathbf{P(x)=nc_x\, p^x\, q^{n-x}}$
Probability of winning 3 matches out of 5, $p(x=3)=5c_3(0.3)^3(0.7)^{5-3}$
$=(10)(0.027)(0.7)^2$
=(10)(0.027)(0.49)
P(x)=0.1323

Example 2.3:
Toss a coin for 12 times what is the probability of getting exactly 7 heads.

Solution:
Number of trails (n)=12
Number of success(x)=7
Probability of single trail p=1/2
P=0.5
q=1-p
=1-0.5
q=0.5
$nc_X=n!/x!(n-x)!$
$12c_7=12!/7!(12-7)!$
12!/7!(5)!=12*11*10*9*8*7!/7!*5*4*3*2*1
=12*11*2*3
$12c_7=792$
$P(x)=12c_x(0.5)^7(0.5)^{12-7}$

$=791(0.007813)(0.5)^5$
$=792(0.0078)(0.0313)$
P(x)=0.19335

The probability of getting exactly 7 head is 0.19

3. Theorem
It X a binomial random variable with parameters p and n then mean, v ariance and moment generating function are respectively given by
$M_x(t)=[q+pe^t]^n$
$M_x(t)=(q+qe^t)^n$
mean $E(x)=np$
variance $=np(1-p)$
variance $=npq$

Proof:
$M_x(t)=E(e^{tx})$
$=\sum_{x=0}^{n} e^t\, nc_x\, p^x q^{n-x}$
$=\sum_{x=0}^{N} e^t\, n!/x!(n-x)!p^x q^{n-x}$
$=(q+pe^t)^n$
$M_x(t)=(q+pe^t)^n$ ……………………(1)

To find mean:
Differenciate with repect to "t"in eqn(1)
$M_x'(t)=n(q+pe^0)^{n-1}pe^0$
$=n(q+p)^{n-1}p \qquad (p+q=1)$
$=np(1)^{n-1}$
$M_{x'}(t)=np$

Mean=np
To find variance :
$M_x'(t)=np(q+pe^t)^{n-1}e^t$
Again differenciate with respect to "t"in eqn(2)
$M_x''(t)=(n-1)(q+pe^t)^{n-1} pe^t pe^t +n(q+pe^t)^{n-1}pe^t$
put t=0
$M_x''(t)=n(n-1)(q+p)^{n-1}pe^0 pe^0+n(q+p)^{n-1}pe^0$
$=n(n-1)(1)^{n-1} p^2 +np$
$= n(n-1)p^2 +np$
$M_x''(t) = n^2p^2+np$
Variance $= [M_x''(t)]-[M_x'(t)]^2$
$= n^2p^2-np^2+np-n^2 p^2$
$= np-np^2$
$= np(1-p)$
Variance $= np (1-p)$
$=npq$

Mean $=np$
Variance $=np(1-p)$
$=npq$
$M_x(t)=(q+pe^t)^n$
Hence the poof

Example 3.1:
The factory produees 500 bears per day. on aver-age 9 out of 10 bears are stitched perfectly .what is the mean and variance for the number for the number of bears that are stitched perfectly each day?

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Solution:
Where n=500
P=9/10
q=1-9/10
q=1/10
μ=mean
=np
=5*9/10
=450
variance=npq
=500*9/10*1/10
Variance=45

Example 3.2:
A die is rolled 312 times. Find the mean,variance & standard deviation of its that will be rolled.

Solution:
Mean=np
Variance=npq
Standard deviation=√npq
N=312
P=1/6
q=1-p
=1-1/6
q =5/6
Mean=np
=312*5/6
mean =52

Variance=npq
=312*1/6*5/6
variance =43.33

Standard deviation=√npq
=√43.33
Standard deviation=6.58

4. Notation
Parameters for binomial distribution
*S and F(Success & Failure) denote the two possible categories of all outcomes.
*P(S)=P(P-Probability of success).
*P(F)=1-p=q(q-Probability of failure).
*n=denote the number of fixed trails.

5. Application of Binomial Distribution
*Binomial distributions describe the possible number of times that a particular event will occur in a sequence of observations.
*They are used when we want to know about the occurrence of an even, not its magnitude.

6. Conclusion
In this paper we derived the derivation of mean, variance and moment generating function of binomial distribution and few example were solved.

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An Study on the Vector Analysis with their Basic Properties

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ABSTRACT: *In this paper, we study about the Vector differential operator del and their vector operators. We discussed about the gradient, the divergence and the curl with examples in easy manner. Further we discussed about Solenoidal and Irrotational Fields.*

Keywords: *Vector differential operator, The gradient, The divergence and The curl.*

1. Introduction

Mathematics is a branch of science that deals with the Logic of shape, Quantity and Arrangement. " VECTOR ANALYSIS" is a branch of Mathematics that deals with quantities that have both Magnitude and Direction. Vector Calculus was developed from quaternion analysis by J. Willard Gibbs and Oliver Heaviside near the end of the 19th Century.

Vector Calculus (or) Vector Analysis, is a branch of Mathematics concerned with Differentiation and Integration of vector fields, primarily in 3-dimensional Euclidean space $\mathbf{R}^3$. The differential expression of divergence, curl and gradient are derived based on one common model. They are involves the limiting value of a differential quantity per unit volume.

The Vector Differential Operator Del:

- The gradient, the divergence, and the curl are the first-order differential operators acting on fields. The vector differential operator del is written by ∇.

$$\nabla \equiv \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

- This vector operator possesses properties analogous to those of ordinary vectors. It is useful in defining three quantities which arise in practical applications and they are also known as the gradient, the divergence and the curl.
- We introduce three field operators which reveal interesting collective filed properties ,
  - The gradient of a scalar field,
  - The divergence of a vector field, and
  - The curl of a vector field.
- The operator ∇ is also known as **nabla.**

The Gradient:

- Let $\emptyset(x,y,z)$ be defined and differentiable at each point (x,y,z) in a certain region of space.
- $\emptyset$ defines a differentiable scalar field.
- Then the gradient of $\emptyset$, written $\nabla\emptyset$ or grad$\emptyset$.
- If U(x,y,z) is a scalar field, ie a scalar function of position **r = [x,y,z]** in 3 dimensions, then its **gradient** at any point is defined in Cartesian co-ordinates by

$$\text{grad}U = \frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}$$

It is usual to define the **Vector Operator**

$$\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Which is called "del" or "nabla". Then

$$\text{grad}U \equiv \nabla U$$

- The component of $\nabla\emptyset$ in the direction of a unit vector **a** is given by $\nabla\emptyset \cdot \boldsymbol{a}$ and is called the directional derivative of $\emptyset$ in the direction **a.** Physically, this is the rate of change of $\emptyset$ at (x,y,z) in the direction **a.**

Problems based on the Gradient

1. If $\emptyset$(x,y,z) = $3x^2y - y^3z^2$, find $\nabla\emptyset$ (or grad $\emptyset$) at the point (1,-2,-1).

Solution:

$\nabla\emptyset = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)(3x^2y - y^3z^2)$

$= \hat{\imath}\frac{\partial}{\partial x}(3x^2y - y^3z^2) + \hat{\jmath}\frac{\partial}{\partial y}(3x^2y - y^3z^2) + \hat{k}\frac{\partial}{\partial z}(3x^2y - y^3z^2)$

$= 6xy\,\hat{\imath} + (3x^2 - 3y^2z^2)\,\hat{\jmath} - 2y^3z\,\hat{k}$

$= 6(1)(-2)\,\hat{\imath} + \{3(1)^2 - 3(-2)^2(-1)^2\}\,\hat{\jmath} - 2(-2)^3(-1)\,\hat{k}$

$= -12\,\hat{\imath} - 9\,\hat{\jmath} - 16\,\hat{k}$

2. If U = x^2

Solution:

Only $\frac{\partial}{\partial x}$ exists so

$\nabla U = 2x\,\hat{\imath}$

The Divergence :

- Let V(x,y,z) = $V_1\hat{\imath} + V_2\hat{\jmath} + V_3\hat{k}$ be defined and differentiable at each point (x,y,z) in a certain region of space (i.e. V defines a differentiable vector field).
- Then the divergence of **V**, Written $\nabla.\boldsymbol{V}$ *or* $div\,\boldsymbol{V}$, is defined by

$\nabla.V = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right).(V_1\hat{\imath} + V_2\hat{\jmath} + V_3\hat{k})$

$= \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$

Note :

- The analogy with **A.B** = $A_1B_1 + A_2B_2 + A_3B_3$.
- Also note that $\nabla.\boldsymbol{V} \neq \boldsymbol{V}.\nabla$

Problems based on the Divergence :

1. If A = $x^2z\,\hat{\imath} - 2y^3z^2\,\hat{\jmath} + xy^2z\,\hat{k}$, find $\nabla.\boldsymbol{A}$ (*or div* $\boldsymbol{A}$) at the point (1,-1,1).

Solution:

$\nabla.A = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right).(x^2z\,\hat{\imath} - 2y^3z^2\,\hat{\jmath} + xy^2z\,\hat{k})$

$= \frac{\partial}{\partial x}(x^2z) + \frac{\partial}{\partial y}(-2y^3z^2) + \frac{\partial}{\partial z}(xy^2z)$

$= 2xz - 6y^2z^2 + xy^2$

$= 2(1)(1) - 6(-1)^2(1)^2 + (1)(-1)^2$

$= -3$ at (1,-1,1)

2. If A = $x\hat{\imath}$, find $\nabla.\boldsymbol{A}$ (*or div* $\boldsymbol{A}$)

Solution:

$\nabla.A = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right).(x\hat{\imath})$

$= \frac{\partial}{\partial x}(x)$

$= 1$

The Curl:

- If V (x,y,z) is a differentiable vector field then the curl or rotation of v, written $\nabla \times \mathbf{V}$, **Curl V or rot V,** is defined by

$$\nabla \times \mathbf{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times (V_1\hat{i} + V_2\hat{j} + V_3\hat{k})$$

$$= \left(\frac{\partial V3}{\partial y} - \frac{\partial V2}{\partial z}\right)\hat{i} + \left(\frac{\partial V1}{\partial z} - \frac{\partial V3}{\partial x}\right)\hat{j} + \left(\frac{\partial V2}{\partial x} - \frac{\partial V1}{\partial y}\right)\hat{k}$$

Note :

- This is the expansion of the determinant the operators $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ *must precede* V_1, V_2, V_3.

