An EOQ model of Special Sales Promotional Incentive Scheme with planned shortages and equivalent Holding and shortages cost.

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ABSTRACT: In this research paper, the conclusion of the sales promotional incentive scheme with planned shortages obtainable by proportional to the order quantity when demand is certain and stock out are permitted. It is assumed that the holding cost does not change for any order quantity of the retailer. This study will help the supplier to take the decision whether to supply a regular or special stock of quantity for profit. The optimum quantity is dependingon equivalent holding and shortages cost during that period. It include the numerical examples are related to the derived results. The hypothetical numerical study is also part of inventory EOQ model parameter.

Key Words: EOQ model, promotional incentive scheme, planned shortages, holding cost, shortages cost.

INTRODUCTION
In classical inventory EOQ model the offer different market strategies of special sales by the supplier increase the demand, attracts more retailers, gain maximum profit ofinventory. But then the problem, “Is it always beneficial to profit of special order quantity ?” in real life situation many times vendor announce special sales promotional scheme for a short term period.one such scheme is price discount in which vendor gives a special price reduction on the unit cost of the ordered quantity during a specified period .Sometimes vendor offer another equivalent scheme in which vendor does not give a discount on the unit cost but vendor supply some more units proportional to the order quantity free of cost. Such scheme is available for short term period and naturally customer will like to buy larger amount of unit, during the short tenure of the scheme, but the purchases of too huge amount of units will significantly growth the inventory holding cost. When shortages are planned earlier then avoid the problem of deciding what optimal order size to be placed during such sales promotional scheme to get maximum profit in future.

Dixit and Shah (2005) contributed a review article on inventory models when a temporary price discount is offered by the supplier to the seller to study the relationship between price discounts and demand policy. The all-unit extent discounts organization policy is argued by Arcelus et al. (2003), Shah et al. (2005), Bhaba and Mahmood (2006), Abad (2007), Dye et al. (2007), Shah et al. (2008), Mishra and Shah (2009). They supposed that the price discount rate is independent of the special order quantity. Though, in market, it is detected that the supplier offers a quantity discount to inspirebigger orders. For the larger order, the higher price discount rate is given by the supplier. Such as a outcome, the retailer has to settle the trade-off for securing price savings in contrast toadvanced total holding cost. Ouyang et al. (2009) conversed the sound of a provisional price rebate on a retailer's demand policy by assuming that the price discount rate is linked to special order quantity. They assumed that the demand is constant and deterministic.

In this EOQ model, we define through the impression of planned shortages on such variety of sales promotional incentive scheme when stock out are permitted. This study will help the vendor to take decision about stock of quantity during sales promotion tool. A mathematical model for this model is defined. Here assumed that if order for Q units during the period of sale promotional scheme the supplier supply (1+b) Qˈunits.  The measure of effectiveness to be used will be the profit due to large amount of purchase.

Assumptions:-
1. The demand rate units per time unit is certain, known & constant.
2. Lead time is fix
3. Holding Cost (C₁) per order is per time unit constant and does not change for different order quantity.
4. Stock act are permitted and shortages or Back ordering cost (C₂) per unit known and constant.
5. Replenishment rate is infinite.
6. The ordering cost ($C_3$) independent and constant during the period under consideration.
7. Through the sales promotional scheme at the peak one order should be placed.
8. The unit cost ($C$) does not depend upon the quantity ordered or received and is constant during the period under consideration.