Problems based on the Curl:

1. If A = $xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$, find $\nabla \times \mathbf{V}$ (**or Curl A)** at point (1,-1,1).

Solution:

$$\nabla \times \mathbf{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times (xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k})$$

$$= \left[\frac{\partial}{\partial y}(2yz^4) - \frac{\partial}{\partial z}(-2x^2yz)\right]\hat{i} + \left[\frac{\partial}{\partial z}(xz^3) - \frac{\partial}{\partial x}(2yz^4)\right]\hat{j} + \left[\frac{\partial}{\partial x}(-2x^2yz) - \frac{\partial}{\partial y}(xz^3)\right]\hat{k}$$

$$= (2z^4 + 2x^2y)\hat{i} + 3xz^2\hat{j} - 4xyz\hat{k}$$

$$= 3\hat{j} + 4\hat{k} \text{ at } (1,-1,1)$$

Solenoidal and Irrotational Fields:

- The with null divergence is called solenoidal, and the field with null-curl is called irrotational field.
- The divergence of the curl of any vector field A must be zero, i.e.

$\nabla(\nabla \times A) = 0$

- Which shows that a solenoidal field can be expressed in terms of the curl of another vector field or that a curly field must be a solenoidal field.
- The curl of the gradient of any scalar field $\emptyset$ *must be zero, i.e.,*

$\nabla(\nabla\emptyset) = 0$

- Which shows that an irrotational field can be expressed in terms of the gradient of another scalar field, or a gradient field must be an irrotational field.

Directional Derivative:

- The directional derivative is the rate at which the function changes at a point in the direction. It is a vector form of the usual derivatives, and can be defined as :-

$\nabla_U f = \nabla f \cdot \frac{u}{|u|}$

where ∇ is called "nabla" or "del" and denotes a unit vector.

Conclusion

In this paper, we explained about vector differential operator del and their vector operators : The gradient, The divergence and The curl with simple examples and we also discussed about Solenoidal and Irrotational Fields.

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An Application of Probability Theory for Baye's Theorem

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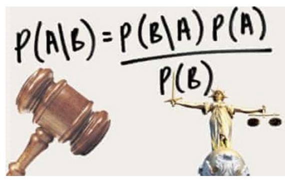
ABSTRACT: *This paper discuss about the observed outcome of the set of possible cases of the given conditional probability and explains the computed steps from the knowledge of the probability of each case.*

Keywords: *Probability, person's age, prediction, sample space, event.*

1. Introduction



In probability related to the theory and statistics, Baye's theorem (alternatively Baye's rule) explains about the probability of an event, bases on events prior knowledge and the conditions that might be related to the event.

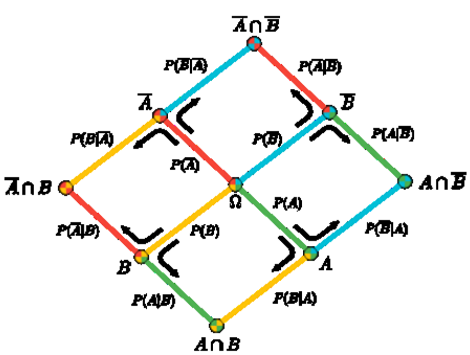
1.2 Definition:

The probability that the event occur if it is certain given that the another event is occurred is called conditional probability.

1.3 Example:

If tumor is connected to age of the person by using Baye's theorem, a person's age can be used to more accurately predict the probability that they have tumor or not, compared to the rating of the probability of tumor can't be predicted without knowledge of the person's age.
Baye's theorem is named after reverend Thomas Baye's (1701 - 1763), who first give an equation that allows new proof to renovate faith is his its Essay towards solving a problem in a doctrine of chances (1763). It was further expand by Pierre – Simon Laplace, who first issue the modern formulation in his 1812" Theorie Analytique Des Probabilities". Sir Harold Jeffreys Put baye algorithm and Laplace's formulation on an axiomatic basis. Jeffreys wrote that Baye's theorem is to the theory of probability what the Pythagorean Theorem is to geometry.

2. Statement



$$P(A|B)\cdot P(B) = P(A\cap B) = P(B|A)\cdot P(A)$$

Let A1,A2,…………,Ai be a partition of the sample space, and let B be any other event. Suppose that we know the probabilities P(A), and the conditional probability P(B/Ai) of event B within each of the sets Ai of the partition. The problem is to decide the conditional probabilities P(Ai/B) of the Ai when B is known to have occurred.

$P(Ai\backslash B) = \frac{P(B\backslash Ai)\,P(Ai)}{P(B)}$

This is known as Baye's rule. This rule also forms as the statistical method is called Bayesian procedure. P(Ai\B) is also called posteriori probability.

2.1 Derivation:

Its derived from the definition of conditional probability:

$P(A\backslash B) = \frac{P(A\cap B)}{P(B)}$, if $P(B) \neq 0$,

$P(B\backslash A) = \frac{P(B\cap A)}{P(A)}$, if $P(A) \neq 0$,

Where $P(A\cap B)$ is the Joint Probability of both A and B being true,

$P(B\cap A) = P(A\cap B)$

$P(A\cap B) = P(A\backslash B)\,P(B)$

$= P(B\backslash A)\,P(A)$

$P(A\cap B)= P(B\cap A)$

$P(A\backslash B)\,P(B) = P(B\backslash A)\,P(A)$

$P(A\backslash B) = \frac{P(B\backslash A)P(A)}{P(B)}$, if $P(B) \neq 0$.

3. Baye's Theorem:

Example 3.1

The total return of a shop is construct on three engine which publication for 40%, 60%, and 90% of the output, respectively. The fraction imperfect items construct in 10% for the first engine, 6% for the second engine, and 2% for the third engine.

a) What fraction of the total return is imperfect ?
b) If an item is pick at random from the entire output and is found to be imperfect, what is the probability that is was made by the third engine?

Solution

Let A_i be the event that a randomly chosen item was made by the ith engine (i = 1,2,3). Let B be the event that a randomly chosen item is imperfect. Then,

$P(A_1) = 0.4$, $(A_2) = 0.6$, $P(A_3) = 0.9$.

If the item was made be engine A_1, the probability that it is imperfect is 0.1; that is, $P(B\backslash A_1) = 0.1$. We thus have

$P(B\backslash A_1) = 0.1$, $P(B\backslash A_2) = 0.06$, $P(B\backslash A_3) = 0.02$.

In (a) we have to find P(B). This may be defined by

$P(B) = \sum(P(B\backslash A_i)P(A_i)$

$= (0.1)(0.4) + (0.06)(0.6) + (0.02)(0.9)$

$P(B) = 0.094$

Hence 9.4% of the total output of the factory is imperfect.

In (b) we are given that B has happen, and wish to find the conditional probability of A_2. By Baye's Theorem,

$P(A_3\backslash B) = \frac{P(B\backslash A3)P(A3)}{P(B)}$

$= \frac{(0.02)(0.9)}{0.094}$

$P(A_2\backslash B) = \frac{9}{47}$

An item is chosen at random from the entire output and is found to be imperfect, the probability that it was made by the third machine is $\frac{9}{47}$.

Example 3.2:
Measurement at the Golden Triangle Universities on a certain day indicated that the source of incoming jobs is 25 percent from University Of Cambridge 24 percent from University Of Oxford and 60 percent from University Of London. Suppose that the probabilities that a job initiated from these Universities requires (operator intervention for tape) set – up are 0.02, 0.08 and 0.04 respectively. Find the Probability that a job chosen at random at GTU is a set - up job. Also find the probability that a randomly chosen job comes from the University of UOO, given that it is a set - up job.

Solution:
Define the events B_i = "Job is from University i" (I = 1,2,3 for UOC, UOO and ULO respectively) and A = "Job requires set – up". Then by the theorem of total probability.
$P(A) = P(A\backslash B_1)\ P(B_1) + P(A\backslash B_2)\ P(B_2) + P(A\backslash B_3)\ P(B_3)$
$= (0.02)(0.25) + (0.08)(0.45) + (0.04)(0.6)$
$= 0.005 + 0.036 + 0.024$
$P(A) = 0.065$

Now the second event of interest is $(B_2\backslash A)$ and from Baye's rule
$P(B_2\backslash A) = \frac{P(A\backslash B2)P(B2)}{P(A)}$
$P(B_2\backslash A) = \frac{(0.08)(0.45)}{0.065}$
$= \frac{0.036}{0.065}$
$P(B_2\backslash A) = 0.5538$

Note that the knowledge that the job uses multitasking increases the chance that it came from UOO from 45% to above 55%.

Example 3.3:
Two boxes containing rings are placed on a stand. The box are labeled A and B. Box A contains 8 green rings and 5 purple rings. Box B contains 6 green rings and 15 white rings. The boxes are arranged so that the probability of selecting box A is $\frac{1}{6}$ and the probability of selecting box B is $\frac{4}{9}$. Hari is blind folded and asked to select a ring. He will win a TV if he selects purple ring.
a) What is the probability that hari will win the TV (that is he will select a purple ring ?
b) If hari wins the TV, what is the probability that the purple marble was selected from the first box ?