**Notations:**

- $Q$: Order quantity
- $Q':$ Special order Quantity in units.
- $Q^*$: The Economic order quantity per units
- $C_1$: The inventory holding cost charges special of interest Charges per unit
- $C_2$: The inventory shortages or back-ordering cost.
- $C_3$: The Replenishment cost per order
- $R$: Inventory fix rate
- $I_c$: The interest charges per rupee investment.
- $T_o$: The time at which the sales promotional scheme will be started.
- $T_1$: The time of which the sales promotional Scheme will be ended.
- $(1+b)Q:=$ The System received units.
- $D$: The annual demand rate
- $d$: Uniformly demand rate
- $M$: Maximum Inventory level for EOQ
- $M' =$ Maximum Inventory level for special order quantity
- $S$: Maximum shortages level or maximum back-order quantity units
- $L_T$: Lead time
- $N$: Number of Quantity received per unit
- $F_b$: Factor for equivalent Holding cost for back ordering.
- $E C_1 C_2 =$ Equivalent Holding and shortages cost when Back order are permitted under Special Sales promotional Incentive Scheme
- $Q-S$: Shortages before quantity added.

**The mathematical model:**

This EOQ model deal with special sales promotional scheme with planned shortages

(*) Holding Cost:--

- During period 'To', Inventory fix rate 'R'. $Q^* = EOQ$
- Order Quantity: $Q$ and Special order Quantity: $Q'$
- Cycle during period: $D/Q$

Maximum Inventory for EOQ: $M = Q^* - S = R T_o$

Maximum Inventory for special sale quantity: $M' = Q' - S = R T_o$

Average Inventory During $T_o = \frac{R T_o}{2} = \frac{Q^* - S}{2}$

Average Inventory During $T_o = \frac{R T_o}{2} = \frac{Q' - S}{2}$

$M = \frac{R T_o}{T} = \frac{T o}{T}$

$M = \frac{d T o}{d T} = \frac{T o}{T}$

$MT = \frac{T (Q' - S)}{Q}$ Heres $N = \frac{1}{T}; NT = 1$

$MT = \frac{T (Q^* - S)}{Q}$ Heres $N = \frac{1}{T}; NT = 1$

$To = \frac{Q}{Q} = \frac{Q}{Q}$
$$C_1(Q') = \left[ \frac{Q' - S}{2} \right] \left[ C_1 T_o \right]$$

$$C_1 (Q') = (\text{Average inventory}), \text{ (Inventory Holding cost per unit time } T_o \text{ for EOQ})$$

$$C_1 (Q') = \left[ \frac{Q' - S}{2} \right] \left[ C_1 T_o \right]$$

(*) **Annual Holding Cost: - special sale**

$$C_1(Q') = \left( \frac{\text{Holding cost per cycle}}{\text{cycle per year}} \right)$$

$$= \left[ \frac{Q' - S}{2} \right] \left[ C_1 T_o \right] N = \left[ \frac{Q' - S}{2} \right] \left[ C_1 \right] \left[ \frac{T(Q' - S)}{Q} \right] N$$

$$= \frac{(Q' - S)^2 C_1 N T}{2Q}$$

$$= \frac{(Q' - S)^2 C_1}{2Q}$$

**Annual Holding Cost: - EOQ**

$$C_1(Q^*) = \left( \frac{\text{Holding cost per cycle}}{\text{cycle per year}} \right)$$

$$= \left[ \frac{Q^* - S}{2} \right] \left[ C_1 T_o \right] N = \left[ \frac{Q^* - S}{2} \right] \left[ C_1 \right] \left[ \frac{T(Q^* - S)}{Q} \right] N$$

$$= \frac{(Q^* - S)^2 C_1 N T}{2Q}$$

$$= \frac{(Q^* - S)^2 C_1}{2Q}$$

(*) **Shortages cost:**

-- For the period of 'T1' shortages up rate 'R' maximum shortages S = R T1

$$\frac{R T_1}{S} = \frac{s}{2}$$

$$S = R T_1 & Q = R T, T = T_o + T_1$$

$$\frac{S}{Q} = \frac{R T_1}{R T} = \frac{T_1}{T} = \frac{S T}{Q}$$

$$C_2 (Q) = (\text{average shortages}) (\text{shortages cost / unit for time } T_1)$$