Solution:
Let E be the event of drawing a purple marble. The prior probability are $P(A) = \frac{1}{6}$ and $P(B) = \frac{4}{9}$.
a) The probability that hari will win the TV is
$P(E) = P(E\cap A) + P(E\cap B)$
$= P(E\backslash A)\ P(A) + P(E\backslash B)\ P(B)$
$= (5/13)\ (1/6) + (15/21)\ (4/9)$
$= (5/78) + (60/189)$
$P(E) = \frac{50}{2457}$

b) Given that hari won the TV, the probability that the purple ring were selected from box A is
$P(A\backslash E) = \frac{P(E\backslash A)P(A)}{P(E\backslash A)P(A)+P(E\backslash B)P(B)}$
$= \frac{\left(\frac{5}{13}\right)\left(\frac{1}{6}\right)}{\left(\frac{5}{13}\right)\left(\frac{1}{6}\right)+\left(\frac{15}{21}\right)\left(\frac{4}{9}\right)}$

$= \frac{\left(\frac{5}{78}\right)}{\left(\frac{5}{78}\right)+\left(\frac{60}{189}\right)}$

P(A\E) = $\frac{63}{20}$

That the P(E\A) is the probability of selecting a purple ring from A whereas P(A\E) is the probability that the purple ring was selected from A.

4. Conclusion

In this paper we have derived the derivation of baye's theorm and solved some problem using Baye's theorem in various practical problems.

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Basic Concepts of Mathematical Statistics

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ABSTRACT: *In this chapter we discuss about the two views of probability that is mathematical view and statistical view and further discussed about some basic concepts of probability.*

Keywords: *Probability, Trial, Random experiment, Random Variable, and Samples.*

1. Introduction

During his lecture in 1929, Bertrand Russel said, "probability is the most important concept in modern science, especially as nobody has the slightest notion what it means." Most people have some vague ideas about what probability of an event means. The interpretation of the word probability involves synonyms such as chance, odds, uncertainty, Prevalence, expectancy etc. "We use probability when We want to make an affirmation, but are not quite sure," Writes J.R Lucas.
A complete discussion of these interpretations will take us to area such as philosophy, theory of algorithm and randomness, etc.
The objective school defines probabilities to be "long run" relative frequencies. This means that one should compute a probability by taking the number of favourable outcomes of an event and dividing it by total number of the possible outcomes of the event, and then taking the limit as the number of trials becomes large. Some statisticians object to the word "long run". The philosopher and statisticians John Keynes said "in the long run we are all dead." The objective school use the theory developed by Von mises (1928) and Kolmogorow (1965).
Probability theory, a branch of mathematics concerned with the analysis of random phenomena. The word probability has several meanings in ordinary conversation. Two of these are particularly important for the development and applications of the mathematical theory of probability. One is the interpretation of probability as relative frequencies, for which simple games involving coins, cards, dice and roulette wheels provide examples. Probability theory is used to evaluate the reliability of conclusions and inferences based on data.

Random Experiment

A Random Experiment is an experiment, trial, or observation that can be repeated numerous times under the conditions. The outcome of an individual random experiment must be independent and identically distributed. It must in no way be affected by any previous outcomes and cannot be predicted with certainty.

Example

The mobile phone is falls down. The experiment can yield two possible outcomes, the mobile glass is broken or unbroken.

Sample Space

In probability theory, the sample space of an experiment or random trial is the set of all possible outcomes or results of that experiment. A sample space is usually denoted by S, and the possible ordered outcomes are listed as elements in the set. In General sample space is dented by S, Ω or U (for "universal set"). The set of all sample points is called sample space.

Example

If selecting an age from a group of people the possible outcomes are old, middle and young.
Sample space S={old, middle, young}

Sample Point

In a probabilistic experiment, A sample point is the most basic outcome of an experiment.

Example
A coin is tossed the sample space is S={H,T} and the sample point are Head and Tail.

Trial and Events
Any particular performance of a random experiment is called a trial.
The outcomes or the set of combination of outcomes are termed as Events.

Example
Throw a die is a trial and get in unique of space is {1,2,3,4,5,6} is an event.

Types of Event
I) Exhaustive event.
II) Favourable event.
III) Disjoint (or) Mutually exclusive event.
IV) Equally likely event.
V) Independent event.
VI) Dependent event.
VII) Complementary event.
VIII) Impossible event.
IX) Compound event.

Probability
Mathematical View
Probability is the likelihood of something happening in the future. It is expressed as a number between zero (can never happen) to one (will always happen). It is expressed as a form of fraction, a decimal, a percentage, or as odds. If all outcomes of an experiment are equally likely. Then the probability of happening of K is given by P(K).

$$P(K) = \frac{\text{Favourable number of cases}}{\text{Exhaustive number of cases}}$$

$$P(K) = \frac{M}{N}.$$

Table of Typical Probabilities

Event	Probability		
	...as a fraction	...as a decimal	...as odds
The sun sets in the evening.	1	1.	0.
A tossing a coin turns up tail.	1/2	0.5	1:1
A card drawn out of a deck is a club.	1/4	.25	3:1
A card drawn out of a deck is a claver, heart or spade.	3/4	.75	1:3
A kid flies by flapping his arms.	0	0	Infinite(∞)

Properties of Probability
I) The Sum of the probabilities of an event and its complementary is 1, so the probability of the complementary event is,
$P(\bar{A})=1-P(A)$.
II) The probability of not possible event is zero
$P(\emptyset)=0$.
III) The probability of the union of two events is equal to the sum of their probabilities and minus the probability of their intersection.
$P(S \cup K)=P(S)+P(K)-P(S \cap K)$.
IV) I an event is a subset of another event, its probability is less than or equal to it.
If $S \subset V$, then $P(S) \le P(V)$
V) If A_1, A_2, A_3,……A_k are mutually exclusive between than, then
$P(D_1 \cup D_2 \cup A_3 ... \cup D_k)=P(D_1)+P(D_2)+P(D_3)+....+P(D_k)$

VI) If the sample space S is finite and an event is
$S=\{C_1, C_2,....C_n\}$ then
$P(S)=P(C_1)+P(C_2)+....+P(C_n)$.

Example of Probability

A coin is tossed twice by finding the probability of getting I) Two head II) One head III) At least one head IV) No head.

Sol:

$$Probability = \frac{\text{Total number of favourable cases}}{\text{Total number of Exhaustive cases}}.$$

Sample space S={HH,HT,TH,TT}
Total number of Exhaustive cases=4.

a) Two head
Number of favourable cases=1.
P (Two head) = 1/4 =0.25.

b) One head
Number of favourable cases=2.
P (One head) = 2/4 =0.5.

c) At least one head
Number of favourable cases=3.
P (At least one head) = 3/4 =0.75.

d) No head
Number of favourable cases=1.
P (No head) = 1/4 =0.25.

Statistical View

A probability distribution is a table or an equation that links each outcome of the statistical experiment with its probability of occurrence. In 'n' trail an event E happen m times then the probability of P of happening of E is given by
$P(E)=\lim_{n\to\infty} \frac{m}{n}$.

Example

Consider a simple experiment in which we toss a coin two times. An outcome of the experiment might be the number of tail that we see in two coin tosses. The table below associates with its probability.

Number Of Tail	Probability
0	0.25
1	0.50
2	0.25

Random Variable

In probability and statistics, a random variable, random quantity, aleatory variable, or stochastic variable is a variable whose possible values are outcomes of a random phenomenon. A stochastic variable, usually denoted by X, is a variable whose possible values are numerical outcomes of a random occurrence. They are two types of random variable.

I) Discrete random variables.
II) Continuous random variables.

Example

Consider the experiment of tossing a coin twice. Then the sample space, S={HH,HT,TH,TT} then the sample point of the experiment S_1=HH, S_2=HT, S_3=TH and S_4=TT.

Keywords:

Probability, Trial, Random experiment, Random Variable, and Samples.

Notations:

S – Sample space.
{ } – Set bracket.
U – Union.
▯ - Intersection.
P(A), P(C), P(D), P(E), P(K) – Probability value.
C – Contain.
M – Total number of Favourable cases.
N – Total number of Exhaustive cases.

Conclusion

This chapter provides an explanation of probability for processes with a finite number of possible outcomes. It explains the meaning of probability as wells as how to calculate probability and odds. The theory of probability is beneficial because we can compare the outcomes with observed events and evaluate the efficiency of our research.

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Groups and Homomorphism in the Symmetry of Triangles

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ABSTRACT: *In this paper we discuss about the concept of groups and homomorphism in the symmetry of equilateral triangle and isosceles triangle . And further we prove that equilateral triangle and isosceles triangle are homomorphic to each other .*

Keywords: Groups, homomorphism ,symmetry, sets, elements, triangles, mapping, binary operation

1. Introduction

The concept of groups and homomorphism comes under the branch abstract algebra which is also known as modern algebra. Abstract algebra is the study of algebraic structures include groups, rings, fields ,modules, vector spaces, lattices and algebras. Here we are going to discuss about the presences of group in triangles specifically using their symmetry. The symmetry of equilateral triangle forms group with six elements under algebraic mulitiplication .The isosceles triangle also forms a group under algebraic multiplication.
We further introduce the concept of homomorphism which is the mapping between two algebraic structures preserves Groups, homomorphism ,symmetry, sets, elements, triangles, mapping, binary operation.
The concept of groups and homomorphism comes under the branch abstract algebra which is also known as modern algebra. Abstract algebra is the study of algebraic structures include groups, rings, fields ,modules, vector spaces, lattices and algebras. Here we are going to discuss about the presences of group in triangles specifically using their symmetry. The symmetry of equilateral triangle forms group with six elements under algebraic mulitiplication .The isosceles triangle also forms a group under algebraic multiplication.
We further introduce the concept of homomorphism operation in those structures. Let as consider the symmetry of triangles as group and staisfing the homomorphic condition between these two triangles.

Basic Definitions

Definition 1.1:Groups

l orders,academic p, A non empty set of element G is said to form a group if in G there is defined by a binary operation (*).The group must satisfies the following axioms
i)closure axiom:
let a,b be any two elements belongs to group G then,
a*b also belongs to the group G.

ii)identity axiom:
let e be the identity element belongs to G then , it should satisfy the condition a*e=e*a=a.where the element belongs to G.

iii)inverse axiom:
let a,a^{-1},e belongs to group G then,
a*a^{-1}=a^{-1}*a=e,where $a^{-1}$is the inverse of a.

iv)associative axiom:
let a,b,c be the elements belongs to group G then,
a*(b*c)=(a*b)*c.

Example

Let G consist of the real numbers 1,-1 under the multiplication of real numbers. G is then a group of order 2.

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Definition1.2: Homomorphism
Homomorphism is a structure preserving map between two algebraic structures of same type that preserves the operation of the structures.

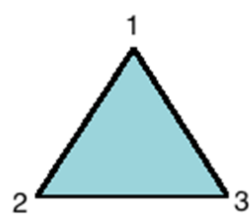
Example
A group homomorphism is a map between two groups which preserves same group operation.

Definition1.2: Group homomorphism
A group homomorphism is a map between groups that preserves the group operation .This implies that the group homomorphism maps the identity element of the first group to the identitiy element of second group and maps the inverse of an element of the first group to the inverse of image of this element.

Example
Let (G,*) and (H,*) be a two groups.
x,y be two elements belongs to G then, x*y=z
consider the function F which defines the mapping between G and H.
- F(x)*F(y)=F(Z)
- F(x)*F(y)=F(x*y)

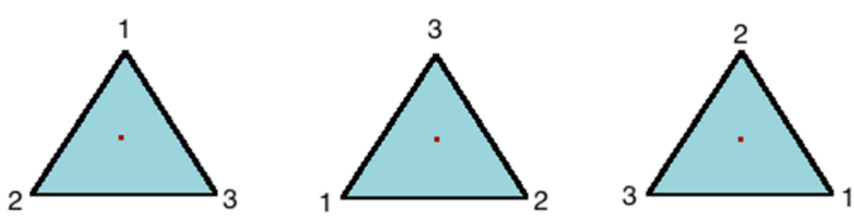
Definition 1.3: symmetry
symmetry means that one shape becomes exactly like another when you move it in some way : turn, flip or slide.

1.3.1: Group Of Symmetries In Equilateral Triangle
Consider the equilateral triangle whose vertices are labeled points:



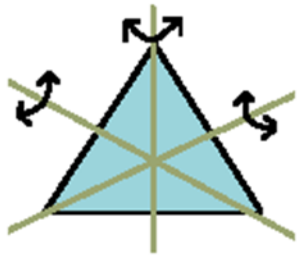
Consider a point fixed in the center of this triangle. There are two types of symmetries we can look a : the first is the counterclockwise rotational symmetries we can rotate the triangle by 0^0 (or equivalently 360^0),120^0 or 240^0 as illustrated in the following images:

Counterclockwise Rotational Symmetries of the Equilateral Triangle

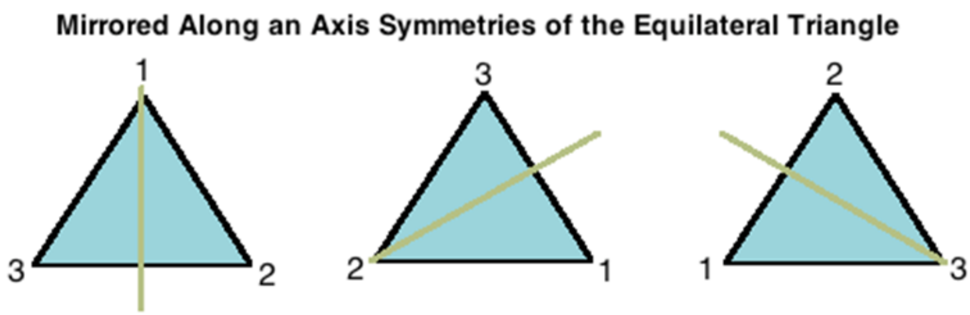


Rotation by 0 degrees. Rotation by 120 degrees. Rotation by 240 degrees.

The second type of symmetries we can look at: are axial symmetries along specified axes. There are three axes which we can mirror the equilateral triangle onto itself.



Mirroring the equilateral triangle around each of these axes produces a symmetry.



We will see that these six symmetries form a group under multiplication as follows:

Let as named the symmetries of triangles as e, r, r^2, f, rf, rf^2. And P be the set whose elements are the symmetry of equilateral triangle.
P={e, r, r^2, f, rf, rf^2 } then (P,.) forms a group by satisfying the following axioms.

- Closure axiom:

$r.r^2 = r^3 = e$ belongs to P
r.f = rf belongs to P
$r.rf = r^2f$ belongs to P
$r.r^2f = r^3f$ belongs to P
$r^2.r^2f = rf$ belongs to P
Henc closure axiom is satisfied.

- Identity axiom:

e=1 be the identity element then,
e.r = r.e = r belongs to P
Hence identity axiom is satisfied.

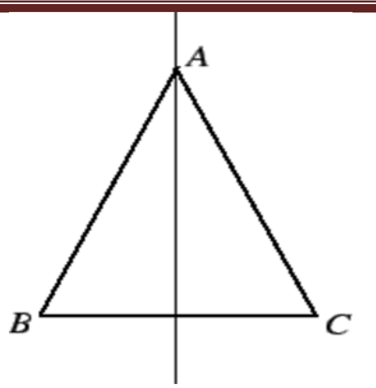
- Inverse axiom:

r^{-1},r belongs to P then,
$r^{-1}.r = r.r^{-1} = e$ belongs to P.
where $r^{-1}$is the inverse of r.
Hence inverse axiom is satisfied.Type equation here.

- Associative axiom:

$r.(r^2.f) = (r.r^2).f$
Since multiplication is associative.

1.3.2: Group of symmetries in isosceles triangle
Isosceles triangle has only two symmetries.



Let the symmetries named as e and f. let Q be the set whose elements are the symmetry of isosceles triangle. Q = {e, f} forms group under multiplication by satisfying the following axioms:

- Closure axiom:

e.f = f belongs to Q.
Hence it is satisfied.

- Identitiy axiom:

e=1 is the identity element.
e.f = f.e = f belongs to Q
Hence it is satisfied,

- Inverse axiom:

f^{-1} be the inverse of f then,
$f.f^{-1} = f^{-1}.f$ = e belongs to Q.
Hence satisfied.
Thus (Q,.)forms a group.

1.4:Equilateral triangle is homomorphic to isosceles triangle

Let (P,.) and (Q,.) be the groups of symmetries of equilateral triangle and isosceles triangle respectively.
Consider the mapping F from P to Q then,
F(x) = e,
where x is any element from P.
F(r.f) = F(r) . F(f)
F(r.f) = e
Hence it preserves a homomorphic condition under the operation multiplication.
Therefore symmetries of equilateral triangle is homomorphic to the symmetries of isosceles triangle.
Hence proved.
Notations used:

- * : binary operation.
- F : defines a function.
- {} : set bractets
- (P,.) , (Q,.) , (G,.) , (H,.) : groups.
- e: identity element used in group.

Conclusion

This chapter clearly explains the symmetry groups and throws light to the concept of homomorphism in field of symmetry groups of triangles .

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Theoritical Concepts of Graph Theory

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ABSTRACT: *Many analysis queries in math arise from concerns of world issues. Part of the duty of a man of science is to raise this kind. During this speak we are going to examine the surprising Origins of many queries from the sphere of graph theory. These examples can give some insight into however mathematicians meaningfully guide issues from the concrete to the abstract.*

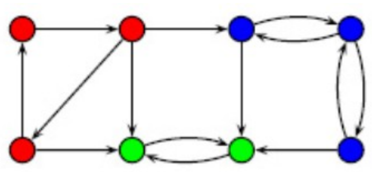
Keywords: *Connectivity, Eulerian, Hamiltonian*

1. Introduction

The city of KÄonigsberg was set on the Pregel stream in geographic region. The stream divided the town into four separate landmasses, together with the island of Kneiphopf. These four regions were joined by seven bridges as shown within the diagram. Residents of the town questioned if it were potential to go away home, cross every of the seven bridges precisely once, and come home. Swiss scientist Leonhard Mathematician (1707-1783) considered this downside and therefore the technique he wont to solve it'sthought of by several to be the birth of graph theory.

Connected Components

In a directed graph G = (V; E), u and v are strongly connected if there exists a walk from u to v and from v to u. This is an equivalence relation and hence leads to equivalence classes, which are called the connected components of the graph
G.



For example:

For n>1 the graph k_n^- consisting of n points and no lines is disconnected. The union of two graphs is disconnected.

Proposition:

A graph G with p points and $\delta \geq \frac{p-1}{2}$ is connected.

Proof:

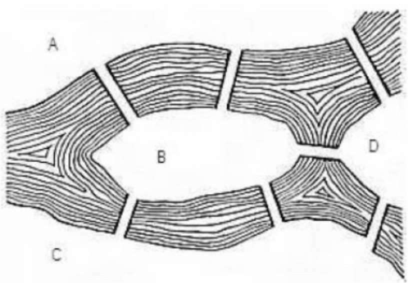
Suppose G is not connected. Then G has more than one component. Consider any component $G_1=(V_1, X_1)$ of G. Let $v \in V_1$. Since $\delta \geq \frac{p-1}{2}$ there exist at least $\frac{p-1}{2}$ points in G_1 adjacent to v_1 and hence V_1 contains at least $\frac{p-1}{2}+1=\frac{p+1}{2}$ points.