$$\frac{S}{2} C_2 T_1$$

(*) **Annual shortages cost:**

$$C_2 (Q) = \left( \frac{\text{shortages cost}}{\text{cycle}} \right) \left( \frac{\text{cycle}}{\text{year}} \right)$$

$$= \left( \frac{S}{2} \right)(C_2 T_1) . N$$

$$= \left( \frac{S}{2} \right) C_2 \left( \frac{S T}{D} \right) . N$$
Before the time $T_0$ an order of $Q^*$ units is to be placed every $\frac{Q^*}{D}$ units of time and the total cost of the system with equivalent holding and shortages cost and interest charges will be

$$K(Q^*) = C.Q^* + \sqrt{2C_3D}bC_1eb$$

The objective of units model is to determine special order size $Q^*$ to get maximum profit. Let $K(Q')$ denotes the total cost of the system during period $T_0$ to $T_1$. When an order of $Q'$ units is placed at time $T$ and the system receives $(1+b)Q'$ units with planned shortages.

$$K(Q') = CQ' + \left(\frac{(Q^*-S)^2}{2DQ}C_1Q'^2 + \frac{C_2S^2}{2Q} + \frac{C_3D}{Q}\right)\left(\frac{Q^*}{D}\right)$$

$$K(Q') = CQ' + \left[(1+b)^2\frac{(Q^*-S)^2}{2DQ}(C_1 + Clc + C_2 S^2)\right] + \frac{C_2S^2}{2Q} + \frac{C_3D}{Q}$$

The profit $P_1(Q')$ due to taking advantage of the special sale promotional scheme is

$$P_1(Q') = \left(\frac{(Q^* - S)^2}{2DQ}C_1Q'^2 + \frac{C_2S^2}{2Q} + \frac{C_3D}{Q}\right)\left(\frac{Q^*}{D}\right) - \left(\frac{(1+b)Q'}{D}\right)$$

$$P_1(Q') = \left(\frac{(Q^* - S)^2}{2DQ}(C_1 + Clc + C_2 S^2)\right)\left(\frac{Q^*}{D}\right) - \left(\frac{(1+b)Q'}{D}\right)$$
For maximization, \( \frac{\partial P_1(Q')}{\partial Q'} = 0 \) gives

\[
bC + \frac{(1 + b)Q'}{DQ} \sqrt{2[(Q' - S)^2(C_1 + Clc + C_2S^2)]C_3D} - \frac{(1 + b)^2 [2(Q' - S)C_1 + Clc + C_2S^2]}{DQ} = 0
\]

\[Q^* = \frac{bCDQ}{(1 + b)^2[(Q' - S)^2C_1 + Clc + C_2S^2]} + \frac{(1 + b)^2[(Q' - S)^2C_1 + Clc + C_2S^2]}{(1 + b)^2[(Q' - S)^2C_1 + Clc + C_2S^2]}
\]

And the maximum profit will be

\[
P_1(Q'^*) = bC + \frac{(1 + b)Q'}{DQ} \sqrt{2[(Q'^* - S)^2(C_1 + Clc + C_2S^2)]C_3D} - \frac{(1 + b)^2 [2(Q'^* - S)C_1 + Clc + C_2S^2]}{DQ} - \frac{C_3D}{Q'}Q'^*
\]

**Special cases:**

Consider the case, when an order of Q* units is placed during the short tenure of sales scheme. Then the model is

Let \( K_2(Q^*) \) denotes the total cost of the system during To to T1 when an order of Q* unit is placed at time To and then at the end of time unit several orders of Q* units are placed at every units of time

\[
K_2(Q^*) = \frac{(1 + b)Q^*}{D} + \frac{[(1 + b)^2[(Q^* - S)^2C_1 + Clc + C_2S^2]}{2DQ} \frac{C_3D}{Q'} + \frac{[(1 + b)[(Q^* - S)^2C_1 + Clc]Q^*(Q' - Q^*)]}{2DQ} + \frac{(1 + b)C_3D(Q' - Q^*)}{QQ^*}
\]
The profit due to purchasing large quantity