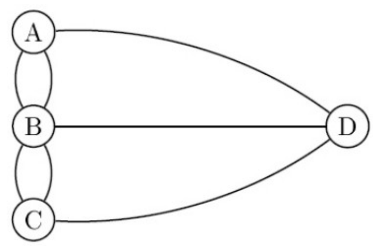
Thus each component of G contains at least $\frac{p+1}{2}$ points and G has at least two components. Hence number of points in G$\geq p+1$ which is a contradiction.
Hence G is connected.

Proposition:
A graph G is connected iff for every any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of $V_{2.}$

Proof:
Suppose G is connected.
Let $V=V_1 U V_2$ be a partition of V into two subsets.
Let $u \in V_1$ and $v \in V_2$. Since G is connected, there exists a u-v path in G, say $u=v_1,v_2.....v_n=v$.
Let i be the least positive integer such that $v_i \in V_2$. Then $v_{i-1} \in V1$ are adjacent. Thus there is a line joining $v_{i-1} \in V1$ and $v_i \in V_2$.
To prove the conserve suppose G is not connected. Then G contains at least two components.
Let V1 denote the set of all components and V2 the remaining vertices of G. Clearly $V=V_1 U V_2$ is a partition of V and there is no line joining any point of V_1 to any point of $V_{2.}$
Hence the proof.

Eulerian Graphs:
A closed trail containing all points and lines is called an eulerian tour. A graph having n eulerian trail is called an eulerian graph.





Proposition:
If G is a graph in which the degree of every vertex is at least two then G contains a cycle.

Proof:
Construct a sequences $v,v_1,v_2,....$ of vertices as follows. Choose any vertex v. Let v_1 be any vertex adjacent to v. Let v_2 be any vertex adjacent to v_1 other than v.
At any stage, if vertex v_i, $i \geq 2$ is already chosen, then choose v_{i+1} to be any vertex adjacent to v_i other than v_{i-1}.
Since degree of each vertex is at least 2, the existence of v_{i+1} is always guaranteed.
Since G has only a finite number of vertices, at some stage we have to choose a vertex which has been chosen before.
Let v_k be the first such vertex and let $V_k = V_i$ where i<k. Then $v_i v_{i+1.....}, v_k$ is a cycle.
The following theorem answers the problem. In what type of graph G is it possible to find a closed trail running through every edge of G.
Hence the proof.

Hamiltonian Graphs:
A spanning cycle in a graph is called a Hamiltonian cycle. A graph having a Hamiltonian cycle is called is a

Hamiltonian graph.
A Hamiltonian cycle is a cyclic sub graph G h = (V; Eh) of
G = (V; E) which passes exactly once through all nodes

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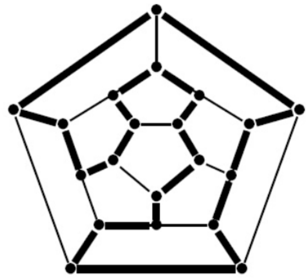

It is a so-called hard problem and there is no general condition for its existence (in contrast with the Eulerian path problem).
It exists for Platonic solids and complete graphs, But not for the Petersen graph.

Proposition:
Every hamiltonian graph is 2-connected.

Proof:
Let G be a hamiltonian graph and let Z be a hamiltonian cycle in G. For any vertex v of G Z-v is connected and hence G-v is also connected.
Hence G has no cut points and thus G is 2-conneted.
The following theorem gives a simple an useful necessary condition for hamiltonian graphs.

Proposition:
If G is hamiltonian, then for every nonempty proper subset S of V(G), ω (G-S)$\leq$ $|S|$ where ω(H) denotes the number of components in any graph H.

Proof:
Let Z be a Hamiltonian cycle of G. Let S be any nonempty proper subset of V(G). Now ω (Z-S)$\leq$ $|S|$. Also Z-S is a spanning sub graph of G-S and hence ω (G-S) $\leq$ ω (Z-S). Hence ω (G-S)$\leq$ $|S|$.
Hence the proof

Conclusion
Graph theory is associate degree exceptionally wealthy space for programmers and designers. Graphs may be accustomed solve some terribly complicated issues, like least value routing, mapping, program analysis, and so on. Network devices, like routers and switches, use graphs to calculate best routing for traffic.

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Concept on Prime and Maximal Ideal

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ABSTRACT: *Let R be a ring with two well defined binary operations '+' and '.' and N be an ideal of R. R/N is given by* $(N+x)+(N+y)=N+(x+y)$ *and* $(N+x)(N+y)=N+xy$. *If R is a ring and N is an ideal of R then R/N is also a ring. Further of R is commutative then R/N is also commutative. Finally, we can apply these results to prove an ideal of R is prime ideal as well as maximal ideal.*

Keywords: *Commutative ring, Prime ideals, Maximal ideals, Integral domain, Field.*

1. Introduction

A ring R is said to be commutative ring if R is commutative towards multiplication. A ring R is said to be commutative ring with unity if r is commutative and it has unit element on multiplication as well. 1 is a unit element. i.e. $1 \in R$

R/N is commutative and $1+N$ is identity for R/N.

A subset N of R is said to be maximal ideal if $N \neq R$. If B is an ideal of R such that $N \subseteq B \subseteq R$ then either $B=N$ or $B=R$. There is no proper ideal of R properly containing N.

An ideal $N \neq R$ is said to be prime ideal if $xy \in N$ which implies that either $x \in N$ or $y \in N$. R is an integral domain which implies that R/N is also an integral domain. N is a prime ideal if and only if R/N is an integral domain.

If R is a commutative ring with unit element and N is an ideal of R, then N is a maximal ideal of R if and only if R/N is a field. R/N is a field if every non zero element of R/N has a multiplicative inverse in R/N.

Derivation in Ideals

Theorem:

Prove that every maximal ideal is a prime ideal if and only if every prime ideal is maximal ideal.

Proof:

Suppose N is a maximal ideal then we have to claim that N is a prime ideal.

Let R be commutative ring with unit element 1. Let $a \in R$. Let N be a maximal ideal of R. Since $N \neq R$.

Since R is commutative ring with unity. R/N is a commutative with unit element $1+N$.

Now, we have to prove that R/N is a field. We supposed to prove that every non- zero element of R/N has a multiplicative inverse in R/N.

Let $x+N \in R/_N$, x not belongs to N. Let $x+N$ is a non- zero element of R/N. Let us consider $Rx=\{ax \mid a \in R\}$.

Rx is an ideal of R and $x=1.x \in Rx$

Now, $N+Rx$ is an ideal of R and $N \subseteq N+Rx \subseteq R$.

Since x not belongs to N and $x \in N+Rx$, we find that $N+Rx$ properly contains N.

Since N is maximal ideal, we have $N+Rx=R$. Since $1 \in R=N+Rx$, there exists an element $y \in R$ and $n \in N$ such that

$1=n+yx$

$1+N=n+yx+N$

$=yx+N$

$=(y+N)(x+N)$, since $n \in N$

Which implies that $y+N$ is a multiplicative inverse of $x+N$.

Hence R/N is a field.

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We know that,
" Every field is an integral domain".
Therefore, R/N is an integral domain.
Hence N is a prime ideal.
Conversely, suppose that N is a prime ideal then we have to prove that it is a maximal ideal.
Let N be a prime ideal. Assume that B is an ideal of R which contains N. Let 1 be an identity element.
Let $a \in N$. Then $a^n \in N$ and so $a + a^n = a(a^{n-1} + 1) \in N.$
Since $a \in N$ and N is a prime ideal, $a^{n-1} + 1$ *not belongs to* $N.$ But $a^{n-1} + 1 \in B$ which implies that $a^n \in B$ and $1 \in B$. Since $1 \in B$ we must have $B = R$. Since N is properly contained in $B = R.$
Thus, N must be maximal.
Therefore, N is a maximal ideal.
Hence every maximal ideal is a prime ideal if and only if every prime ideal is maximal ideal.

Conclusion

There is no proper ideal of R proper containing N. Hence N is maximal in R. Every prime ideal is maximal. In case of (0) is a prime ideal of but not a maximal ideal of Z

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Correlation on Mathematical Statistics

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ABSTRACT: *So far we have studied problems relating to one variable only .In practice we come across a large Number of problems involving the use of two or more than two variables.If quantities vary in Such a way that movements in one are accompanied by movements in the other ,these quantities Are correlated.The degree of relationship between the variables under consideration is measuredThrough the correlation analysis . The measure of correlation called the correlation coefficient orCorrelation index summarizes in one figure the direction and degree of correlation.*

Keywords: *Rank, Coefficient, Operations, Linear, Non-linear, Correlation, Demand, Multiple, Ratio, Simple, Partial.*

1. Introduction

Correlation analysis helps us in determining the degree of relationship between two or more variables it does not tell us anything about cause and affect relationship.even a high degree of correlation does not necessarily mean that a relationship of cause and affect exits between the variables or simply stated.

Definitions

Correlation is a statistical tool which studies the relationship between the two variables. correlation analysis involves various method and technique for studying and measuring the extent of the relationship between the two variables.

Types of Correlation

1. Positive or negative
2. Simple ,partial and multiple
3. Linear or non -linear

- 1. **Positive :** Two variables are called positively correlated,if for an increase value in one variable and there is increase value in the another variable.
- **Negative :**Two variables are called negatively correlated,if for an increase in the value of one variable there is decrease in the value of another variable.
- **2. Simple:** The correlation between the variables is called simple correlation.
- **Partial:** The correlation have more than two or three variables are called multiple or partial.
- **3.linear** and non-linearif the amount of change in one variable tends to bear the constant ratio to the amount of change in the other variable than the correlation are called linear.If the amount of change in one variable does not bears a constant ratio to the amount of change in other variable.

Properties of Correlation

1. The correlation does not affect by the change of origin and scale of references.
2. The value of correlation lies between.

Limitations

1. X and Y must have jointly connected.
2. we cannot establish the exact degree of correlation between the variable.

Formulas

1. Rank correlation :

$$\rho = \frac{1-6\sum d^2}{n(n^2-1)}$$

$$\rho = 1 - \frac{\frac{6[\sum d^2 + \sum m(m^2-1)]}{12}}{n(n^2-1)}$$

2. Coefficient of correlation :

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\ \sqrt{\sum(y-\bar{y})^2}}$$

3. karl Pearson's coefficient:-

$$r = \frac{\sum dXdY - \frac{\sum dX \sum dY}{N}}{\sqrt{\sum dX^2 - \frac{(\sum dX^2)}{N}}\ \sqrt{\sum dY^2 - \frac{(\sum dY^2)}{N}}}$$

1. Problem:
Calculate the coefficient of correlation between x and y from the following data

X	1	3	5	8	9	10
Y	3	4	8	10	12	11

Solution:

x	y	$(X-\bar{X})$	$(Y-\bar{Y})$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(x-\bar{X})(y-\bar{Y})$
1	3	-5	-5	25	25	25
3	4	-3	-4	9	16	12
5	8	-1	0	1	0	0
8	10	2	2	4	4	4
9	12	3	4	9	16	12
10	11	4	3	16	9	12

$\bar{X} = \frac{\sum x}{n} = \frac{36}{6} = 6$

$\bar{Y} = \frac{\sum Y}{n} = \frac{48}{6} = 8$

$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2}\ \sqrt{\sum(y-\bar{y})^2}}$

$r = \frac{65}{\sqrt{64}\ \sqrt{10}}$

r= 0.97.

2. Calculate Karl Pearson's coefficient of correlation for the following data of prices and demand (43,105) , (54, 98) , (85, 53) , (91, 49) , (59, 84) , (95, 40) , (68, 73) , (29, 59) , (73, 63) , (72, 52)

Solution :

X	Y	dX	dY	dX^2	dY^2	dXdY
43	105	-25	32	625	1024	-800
54	98	-14	25	196	625	-350
85	53	17	-20	289	400	-340
91	49	23	-24	529	576	-552
59	84	-9	11	81	121	-99
95	40	27	-33	729	1089	-891
68	73	0	0	0	0	0
79	59	11	-14	121	196	-154
73	63	5	-10	25	100	-50
72	52	4	-21	16	441	-84

$dx = X-\bar{X}$ $dy = Y-\bar{Y}$

$\sum dx = 39$ $\sum dy^2 = 4572$

$\sum dy = -54$ $\sum dx^2 = 2611$
$\sum dxdy = -3320$

$$r = \frac{\sum dXdY - \frac{\sum dX \sum dY}{N}}{\sqrt{\sum dX^2 - \frac{(\sum dX^2)}{N}}\ \sqrt{\sum dY^2 - \frac{(\sum dY^2)}{N}}}$$

$$r=\frac{10(-3320)-39(54)(-1)}{\sqrt{10(2611)-39^2}\ \sqrt{10(4572)-(54)^2}}$$

r =-0.959

3. The application for a post were interviewed by the personal manager and the training manager. He was first placed by the personal manager followed by F, D, B, I, C, J, G, A and E in the order the training manager was first F followed by D, H,I, C, B, A, J, E and G in that order calculate the value of spearman's rank correlation coefficient?

	Rank By			
Application	Personal Manager	Training Manager	d	d^2
A	9	7	2	4
B	4	6	-2	4
C	6	5	1	1
D	3	2	1	1
E	1	9	1	1
F	2	1	1	1
G	8	10	-2	4
H	1	3	-2	4
I	5	4	1	1
J	7	8	-1	1

$$\rho = \frac{1-6\sum d^2}{n(n^2-1)}$$

$$= \frac{1-6(22)}{10(99)}$$

ρ = 0.867

Conclusion

The objective of this paper is to justify the importance of statistics especially correlation. This Paper is an overview of basic definitions and problems on correlation.

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Characteristic Function of Normal Distribution

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ABSTRACT: *The most useful theoretical distribution of discrete random variables are binomial and Poisson distributions but one very good reason for this is that so many physical measurements and natural phenomena have actual observed frequency distribution that closely resembles the "NORMAL DISTRIBUTION". In fact the normal distribution plays a central role in statistical theory and practice particularly in the area of statistical reference and can be regarded as cornerstone of modern statistics. In this paper we are going to discuss the "Characteristic Function Of Normal Distribution".*

1. Introduction

Statistics refers to the study and research into the theory and principles underlying statistical methods. There is another meaning of "statistics" for those who are familiar with statistical principles and methods. Normal distribution is the most important probability distribution. It was investigated first in the 18th century, when scientist observed an astonishing degree of regularity in errors of measurement. It is a continuous distribution. The normal distribution occupies a prominent place in statistics because of its elegant properties and applicability to many practical situations. Normal distribution plays a vital role in sampling theory for drawing conclusion about point estimate of parameters. Normal distribution be derived as the limit of the binomial distribution under suitable statistical conditions namely, the sample size n is large and neither p nor q is small. The normal distribution was first discovered by the English mathematician De moivre (1667-1754). It was later rediscovered by the French mathematician Laplace (1749-1827). It was extensively developed by German mathematician Gauss (1777-1855). A normal distribution is determined by the parameters mean and standard deviation. For different values of mean and standard deviation we get different normal distribution. The normal distribution is the most frequently used of all probability distributions.

Definition: (Normal Distribution)

A random variable X is said to be normal distribution with mean μ and variance σ^2 is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2\left(\frac{1}{2\sigma^2}\right)} \qquad (-\infty < x < \infty)$$

Where,

- $\sigma > 0$
- μ = mean
- σ = standard deviation
- x = values of the continuous random variable

Standard Normal Variable:

If X is normally distributed random variable μ and σ are respectively its mean (μ)and standard deviation (σ) then,

$Z = \frac{(x-\mu)}{\sigma}$ is standard normal variable

Remarks:

A normal distribution can have different shapes depending on different values of μ and σ. But there is one and only one normal distribution for any given pair of values for μ and σ.

Normal distribution is a limiting case of binomial when,

- $n \to \infty$ and neither p nor q is very small
- normal distribution is a limiting case of Poisson distribution, when its mean m is large
- the mean of normally distributed population lies at the centre of the normal curve

Properties

- The graph of the distribution is symmetrical about x=μ
- The mean, median, mode coincide at x=μ and it is equal to $\frac{1}{\sigma\sqrt{2\pi}}$
- The normal curve approaches the x-axis a symmetrically on other side of the origin
- It has points of inflection at x=μ$\pm\sigma$

Characteristic Function:

Let X be a continuous random variable with probability density function f(x) and t be the real number then the characteristic function X is defined by,

ϕ(t)= E(e^{itx})

= $\int_{-\infty}^{\infty} e^{itx}$ f(x) dx

Now using normal distribution,

F(x) = $\frac{1}{\sigma\sqrt{2\pi}} \quad e^{-(x-\mu)^2\frac{1}{2\sigma^2}}$ dx

We know that,

Characteristic function,

ϕ_x(t)= E (e^{itx})

=$\int_{-\infty}^{\infty} e^{itx}$ f(x) dx

= $\int_{-\infty}^{\infty} e^{itx} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2\frac{1}{2\sigma^2}}$ dx

Put $\frac{x-\mu}{\sigma}$ = u

X-μ = u σ

x = u σ+μ

Differentiating with respect to 'u'

dx = σ du

$$\phi_{x(t)} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{it(\sigma u+\mu)} e^{-\sigma^2 u^2\frac{1}{2\sigma^2}} \, du \, \sigma$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{it\sigma u+it\mu} e^{\frac{-u^2}{2}} \, du \, \sigma$$

$$= \frac{1}{\sqrt{2\pi}} e^{it\mu} \int_{-\infty}^{\infty} e^{it\sigma u-\frac{u^2}{2}} \, du$$

$$= \frac{1}{\sqrt{2\pi}} e^{it\mu} \int_{-\infty}^{\infty} e^{(-2it\sigma u+u^2)\frac{-1}{2}} \, du$$

Add and subtract $\frac{\sigma^2 i^2 t^2}{2}$ we get,

$$\phi_{x(t)} = \frac{1}{\sqrt{2\pi}} e^{it\mu} \int_{-\infty}^{\infty} e^{\frac{-1}{2}(u^2-2it\sigma u)-\frac{\sigma^2 i^2 t^2}{2}+\frac{\sigma^2 i^2 t^2}{2}} \, du$$

$$= \frac{1}{\sqrt{2\pi}} e^{it\mu} \int_{-\infty}^{\infty} e^{\frac{-1}{2}(\frac{u^2}{1}-\frac{2it\sigma u}{1}+\frac{\sigma^2 i^2 t^2}{2})} e^{\frac{\sigma^2 i^2 t^2}{2}} \, du$$

$$= \frac{1}{\sqrt{2\pi}} e^{it\mu} \int_{-\infty}^{\infty} e^{\frac{-1}{2}(\mu-\sigma it)^2} e^{\frac{-\sigma^2 t^2}{2}} \, du$$

$$= e^{it\mu} e^{\frac{-\sigma^2 t^2}{2}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-1}{2}(\mu-\sigma it)^2} \, du$$

$$= e^{it\mu} e^{\frac{-\sigma^2 t^2}{2}}$$

$$\phi_{x(t)} = e^{\frac{2it\mu-\sigma^2 t^2}{2}}$$

Differentiate with respect to't'

$\phi'_{x(t)} = e^{\frac{1}{2}(2it\mu-\sigma^2t^2)} \frac{1}{2}(2it\mu-2\sigma^2t)$

Put t = 0

$\phi'_{x(t)} = e^{\frac{1}{2}(2i(0)\mu-\sigma^2(0)^2)} \frac{1}{2}(2i\mu-2\sigma^2(0))$

$= i\mu$

$\frac{\phi_{x(0)}}{i} = \mu$

$\phi''_{x(t)} = e^{\frac{1}{2}(2it\mu-\sigma^2t^2)} (\frac{1}{2}(2i\mu-2\sigma^2t^2)^2 + e^{\frac{1}{2}(2it\mu-\sigma^2t^2)} \frac{1}{2}(0-2\sigma^2)$

Put t = 0

$\phi''_{x(0)} = e^0 (\frac{1}{4}(4i^2\mu^2) + e^0(i^2\sigma^2)$

$= i^2(\mu^2+\sigma^2)$

$\frac{\phi''_{x(0)}}{i^2} = \mu^2+\sigma^2$

Mean : $\mu_1 = 0$

Variance : $\mu_2 = \sigma^2$

NOTE:

If $\alpha_1=\alpha_2=$…………………α_{n-1}. We get the additive property that the sum of n independent normal random variables is also a normal random variable.