\[
P_2(Q^*) = K_2(Q^*) - K(Q^*)
\]

\[
P_2(Q^*) = \left[ \frac{bc + \frac{(1 + b)}{2DQ} \sqrt{2[(Q^* - S)^2 C_1 + Clc + C_2 S^2]C_1D}}{Q^*} \right] Q^* - \left[ \frac{(1 + b)^2 2(Q^* - S)C_1 + Clc + C_2 S^2}{2DQ} \right] Q^* - \frac{C_3 D}{Q} - b \left[ K(Q^*) - \frac{(2 + b)(Q^* - S)^2 C_1 Q^*}{2DQ} \right]
\]
\[
\left[ \frac{(1 + b)^2 (Q^* - S)^2 C_1 + C1c}{2DQ} \right] Q' - \frac{C_3 D}{Q}
\]
\[
P_2(Q') = \left[ bC + (1 + b) \sqrt{2(Q^* - S)^2 (C_1 + C1c + C_2 S^2)}C_3 D \right] Q' - \]
\[
\left[ \frac{(1 + b)^2 (Q' - S)^2 C_1 + C1c}{2DQ} \right] Q' - \frac{2C_3 D}{Q}
\]
\[
\frac{\partial P_2(Q')}{\partial Q'} = 0 \text{ Gives}
\]

For maximization
\[
bc + \frac{(1 + b)}{DQ} \sqrt{2 [(Q^* - S)^2 C_1 + C1c + C_2 S^2]}C_3 D - \]
\[
\left[ \frac{(1 + b)^2 [2(Q' - S)^2 C_1 + C1c + C_2 S^2]}{DQ} \right] = 0
\]
\[
Q^* = \frac{bCDQ}{(1 + b)^2 [2(Q' - S)^2 C_1 + C1c + C_2 S^2] +}
\]
\[
(1 + b) \sqrt{2 [(Q^* - S)^2 C_1 + C1c + C_2 S^2]}C_3 D
\]
\[
(1 + b)^2 [2(Q' - S)^2 C_1 + C1c + C_2 S^2]}
\]
And, the optimum maximum profit is
\[
P_2(Q^*) = \left[ bc + \frac{(1 + b)}{DQ} \sqrt{2 [(Q^* - S)^2 C_1 + C1c + C_2 S^2]}C_3 D \right] Q^*1
\]
\[
- b \left[ K(Q^*) - \frac{(2 + b) [(Q^* - S)^2 C_1]}{2DQ} \right] Q^*2 - C_3 D
\]
\[
\text{Here, if } b = 0 \text{ then, } Q^* = \sqrt{\frac{2C_3 D}{[C_1 + C1c + C_2 S^2]}}
\]
And, \(P_2(Q^*) = 0\)
Now, we define the condition for optimum. The cost of item is also include in Total cost of Q* associate for inventory.
\[
K(Q^*) = \frac{\partial K(Q^*)}{\partial S} = \frac{\partial}{\partial S}\left( C_1 Q^* + \frac{(Q^* - S)^2 C_1 + C1c}{2DQ} Q^*2 + S^2 \cdot C_2 \cdot \frac{2Q}{Q} + \frac{C_3 D}{Q} \right)
\]
Equating it to zero
\[ 0 = 0 - \frac{(Q^* - S)(C_1 + Clc)Q^*}{2DQ} + \frac{C_s (2S)}{2Q} \]

This is the optimum condition for S. Hence S is replaced by S*

Now,
\[ \frac{Q^*}{S} = \frac{C_1 + Clc + C_s S}{C_1 + Clc} \]

\[ \frac{Q^*}{Q} = \frac{C_1 + Clc}{C_1 + Clc + C_s S} \]

Factor of Back-ordering:
\[ F_b = \frac{C_2 S}{C_1 + Clc + C_s S} \]