Appilication:

- Normal distribution comes as best approximation for the actual observed frequency distribution of many phenomena.
- Normal distribution also finds considerable application in the theory of statistical quality control.
- Normal distribution will be close approximation to distribution of heights or weights individuals in homogeneous population.

Conclusion

In this paper we have discussed the characteristic function of normal distribution.

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Moment Generating Function of Poisson Distribution in Mathematical Statistics

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ABSTRACT: *Computing probabilities by direct use of the binomial distribution for large number of trials is a long and tedious task. On the other hand, the Poisson distribution with less demanding computations can be used as an approximations to the binomial distribution. Poisson distribution is derived as an approximation of the binomial distribution, rather than from the Poisson process. In this paper, we have discuss the Poisson distribution and also compute the mean and variance by using the moment generating function.*

1. Introduction

Poisson distribution is a discrete probability distribution. It was developed by a French Mathematician, Simeon Denis Poisson (1781-1840), in 1837. Poisson distribution maybe expected in cases where the chance of any individual event being a success is small. The distribution is used to describe the behaviour of rare events such as the number of accidents on roads, number of printing mistakes in a book, etc., and has been called "THE LAW OF IMPROBABLE EVENTS". In recent years the statisticians had a renewed interest in the occurrence of comparatively rare events, such as serious floods, accidental release of radiation from a nuclear reactor. The Poisson distribution is used in practice in a wide variety of problems where there are infrequently occurring events with respect to time, area, volume or similar units. One great advantage of the Poisson distribution is that we need only the value of mean in order to compute the values of various constants. The Poisson distribution is based on the same assumptions as the binomial distribution. This means that in the Poisson experiment we deal with either success or failure, that the success throughout the entire process remains constant. The sub intervals should be sufficiently small so that no more than one success can occur in a sub interval. The number of sub intervals will be the number of trials.

Definition : (Poisson Distribution)

The approximation of binomial when 'n' is large and p is close to zero is called the POISSON DISTRIBUTION. The Poisson distribution is defined as

$$P(r) = \frac{e^{-m}.m^r}{r!}$$

Where,
r = any positive integer for which probable frequency is to be calculated (0, 1, 2,....)
e = the base of the natural logarithms and has a value of 2.7183
m = positive constant equal to the mean of the distribution.

Derivation of Poisson Distribution:

In case of binomial distribution the probability of 'r' successes is given by,

$$P(r) = n c_r \, p^r \, q^{(n-r)}$$

$$= \frac{n(n-1)(n-2)\,.....(n-r+1)}{r!} \, p^r q^{(n-r)}$$

Put, $p = \frac{m}{n}$ and $q = 1\text{-}p = 1\text{-} \frac{m}{n}$

Thus we get,

$$P(r) = \frac{n(n-1)(n-2)\,......(n-r+1)}{r!} \left(\frac{m}{n}\right)^r \left(1-\frac{m}{n}\right)^{n-r}$$

$$= \frac{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)....\left(1-\frac{r-1}{n}\right)}{r!} \, \frac{m^r\left(1-\frac{m}{n}\right)^n}{\left(1-\frac{m}{n}\right)^r}$$

For fixed r, as n$\to \infty$
$\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{r-1}{n}\right)$ $(1-\frac{m}{n})^r$ all tend to 1 and $(1-\frac{m}{n})^n$ to e^{-m}.

Thus in the limiting case,
P(r) = $\frac{e^{-m}.m^r}{r!}$

This is called the "POISSON PROBABILITY DISTRIBUTION".

Note:
The Poisson distribution is characterized by a single parameter 'm'.

Form of Poisson Distribution:
Just like binomial distribution, the variate of the Poisson distribution is also discrete one, that is, it takes only integral values. The probabilities of 0, 1, 2,….. Successes can be found out by successive terms of the expansion:

P = $e^{-m}\left[1+\frac{m}{1!}+\frac{m^2}{2!}+\cdots+\frac{m^r}{r!}+\cdots\right]$

Moment Generating Function:
Let r be a discrete random variable with probability distribution. Then the function M_r, called the moment generating function of r is defined by

$M_r(t) = \sum e^{tr}\ p(r)$, If it exists.

Moment Generating Function of Poisson Distribution:
The Poisson distribution is given by,
$P(r) = \frac{e^{-m}.m^r}{r!}, r = 0,1,2,\ldots\ \ M_r(t) = \sum e^{tr} P(r)$
$= \sum_{r=0}^{\infty} e^{tr}\ \frac{e^{-m}m^r}{r!}$
$= e^{-m}\ \sum_{r=0}^{\infty} \frac{(e^t m)^r}{r!}$
$= e^{-m}.(e^{me^t})$
$M_r(t) = e^{m(e^t-1)}$

Which is the moment generating function of Poisson distribution. And also we can compute the mean and variance of Poisson distribution from the moment generating function.
MEAN : $\mu_1 = m$
VARIANCE: $\mu_2 = m$

Role of the Poisson Distribtion:
- Number of traffic arrivals such as trucks at terminals, aeroplanes at airports, ships at docks and so forth.
- In problems dealings with the inspection of manufactured products with the probability that any one piece is defective is very small and the lots are very large and to model the distribution of the number of persons joining a queue (a line) to receive a service or purchase of a product.
- In general, the Poisson distribution explains the behaviour of those discrete variates where the probabilities of occurrence of the event is small and the total number of possible cases is sufficiently large.

Examples:
Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2% of such fuses are defective ($e^{-4} = 0.0183$).

Solution:
Given that,
P = probability that a fuse is defective = $\frac{2}{100}$
n = 200
m = np =4
Probability that at most 5 defectives will be found in a box of 200 fuses.

$P(r \leq 5) = e^{-4}\left(1 + \frac{4}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!}\right)$
$= (0.0183)\left(\frac{643}{15}\right)$
$P(r \leq 5) = 0.785$

Conclusion:
In this paper, we discussed the moment generating function of Poisson distribution. This distribution plays a vital role and some of them are noted.

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Derivation for Bessel's Formula in Numerical Methods

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ABSTRACT: *Many famous mathematicians have their names associated with procedures for interpolation: Gauss, Newton, Bessel, and Stirling. The need to interpolate began with the early studies of astronomy, when the motion of heavenly bodies was to be determined from periodic observations. Interpolation technique is used in various disciplines like business, economics, population studies, price determination, etc.,*
In this paper, we are going to discuss the derivation for Bessel's formula using Gauss forward interpolation formula.

Keywords: *Δ= forward Del, interpolation*

1. Introduction

When using numerical methods or algorithms and computing with finite precision, errors of approximation or rounding and truncation are introduced.
The estimation of values between well-known discrete points are called interpolation. Interpolation is the process of finding the most appropriate estimate for missing data. For making the most probable estimate, it requires the following assumptions.

- The frequency distribution is normal and is not marked by sudden ups and downs.
- The changes in the series are uniform within a period. It is used to fill in the gaps in the statistical data for the sake of continuity of information.

The Bessel functions have been known since the 18th century when mathematics and scientists started to describe physical processes through differential equations.
This is very useful formula for practical interpolation, and its uses the differences as shown in the following table where the brackets mean that the average of the values has to be taken.

..
..
..
$x_1 y_{-1}$

$$\begin{matrix} x_0 \\ x_1 \end{matrix} \quad \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} \Delta y_0 \quad \begin{pmatrix} \Delta^2 y_{-1} \\ \Delta^2 y_0 \end{pmatrix} \Delta^3 y_{-1} \quad \begin{pmatrix} \Delta^4 y_{-2} \\ \Delta^4 y_{-1} \end{pmatrix} \Delta^5 y_{-2} \quad \begin{pmatrix} \Delta^6 y_{-3} \\ \Delta^6 y_{-2} \end{pmatrix}$$

..
..
..

Definition

There are many special functions which arise as solutions to differential equations. Here we will look at how one important class of functions, Bessel functions, arise through a series solution to a differential equation by using gauss forward interpolation formula.

$$y = y_0 + uc_1 \Delta y_0 + uc_2 \Delta^2 y_{-1} + (u+1)c_3 \Delta^3 y_{-1} + \cdots$$

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \cdots$$

Derivation for Bessel's Formula

By using Gauss forward formula,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \cdots$$

We know that,

$$\Delta y_0 = y_1 - y_0$$

$$y_0 = y_1 - \Delta y_0 \quad \rightarrow (a)$$

$$\Delta y_{-1} = y_0 - \Delta y_{-1}$$

$y_{-1} = y_0 - \Delta y_{-1}$

multiply by Δ, $\Rightarrow \Delta y_{-1} = \Delta y_0 - \Delta^2 y_{-1}$

Multiply by Δ^2, $\Rightarrow \Delta^2 y_{-1} = \Delta^2 y_{-1} - \Delta^3 y_{-1} \quad \rightarrow (b)$

$$y(x) = \frac{y_0}{2} + \frac{y_0}{2} + u\Delta y_0 + \frac{u(u-1)}{2!}\frac{\Delta^2 y_{-1}}{2} + \frac{u(u-1)}{2!}\frac{\Delta^2 y_{-1}}{2} + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-1} + \ldots$$

$$y(x) = \frac{y_0}{2} + \frac{1}{2}(y_1 - \Delta y_0) + u\Delta y_0 + \frac{u(u-1)}{2!}\frac{\Delta^2 y_{-1}}{2} + \frac{u(u-1)}{2!}\frac{1}{2}(\Delta^2 y_0 - \Delta^3 y_{-1}) + \frac{u(u+1)(u-1)}{3!}\Delta^3 y_{-1} + \cdots$$

$$y(x) = \frac{y_0}{2} + \frac{y_1}{2} - \frac{\Delta y_0}{2} + u\Delta y_0 + \frac{u(u-1)}{2!}\frac{\Delta^2 y_{-1}}{2} + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_0}{2} - \frac{\Delta^3 y_{-1}}{2}\right] + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \cdots$$

$$y(x) = \frac{y_0}{2} + \frac{y_1}{2} - \frac{\Delta y_0}{2} + u\Delta y_0 + \frac{u(u-1)}{2!}\frac{\Delta^2 y_{-1}}{2} + \frac{u(u-1)}{2!}\frac{\Delta^2 y_0}{2} - \frac{u(u-1)}{2!}\frac{\Delta^3 y_{-1}}{2} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \cdots$$

$$y(x) = \frac{y_0 + y_1}{2} + \Delta y_0\left(u - \frac{1}{2}\right) + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \Delta^3 y_{-1}\left[\frac{(u+1)u(u-1)}{3!} - \frac{u(u-1)}{2! \times 2}\right] + \cdots$$

$$y(x) = \frac{y_0 + y_1}{2} + \Delta y_0\left(u - \frac{1}{2}\right) + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \frac{u(u-1)}{2!}\left(\frac{(u+1)}{3} - \frac{1}{2}\right)\Delta^3 y_{-1} + \cdots$$

$$y(x) = \frac{y_0 + y_1}{2} + \Delta y_0\left(u - \frac{1}{2}\right) + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \frac{u(u-1)}{2!}\left(\frac{2u+2-3}{6}\right)\Delta^3 y_{-1} + \cdots$$

$$y(x) = \frac{y_0 + y_1}{2} + \Delta y_0\left(u - \frac{1}{2}\right) + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \frac{u(u-1)}{2!}\left(\frac{2u-1}{6}\right)\Delta^3 y_{-1} + \cdots$$

$$y(x) = \frac{y_0 + y_1}{2} + \Delta y_0\left(u - \frac{1}{2}\right) + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \frac{u(u-1)}{2!}\frac{2(u-\frac{1}{2})}{6}\Delta^3 y_{-1} + \cdots$$

$$y(x) = \frac{y_0 + y_1}{2} + \Delta y_0\left(u - \frac{1}{2}\right) + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \frac{u(u-1)(u-\frac{1}{2})}{6}\Delta^3 y_{-1} + \cdots$$

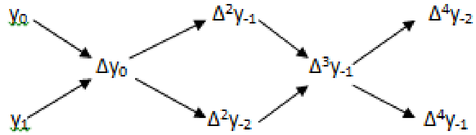
Bessel's formula:

$$y(x) = \frac{y_0 + y_1}{2} + \Delta y_0\left(u - \frac{1}{2}\right) + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \frac{u(u-1)(u-\frac{1}{2})}{6}\Delta^3 y_{-1} + \cdots$$

Note

This formula involves the odd differences below the central line and averages of the even differences above the central line.

i.e.,



Example:
From the following table find y_{25}. Using Bessel formula

X	10	20	30	40
Y	564	446	343	234

Solution:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
10	564	-118		
20	446	-103	15	
30	343	-109	-6	-21
40	234			

u = $\frac{(x-x_0)}{h}$ = $\frac{(25-20)}{10} = \frac{5}{10} = 0.5$

By Bessel's formula,

$$y(x) = \frac{y_0 + y_1}{2} + \Delta y_0 \left(u - \frac{1}{2}\right) + \frac{u(u-1)}{2!}\left[\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right] + \frac{u(u-1)(u-\frac{1}{2})}{6}\Delta^3 y_{-1} + \cdots$$

$$y_{25} = \frac{446+343}{2} + \left(0.5 - \frac{1}{2}\right)(-103) + \frac{(0.5)(0.5-1)}{2!}\frac{(15-6)}{2} + \frac{(0.5)(0.5-1)(0.5-\frac{1}{2})}{3!}(-2)$$

$$y_{25} = 394.5 + \frac{(0.5)(-0.5)}{2}\left(\frac{9}{2}\right)$$

$$y_{25} = 394.5 + (-0.125)(4.5)$$

$$y_{25} = 394.5 - 0.5625$$

y_{25} =393.44

Applications of Bessel Functions:
Applications of Bessel functions include mechanics, electrodynamics, electro engineering, solid state physics, and celestial mechanics.

Conclusion
In this paper we discussed the derivation for Bessel's formula by gauss forward interpolation formula with solved examples.

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Application of Queuing theory in Signal Processing

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ABSTRACT: *In this paper M/M/1/ /FCFS, Static Queue system, its characteristics , steady state solution of this queue model, parallel channels , analogous to continuous system, how the data arriving as packets, standard parametric Estimation of due to delay in arrivals of signal is discussed.*

Keywords: *Birth and Death Process, Markov Chain, Data Packets*

1. Introduction

1.1 Model Defintion and Assumption

Birth-and –Death Process

Markovian(or) Exponential inter arrival time or Service time number of arrivals is described by a poisson probability distribution, the service time distribution , a Continuous time Markov chain is a birth-death process in state I can make transtion only to stats i-1 or i+1, Birth and death process the state represent the number of data packets . Data set from I to i+1 is a Birth since the buffer increases by one. A transion from 1 to i-1 is considered as death.

1.2 Service Rate, Arrival Rate

The transition probability qi,i-1 is denoted by ui and is called the Service Rate, in state i since the transition from i to i-1 , occurs only if a Data arrival complete to reach the transmission as signals ^I = Qi,i+1 is called the arrival Rate in state I since a transmission from State I to i+1, corresponding to the arrival of a Signal. The birth and death process can be described in terms of the signal transition Rates ^i and Ui

2. Analysis

2.1 Markov Chain

Markov chain , in which xn+1 depends on xn but not on the earlier values X0 .....Xn-1 of the random sequence, Poisson Process , Birth and death Process , to predict the next variable xn+1 in the random sequence, we call xn is called the set of States(or) State space, to find n step transition probabilities are given by the Matrix P(n), which has I,j the element

Pij(n) = P(xn+m =j/xm=i)

The I,j the elment of p(n) tells us the probability of going from state I to state j in exactly n steps. For n =1, p(1) =P, the state transition Matrix.

2.2 Chapman Kolmogrov Equation

The chapman –Kolmogorov Equation gives a recursive procedure for calculculating the n-step transition probabilities, the quations are based on the observations, that going from I to j in n+m steps required being in some state k after n steps. For a finite Markov Chain, the n step transion probability satisfy Pij(n+m) = Pik(n)Pkj(m), P(n+m) = P(n)P(m)

2.3 Two State Transition Matrix

P = 1-p p
q 1-q

The n- step transion matrix is a complete description of the evolution of probabilities in Markov Chain, given that the system is in state i. We learn the probability the system is in State j n step is pij(n), the n step transion matrix provides more information than we need. When working with a MC Xn/n>o}, we may need to know only the state probabilities P{xn=i}, since each Xn is a random Variable, we could express the set of probabilities in terms of PMF Pxn(i)

3. Conclusion

For a birth-death queue with arrival rates ^I and Service rates ui, the stationary probabilities pi satisfy pi-1, ^i-1 = pi,ui, Pi =1

A s recalling Anologus to Discrete time , In each time slot, a route can either store an arriving data packet in its buffer or forward a stored packet(and remove that packet from its buffer? In each time slot, a new packet arrives with probability p, independent of arrivals in all other slots. This packet is stored as long as the router is storing fewer than c packets. If c packets are already buffered, then the new packets is discarded by the routes. If no new packets arrives and n<o, Packets are buffered by sthe routes, then the router will forwad one buffered packet. That packet is then removed from the buffer. Let Xn denotes number of buffered packets a t time n

The MC for the packet buffer shouw S-S', partion we use to calculate the Stationary probabilities. For a finite Markov Chain with transition Markov Chain with transion Matrix P, the n-step transition Matrix is P(n) =pn. In the figure shows the S-S', yielding

IIi+1 = P/1-P IIi ... (1)

Since equation holds, for i= 0,1 …c-1, we have that Iii= II0li where l = 1/1-p, Requiring the state probabilities to sum to I, We have

Iii = II0 li = II0 1-l/1-l =1

The complete state probabilities are II = 1-l/1-l c-1.l1 , I = 0,1,2,…c

Suppose that the router can either store an arriving data packet in its buffer or forwad a stored packet(and remove the packet from its buffer.

In each time slot, a new packet is stored with probability, p1 independent of arrivals in all other slots.

While computer software package is developed a testing procedure is often put into effect to eliminate the failure on data transmission from new packet arrivals, then one packet will be removed from the buffer and forwarded sketch the Markov chain leads to Statistical Pattern Recongnition and Image processing, Application Queueing theory is essential.

Acknowledgement

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