Subtracting LHS and RHS from 1
\[ 1 - F_b = 1 - \frac{C_2 S}{C_1 + Clc + C_s S} = \frac{C_1 + Clc}{C_1 + Clc + C_s S} \]

\[ \text{Equation (2) & (4)} \]
\[ 1 - F_b = \frac{C_1 + Clc + C_s S}{C_1 + Clc + C_s S} \]

\[ S^* = (1 - F_b) Q^* \]

Now, we define the condition for optimum. The cost of item is also including in Total cost of Q’ associate for inventory.

\[ K(Q) = C.Q' + \frac{(1 + b)^2 (Q' - S)^2 C_1 + Clc}{2DQ} Q'^2 + \frac{S^2 C_2 + C_s D}{Q} \]

\[ \frac{\partial K(Q')}{\partial S} = \frac{\partial}{\partial S} \left( C.Q' + \frac{(1 + b)^2 (Q' - S)^2 C_1 + Clc}{2DQ} Q'^2 + \frac{S^2 C_2 + C_s D}{Q} \right) \]

\[ 0 = - \frac{2(1 + b)(Q' - S)(C_1 + Clc)2Q'}{2DQ} + \frac{C_2 (2S)}{2Q} \]

Equating it to zero,
\[ 0 = 0 - \frac{(1 + b)(Q' - S)(C_1 + Clc)Q'}{DQ} + \frac{C_s S}{Q} \]

\[ = - \frac{(bQ'^2 (C_1 + Clc) - S(C_1 + Clc)Q')}{DQ} + \frac{C_s S}{Q} \]

\[ 0 = - \frac{(bQ'^2 (C_1 + Clc) - S(C_1 + Clc)Q')} + C_s S \]
\[(bQ^2 (C_1 + Clc) = S(C_1 + Clc)Q') + C_2 S\]
\[
\frac{Q'}{S} = \frac{C_1 + Clc + C_2 S}{C_1 + Clc}
\]
This is the optimum condition for S. Hence S is replaced by \(S^*\).
Now,
\[
\frac{Q'}{S} = \frac{C_1 + Clc + C_2 S}{C_1 + Clc} - (1) \quad \text{&} \quad \frac{S^*}{Q'} = \frac{C_1 + Clc}{C_1 + Clc + C_2 S} - (2)
\]

* Factor of Back-ordering:-
\[
Fb = \frac{C_2 S}{C_1 + Clc + C_2 S} - (3)
\]
Subtracting LHS and RHS from 1
\[
1 - Fb = 1 - \frac{C_2 S}{C_1 + Clc + C_2 S} = \frac{C_1 + Clc}{C_1 + Clc + C_2 S} - (4)
\]
\[
\text{Eqbs } (2) \text{ & (4)} 1 - Fb = \frac{C_1 + Clc + C_2 S}{C_1 + Clc + C_2 S} = \frac{S^*}{Q'}
\]
\[
\therefore \ S^* = (1 - Fb) Q' \quad \text{--- (5)}
\]
Here,
\[
EC_{1C_2} = C_1 (\frac{Q'}{S}) + C_2 (\frac{Q'}{S})
\]
\[
= \frac{(Q' - S) C_1}{2Q} + \frac{C_2 S^2}{2Q}
\]
Put the value of \(S = S^* = (1 - Fb)\)
\[
= \frac{(Q' - (1 - Fb) Q')^2 C_1}{2Q} + \frac{C_2 ((1 - Fb)Q')^2}{2Q}
\]
\[
= \frac{Q'^2 (1 - (1 - Fb))^2 C_1}{2Q} + \frac{C_2 Q'^2 (1 - Fb)^2}{2Q}
\]
\[
= \frac{Q'^2}{2Q} \left[ (1 - (1 - Fb))^2 \right] C_1 + \frac{C_2 Q'^2 (1 - Fb)^2}{2Q}
\]
\[
= \frac{Q'^2}{2Q} \left[ Fb^2 C_1 + C_2 (1 - Fb)^2 \right]
\]
Putting Fb & (1-Fb) in terms of C_1 & C_2 from (3) & (4)
\[
= \frac{Q'^2}{2Q} \left[ \left( \frac{C_2 S}{C_1 + Clc + C_2 S} \right)^2 C_1 + C_2 \left( \frac{C_1 + Clc}{C_1 + Clc + C_2 S} \right)^2 \right]
\]
\[
= \frac{Q'^2}{2Q} \left[ C_1 C_2 S^2 + (C_1 + Clc)^2 \frac{C_2}{(C_1 + Clc C_2)^2} \right]
\]
\[
= \frac{Q'^2}{2Q} \left[ (C_1 + Clc) C_2 S \left( C_1 + Clc C_2 S \right) \right]
\]
Observe RHS of equation (7). It is annual holding and shortages cost for EOQ model under consideration. Difference is just factor of back ordering or shortages. For EOQ model annual holding cost:

\[ F_b = 1 \quad \text{then} \quad \frac{Q'^2}{2Q} \quad C_1 + C_{Ic} \]

EOQ model do not have shortages cost. So, we define by analogy equation equivalent holding cost for EOQ, \( EC_1 C_2 \) for EOQ model is the product of holding cost with interest charges and factor of back ordering. \( C_{Ieb} \) notes equivalent holding cost for EOQ model.

\[ EC_1 C_2 = \frac{Q'^2}{2Q} \quad C_{Ieb} + C_{Ic} \]

Total annual inventory for EOQ with equivalent holding and shortages cost

\[ K(Q) = \frac{Q'^2}{2Q} (C_{Ieb} + C_{Ic}) + \frac{C_3 D}{Q} \]

In order to minimize the annual total cost, the partial derivative of total cost, \( K(Q) \) with respect to \( Q \)

\[ \frac{dK(Q)}{dQ} = \frac{d}{dQ} \left[ \frac{Q'^2}{2} (C_{Ieb} + C_{Ic}) + \frac{C_3 D}{Q} \right] \]

\[ = \frac{Q'^2}{2} (C_{Ieb} + C_{Ic}) + (-1) \frac{C_3 D}{Q} \]

Equating it to zero

\[ \frac{Q'^2}{2} (C_{Ieb} + C_{Ic}) + (-1) \frac{C_3 D}{Q} = 0 \]

\[ \therefore Q^* = \frac{2C_3 D}{Q'^2 (C_{Ieb} + C_{Ic})} \]

Hence, the quantity \( Q \) found is the minimum cost point optimum quantity denoting it as \( Q^* \)

For EOQ model with \( EC_1 C_2 \)

\[ \therefore Q^* = \sqrt{\frac{2C_3 D}{Q'^2 (C_{Ieb} + C_{Ic})}} \]

Before the time \( T \) an order of \( Q^* \) units is to be placed every \( \frac{Q^*}{D} \) units of time and the total cost of the system with equivalent holding and shortages cost and interest charges will be

\[ K(Q) = CQ^* + \frac{Q'^2 (C_{Ieb} + C_{Ic})Q^*}{2DQ} + \frac{C_3 D}{Q} \]

The objective of unit model is to determine special order size \( Q' \) to get maximum profit.

Let \( K(Q) \) denotes the total cost of the system during period from \( T \) to \( T_1 \). When an order of \( Q' \) units is placed at time \( T \) and the system receives \((1+b)Q' \) units with planned shortages.
Let, $K_3(Q^*)$ denotes the total cost of the system during the period $T_0$ to $T_1$ when no special order is placed at during the tenure of sale promotional scheme $Q^*$ units are made at every $\frac{Q^*}{D}$ units of time.

$$K_3(Q^*) = \left[ \left( 1 + \frac{Q^*}{Q} \right)^2 \frac{Q'^2(C_{1eb} + Clc)Q'^2}{2DQ} + \frac{C_3D}{Q} \right] \left( 1 + b \frac{Q^*}{Q} \right)$$

$$= (1 + b)CQ' + \frac{(1 + b)Q'^2(C_{1eb} + Clc)Q^*Q'}{2DQ} + \frac{(1 + b)^2}{Q^*}$$

$$= (1 + b)CQ' + \frac{(1 + b)Q'^2(C_{1eb} + Clc)Q^*Q'}{2DQ} + \frac{(1 + b)^2}{Q^*}$$

$$= (1 + b)CQ' + \frac{(1 + b)Q'^2(C_{1eb} + Clc)Q^*Q'}{2DQ} + \frac{(1 + b)^2}{Q^*}$$

$$P_3(Q^*) = K_3(Q^*) - K(Q')$$

$$= (1 + b)CQ' + \frac{(1 + b)Q'^2(C_{1eb} + Clc)Q^*Q'}{2DQ} - CQ' - (1 + b)^2 \left[ \frac{Q'^2(C_{1eb} + Clc)Q'^2}{2DQ'} \right] - \frac{C_3D}{Q'^*}$$

For maximization

$$\frac{\partial P_3(Q^*)}{\partial Q'} = 0$$

$$bC + \frac{(1 + b)}{DQ} \sqrt{4Q'(C_{1eb} + Clc)C_3D} - \left[ \frac{(1 + b)^2 Q'(C_{1eb} + Clc)}{2DQ'} \right] = 0$$

$$bC + \frac{(1 + b)}{DQ} \sqrt{4Q'(C_{1eb} + Clc)C_3D} - \left[ \frac{(1 + b)^2 Q'(C_{1eb} + Clc)}{D} \right] = 0$$

$$\therefore Q^* = \frac{bCDQ}{(1 + b)^2 C_{1eb} + Clc} + \frac{(1 + b)\sqrt{4Q'(C_{1eb} + Clc)C_3D}}{(1 + b)^2 C_{1eb} + Clc} = 0$$

And maximum profit will be

$$P_3(Q^*) = \left[ bC \frac{(1 + b)}{DQ} \sqrt{4Q'(C_{1eb} + Clc)C_3D} \right] Q^* - \left[ \frac{(1 + b)^2 C_1 + Clc}{D} \right] Q^* - \frac{C_3D}{Q'^*}$$

$$P_3(Q^*) = \left[ bC \frac{(1 + b)}{DQ} \sqrt{4Q'(C_{1eb} + Clc)C_3D} \right] Q^* - \left[ \frac{(1 + b)^2 C_1 + Clc}{D} \right] Q^* - \frac{C_3D}{Q'^*}$$

Sensitivity analysis:

Unit cost ($C$) = Rs. 10 per unit
Demand ($D$) = Rs. 12000 unit per year
Interest charges ($Ic$) = Rs. 0.15 per year
Replenishment cost ($C_3$) = Rs. 250 per order
Holding cost ($C_1$) = Rs. 0.25 per unit per year
Shortages cost \( C_2 \) = Rs. 0.30 per unit per year  
Maximum shortages \( S \) = Rs. 30 per unit

**Table:1**

Resulting table shows the different values of optimum order quantity and optimum profit under the case of different values of the sales promotional factor \( b \).

<table>
<thead>
<tr>
<th>( b )</th>
<th>( Q' )</th>
<th>( Q^* )</th>
<th>( P_1(Q^*) )</th>
<th>( P_2(Q^*) )</th>
<th>( P_3(Q^*) )</th>
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<td>39301.73</td>
<td>869637.06</td>
</tr>
<tr>
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<td>152.33</td>
<td>889.21</td>
<td>126370.53</td>
<td>47971.53</td>
<td>965883.37</td>
</tr>
</tbody>
</table>

**Interpretation:**
Previous table clearly shows that the values of sales promotional factor \( b \) increases the related optimum order quantity and profit both increases. It propose that the purchaser has to place order quantity for more units.

**References:**